$K.E = E_1 - e^2 E_1 = (1-e^2) E_1$

$= \frac{1}{2} \frac{m_1 m_2}{m_1+m_2} (1-e^2)(u_1-u_2)$ //

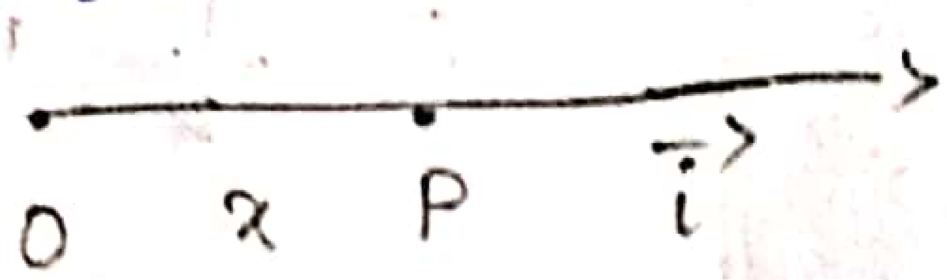
Unit - IV

A Simple Harmonic motion

Equation of motion:

Let O be a fixed point p be the position of the particle at any time t and OP=x let, i be the unit vector in the direction as shown in the figure an the position vector of the particle with reference to O at time t be $\bar{r}$.

$\bar{r} = x\bar{i}$



$\ddot{\bar{r}} = \ddot{x}\bar{i}$

The accleration is proportional to the distance from O. so it's magnitude can be taken as n^2x, where n^2 is the +ve constant. further the acceleration is towards O.

Hence it is $(n^2x)(-i)$ equating the above two quantities we get $\ddot{x}\bar{i} = n^2x(-i)$

This eqn. of motion of the particle the scalar form of this equation

$\ddot{x} = -n^2 x$.

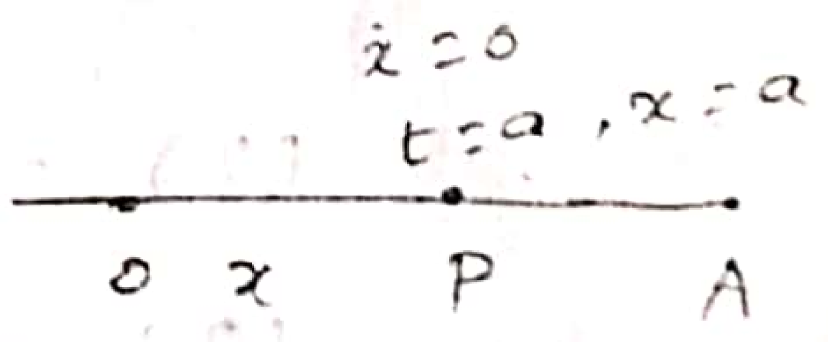
Simple Harmonic motion:

When a particle moves in a straight line so that it's acceleration is always directed towards a fixed point in the line and proportional to the distance from that point, it's motion is called simple harmonic motion.

U-Q 10M BOOK WORK:

The S.H.M is whose eqn is $\ddot{x} = -n^2 x$ to express i) x in t ii) $\dot{x}$ in t iii) $\dot{x}$ in x.

soln: Let O be the fixed point towards which the acceleration is and



let the particle be at rest initially at A, where OA = a.

The eqn of motion may be return as,

$$\ddot{x} + n^2 x = 0$$

$$(D^2+n^2)x = 0$$

It's acoilarly equation

$$m^2+n^2=0$$

$$m^2=-n^2$$

$$m = -n^2$$

$m = \pm ni$ and it's roots are real & imaginary so the general soln of the equation of motion is $x = e^{m_1}(A\cos m_2 t + B\sin m_2 t)$

$$m_1 = 0,\ m_2 = n.$$

$$x = A\cos nt + B\sin nt \longrightarrow (1)$$

From the initial condition when $t=0$, $x=a$

so, $x = A\cos nt + B\sin nt$

$$a = A\cos n(0) + B\sin n(0)$$

$a = A(1) + 0$

$A = a$.

Thus eqn (1) becomes.

$x = A \cos nt + B \sin nt$

$x = a \cos nt + B \sin nt \longrightarrow (2)$

$\dot{x} = -an \sin nt + Bn \cos nt \longrightarrow (3)$

When $t = 0$, $\dot{x} = v = 0$.

$0 = -an \sin n(0) + Bn \cos n(0)$

$0 = Bn$

$B = 0$.

Thus from eqn (2) & (3) become.

(2) $\Rightarrow$ $x = a \cos nt \longrightarrow (4)$

(3) $\Rightarrow$ $\dot{x} = -an \sin nt$ (or)

$v = -an \sin nt \longrightarrow (5)$

squaring Eliminating t from (4) & (5)

$x^2 = a^2 \cos^2 nt$

$v^2 = a^2 n^2 \sin^2 nt$

$\sin^2 nt = \dfrac{v^2}{a^2 n^2}$

$\cos^2 nt = \dfrac{x^2}{a^2}$

$\sin^2 nt + \cos^2 nt = \dfrac{v^2}{a^2 n^2} + \dfrac{x^2}{a^2}$

$1 = \dfrac{v^2}{a^2 n^2} + \dfrac{x^2}{a^2}$

$\dfrac{v^2}{a^2 n^2} = 1 - \dfrac{x^2}{a^2} = \dfrac{a^2 - x^2}{a^2}$

$= \dfrac{a^2 - x^2}{\cancel{a^2}} \cdot \cancel{a^2} n^2$

$v^2 = n^2 (a^2 - x^2) \longrightarrow (6)$

Now eqn (4) (5) & (6) are the required results.

Definition : Maximum speed :

As t increases from 0, x decreases (zero) from A and the speed increases from 0 (zero) the speed is maximum when $x = 0$. This maximum speed is na.

Nature of Motion :

The particle has same speed at point equidistance from zero. so the motion is an oscillatory motion between A and A' with 0, where $OA = OA'$

Acceleration:

One complete motion of the particle from a point on it's path to one extremely of it's path, then to the other extremity and back to the point is called an acceleration.

Vibration :

The motion of the particle from one extremity to the other extremity of its path is called a vibration.

Amplitude :

The maximum distance through which the particle moves on either side of the mean position of the motion is called the amplitude of the motion ($OA = a$ is the amplitude).

In an oscillation on particle travels along a distance equal to 4 times amplitude period:

The time taken by the particle to make one oscillation called the period of the motion. In the above working. let the time taken by the particle to move from A to O be t_0

Then from (A)

$0 = a\cos nt_0$ (or) $nt_0 = \pi/2$ (or) $t_0 = \pi/2n$.

But the period T is four times t_0.

$T = 4 \cdot \pi/2n = 2\pi/n$

It is important to note that the period of a S.H.M is independent of it's amplitude

Frequency:

The number of oscillations per second is called the frequency of the motion that is the frequency is the reciprocal of the period so it is $1/T$ (or) $n/2\pi$

phase and epoch:

The general form of the displacement x of the particle is $x = a\cos(nt+\epsilon)$.

Hence $nt+\epsilon$ is called the phase at time t. The initial phase i.e) The phase when $t=0$ is called epoch, so ϵ is the epoch.

Book work:

To show that in a S.H.M the sum of the K.E and P.E is a constant.

Soln:

K.E at P $= \frac{1}{2} m (\text{velocity})^2 = \frac{1}{2} m [n^2(a^2 - x^2)]$

taking O as the standard point for the calculation of P.E.

$$\text{P.E at P} = \int_x^0 (\text{force})\,dx = \int_x^0 (m\ddot{x})\,dx$$

$$= \int_x^0 m(-n^2 x)\,dx = \frac{1}{2} m n^2 x^2$$

$$\therefore \text{K.E} + \text{P.E} = \frac{1}{2} m n^2 (a^2 - x^2) + \frac{1}{2} m n^2 x^2$$

$$= \frac{1}{2} m n^2 a^2 - \frac{1}{2} m n^2 x^2 + \frac{1}{2} m n^2 x^2$$

$$= \frac{1}{2} m n^2 a^2,$$

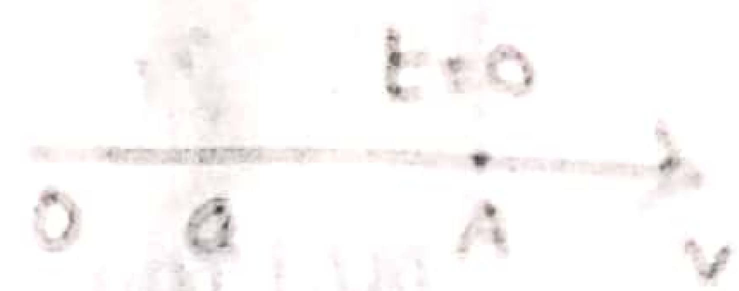
which is a constant

Book work :

If initially the particle is projected from A with a velocity v away from O (OA = a) then to find the S.H.M

soln: Now the initial conditions are when

$t = 0,\ x = a,\ V = \dot{x} = v$ so in



$$x = C\cos nt + D\sin nt$$

$$a = C\cos n(0) + D\sin n(0) \Rightarrow \boxed{C = a}$$

$$x = a\cos nt + D\sin nt$$

$$\dot{x} = -an\sin nt + Dn\cos nt$$

$$V = -an(0) + Dn(1)$$

$$V = Dn$$

$$\boxed{D = V/n},\ \boxed{C = a}$$

$$x = a\cos nt + V/n \sin nt \longrightarrow (1)$$

$$\dot{x} = -an\sin nt + V/n\, n\cos nt$$

$$\frac{\dot{x}}{n} = -a\sin nt + V/n \cos nt \longrightarrow (2)$$

Squaring and adding there two weget (1) & (2) put $\dot{x} = v$

$$x^2 + \frac{v^2}{n^2} = a^2 + \frac{v^2}{n^2}$$

$$x^2 + \frac{v^2}{n^2} = a^2(\cos^2 nt + \sin^2 nt) + \frac{v^2}{n^2}(\cos^2 nt + \sin^2 nt)$$

$$x^2 + \frac{v^2}{n^2} = a^2 + \frac{v^2}{n^2} \longrightarrow (3)$$

$$= a^2 - x^2 + \frac{v^2}{n^2}$$

$$\frac{v^2}{n^2} = \frac{(a^2 - x^2)n^2 + v^2}{n^2} \quad (or)$$

$$v^2 = n^2(a^2 - x^2) + v^2 \,{}_{''}$$

Amplitude:

The amplitude of this motion is the value of x when $x = 0$ so it is (3) =>

$$x = \sqrt{a^2 + v^2/n^2}.$$

Projection of a particle having a uniform circular motion:

In this section we consider a circular motion whose projection on a diametre illustrates a S.H.M.

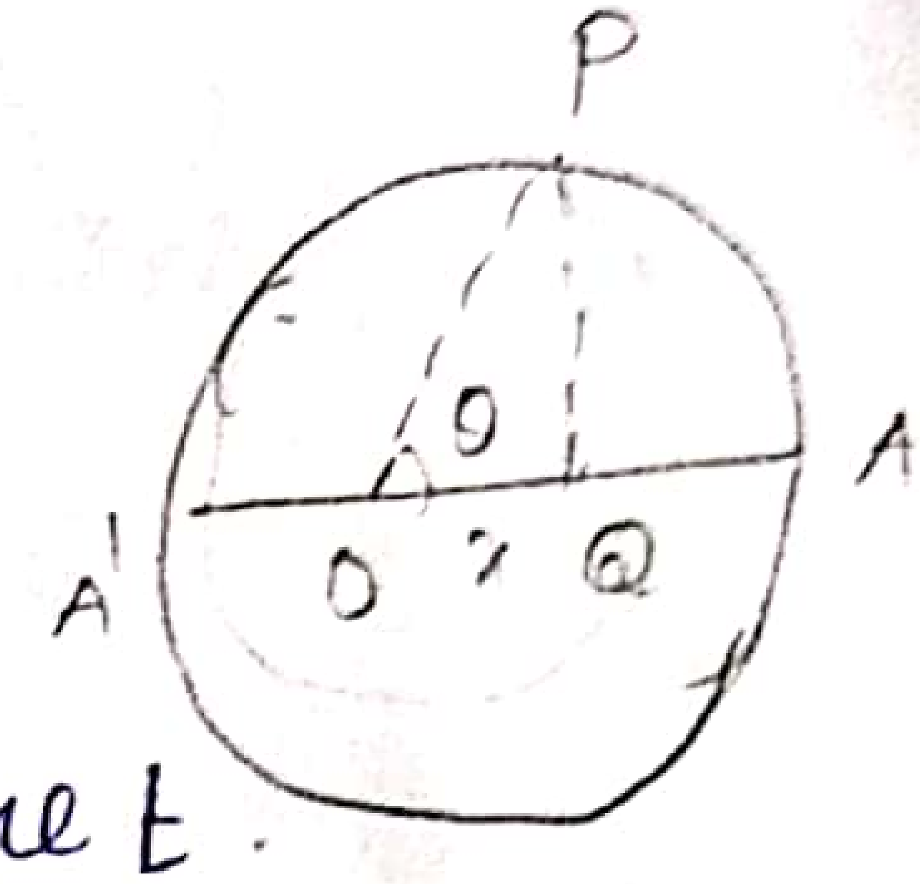
Book work:

A particle moves along a circle with a uniform speed. To show that the motion of its projection on a fixed diameter is simple harmonic.

Soln: Let us have the following assumptions

O: centre of the circle.

a : Radius of the circle.



AA' : A diameter of the circle.

P : position of the particle at time t.

Q : projection of P on AA'

θ : Angle AOP.

ω : Angular velocity $\dot{\theta}$ of P about O which is a constant.

Let the distance of Q, the projection of P, from O be x. Then.

$$x = OQ = a\cos\theta \qquad \cos\theta = \frac{OQ}{OP} = x/a$$

$$\dot{x} = -a\sin\theta\,\dot{\theta} = -a\omega\sin\theta, \qquad OQ = a\cos\theta$$

$$\ddot{x} = -a\omega\cos\theta\,\dot{\theta} = -a\omega^2\cos\theta$$

$$= -\omega^2 x.$$

So the motion of Q along the diameter is simple harmonic whose amplitude and period of oscillation are a, $2\pi/\omega$.

20.08.19

Unit - IV

composition of two simple H.m of same period.

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

Book Work:

1. Two S.T the resultant of whose S.H.m of same period angle the same straight line is also S.H. with same period.

soln: Let the displacement in the two given motion be $x_1 = a_1\cos(nt+\epsilon_1)$, $x_2 = a_2\cos(nt+\epsilon_2)$

Then the resultant displacement x is given by $x = x_1 + x_2$.

$$x = a_1\cos(nt+\epsilon_1) + a_2\cos(nt+\epsilon_2)$$

$$= a_1\cos nt\cos\epsilon_1 - \sin nt\,\epsilon_1 +$$

$$a_2\cos nt\cos\epsilon_2 - \sin nt\sin\epsilon_2$$

$$x = a_1\cos nt\cos\epsilon_1 - a_1\sin nt\,\epsilon_1 + a_2\cos nt\cos\epsilon_2 - a_2\sin nt\sin\epsilon_2$$

$$= (a_1\cos\epsilon_1 + a_2\cos\epsilon_2)\cos nt - (a_1\sin\epsilon_1 + a_2\sin\epsilon_2)\sin nt.$$

$$= (a\cos\epsilon)\cos nt - (a\sin\epsilon)\sin nt.$$

$$= a[\cos nt\cos\epsilon - \sin nt\sin\epsilon]$$

$x = a\cos(nt+\epsilon)$, where $a\cos\epsilon$

$a\cos\epsilon = a_1\cos\epsilon_1 + a_2\cos\epsilon_2 \rightarrow (1)$ and $a\sin\epsilon = -a_1\sin\epsilon_1 + a_2\sin\epsilon_2 \rightarrow (2)$

Squaring adding (1) & (2)

$$a^2\cos^2\epsilon = a_1^2\cos\epsilon_1 + a_2^2\cos\epsilon_2$$

$$a^2\sin^2\epsilon = -a_1^2\sin^2\epsilon_1 + a_2^2\sin^2\epsilon_2$$

$$a^2\cos^2\epsilon + a^2\sin^2\epsilon = a_1^2\cos\epsilon_1 + a_2^2\cos\epsilon_2 + 2a_1a_2\cos\epsilon_1 + \cos\epsilon_2 + a_1^2\sin^2\epsilon_1 + a_2^2\sin^2\epsilon_2 + 2a_1a_2\sin\epsilon_1\epsilon_2$$

$$a^2 = a_1^2 + a_2^2 + 2a_1a_2\cos(\epsilon_1 - \epsilon_2)$$

$$a = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos(\epsilon_1 - \epsilon_2)}$$

and $$\tan\epsilon = \frac{a_1\sin\epsilon_1 + a_2\sin\epsilon_2}{a_1\cos\epsilon_1 + a_2\cos\epsilon_2}$$

$$\epsilon = \tan^{-1}\left(\frac{a_1\sin\epsilon_1 + a_2\sin\epsilon_2}{a_1\cos\epsilon_1 + a_2\cos\epsilon_2}\right)$$

So the resultant motion is also simple harmonic with the same period as the component of motion as the it's amplitude ebochs a and ϵ.

Two S.T the resultant motion of two S.H.M of same period along two $\perp$r lines is along and ellipse.

soln: choose the line of motion as the x, y axis. let us the count the time from the moment when the first particle is at one extreme of it's path.

So that at the time t its displacement is

$x = a\cos nt \longrightarrow (1)$

Let the displacement of the 2nd particle at the time t be $y = b\cos(nt + \epsilon) \longrightarrow (2)$

$y = (b\cos nt\cos\epsilon - \sin nt\sin\epsilon) \longrightarrow (2)$

Elimination of t from these two eqn from gives the equation of the path

Corresponding to the resultant motion at

From (1) => $\cos nt = x/a$

$$\sin^2 nt = 1 - \cos^2 nt$$
$$= 1 - x^2/a^2$$
$$\sin nt = \sqrt{1 - x^2/a^2}$$

From (2) => $y = b\left(x/a \cos\epsilon - \sqrt{1 - x^2/a^2}\sin\epsilon\right)$

$$\left(y - \frac{bx}{a}\cos\epsilon\right)^2 = \left(-b\sqrt{1-x^2/a^2}\sin\epsilon\right)^2$$

$$y^2 + \frac{b^2x^2}{a^2}\cos^2\epsilon - 2y\frac{bx}{a}\cos\epsilon = b^2\left(1 - x^2/a^2\right)\sin^2\epsilon$$

$$\frac{y^2}{b^2} + \frac{b^2x^2}{b^2a^2}\cos^2\epsilon - \frac{2xyb}{ab^2}\cos\epsilon = \left(1 - \frac{x^2}{a^2}\right)\sin^2\epsilon$$

$$\left(\frac{y^2}{b^2} + \frac{x^2}{a^2}\cos^2\epsilon - 2\frac{xy}{ab}\cos\epsilon\right)\left(1 - x^2/a^2\right)^{-1} = \sin^2\epsilon$$

$\left(1 + x^2/a^2\right)$

$$\frac{x^2}{a^2} - 2\frac{xy}{ab}\cos\epsilon + \frac{y^2}{b^2} = \sin^2\epsilon$$

which is the ellipse since the eqn satisfy $h^2 - ab < 0$.

The displacement x of the particle moving along a straight line is given by $x = A\cos nt + B\sin nt$, where A, B, n are constant. S.T the motion is S.H if $A = 3$, $B = 4$, $n = 2$

find this period amplitude, maximum velocity & maximum acceleration.

Soln: Given $x = A\cos nt + B\sin nt$.

$$x = 3\cos 2t + 4\sin 2t \rightarrow (1)$$

$$v = \dot{x} = 3(-2\sin 2t) + 4(2\cos 2t)$$

$$v = 2[(-3\sin 2t) + (4\cos 2t)]$$

$$v/2 = -3\sin 2t + 4\cos 2t \rightarrow (2)$$

eqn $(1)^2 + (2)^2 \Rightarrow x^2 + v^2/4 = 3^2(\cos^2 2t + \sin^2 2t) + 4^2(\sin^2 2t + \cos^2 2t)$

$$x^2 + v^2/4 = 3^2 + 4^2$$

when $v = 0$, x = amplitude

$$x^2 = 3^2 + 4^2$$

$$x = \sqrt{3^2 + 4^2} = \sqrt{25}$$

$$x = 5$$

maximum velocity $= na$
$= 2 \times 5 = 10$.

maximum accleration $= n^2 a$
$= 4 \times 5 = 20$.

(*) A particle is moving with S.H.M and

U.Q 5M why the mean position moving from & to one extreme position its distances at 3 consecutive second are x_1, x_2, x_3

S.T the period is $\dfrac{2\pi}{\cos^{-1}\{(x_1+x_3)/2x_2\}}$

Soln: Let the three consecutive seconds be $t-1, t, t+1$, Then from $x = a\cos nt$

$\therefore x_1 = a\cos n(t-1)$

$x_2 = a \cos nt$

$x_3 = a \cos n(t+1)$

$x_1 + x_3 = a\cos(nt - n) + a\cos(nt+n)$

$= a[2\cos nt \cos n]$

$= 2a\cos nt \cos n$

$= 2x_2 \cos n$

$\cos n = \dfrac{x_1 + x_3}{2x_2}$

$n = \cos^{-1}\dfrac{x_1+x_3}{2x_2}$

The period $= \dfrac{2\pi}{n} = \dfrac{2\pi}{\cos^{-1}\left\{\dfrac{(x_1+x_3)}{2x_2}\right\}}$

The particle is excuteing S.H.M with o as the mean position 'a' as the amplitude when it is at distance $a/2$ from o. is velocity is quadrupled by a blow S.T it's new amplitude is $7a/2$

soln: Let ~~v~~ & 4V be the velocities before and after the below and a_1 be the new amplitude.

Then the using the form

$\dot{x}^2 = n^2(a^2 - x^2)$

$v^2 = n^2(a^2 - (a/2)^2)$

$v^2 = n^2(a^2 - a^2/4)$

$v^2 = n^2(3a^2/4) \longrightarrow (1)$

$(4V)^2 = n^2[a_1^2 - (a/2)^2]$

$16v^2 = n^2(a_1^2 - a^2/4) \longrightarrow (2)$

Eliminating v_2 form (1) & (2)

$$16 \cdot n^2 \left(\frac{3a^2}{4}\right) = n^2 \left(a_1^2 - a^2/4\right)$$

$$\frac{a^2}{4} + 12a^2 = a_1^2$$

$$a_1^2 = \frac{49}{4} a^2 \qquad a_1 = \sqrt{49/4}\, a^2$$

$$a_1 = 7/2\, a.$$

A particle is exciteing a S.H.M of period T with O as the mean position: particle posses through point p with velocity v in the direction of OP. S.T the time which lapses before its ~~to~~ return to OP is

$$\frac{T}{\pi} \tan^{-1} \frac{VT}{2\pi OP}$$

Soln:
Let a particle take a time t_1 to reach end A from P then the time takes to ~~reach~~ return P. $OP = b$, $OA = a$

Now the required time is $2t_1$. let $OP = b$

$OA = a$ then the considering the motion from A to P from $x = a\cos nt_1$, $v^2 = n^2(a^2 - x^2)$

put $x = b$

$$b = a\cos nt_1, \quad v^2 = n^2(a^2 - b^2)$$

$$\cos nt_1 = b/a, \quad \sin^2 nt_1 = 1 - \cos^2 nt$$

$$= 1 - b^2/a^2$$

$$= a^2 - b^2 / a^2$$

$$\sin nt_1 = \frac{\sqrt{a^2 - b^2}}{a}$$

$$\tan nt_1 = \frac{\sin nt_1}{\cos nt_1}$$

$$= \frac{\sqrt{a^2 - b^2}}{a} \times \frac{a}{b}$$

$$\tan nt_1 = \frac{\sqrt{a^2-b^2}}{b} = \frac{n\sqrt{a^2-b^2}}{nb} = \frac{V}{nb}$$

$$T = \frac{2\pi}{n} \Rightarrow n = \frac{2\pi}{T}$$

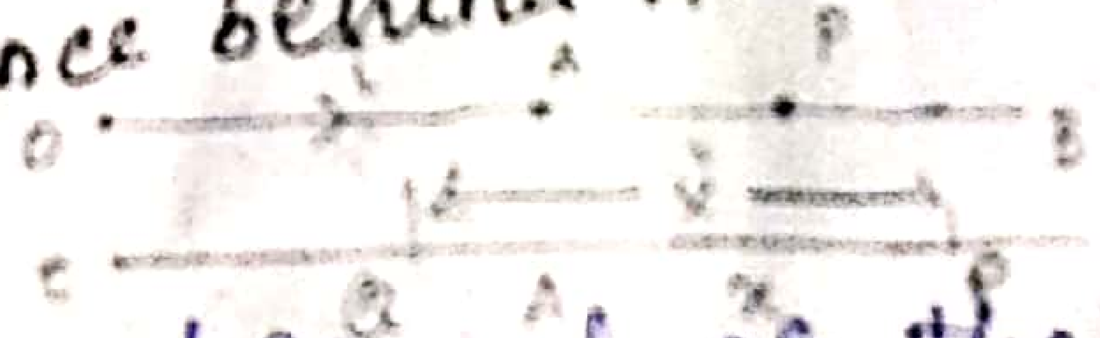
$$\tan nt_1 = \frac{V}{nb}$$

$$\tan \frac{2\pi}{T} t_1 = \frac{V.T}{b.2\pi}$$

$$\frac{2\pi}{T} t_1 = \tan^{-1} \frac{VT}{2\pi b}$$

$$2t_1 = \frac{T}{\pi} \tan^{-1} \frac{VT}{2\pi OP},$$

A particle P of mass m moves in a straight line OP under the force mn^2x (distance from A) directed towards A when A moves along OB with constant acceleration α. S.T the motion of P is S.H of period $2\pi/n$ about to moving centre which always at a distance behind A.



sdn:

let i be the unit vector along the line OA equal to y and AP = x.

Then the position vector of P with respect to fixed point O is $\bar{r} = (x+y)\bar{i}$

The force acting on a particle is

$\bar{F} = mn^2 x \, AP(-i)$

$\bar{F} = -mn^2 x$

So the eqn of the motion

$$\bar{F} = m\ddot{\bar{r}}$$

$$m(\ddot{x} + \ddot{y})\,\bar{i} = -mn^2 x\,\bar{i}$$

$$\ddot{x} + \ddot{y} = -n^2 x$$

$\ddot{y} = \alpha$, so, hence

$$\ddot{x} + \alpha = -n^2 x$$

$$\ddot{x} = -n^2 x - \alpha$$

$$\ddot{x} = -n^2 (x + \alpha/n^2)$$

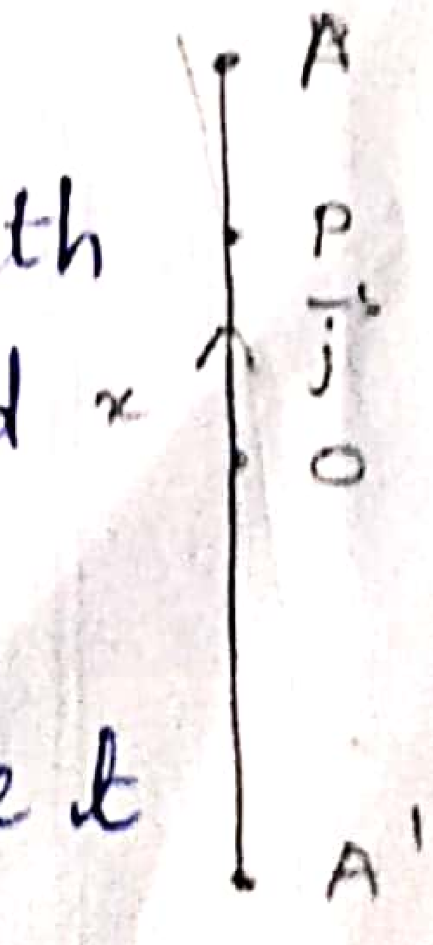
Said $X = (x + \alpha/n^2)$ then x is the distance of P from a paint Q behind A at a distance α/n^2

and $\ddot{x} = -n^2 X$.

which shows that the motion of P is S.H with period $2\pi/n$ and with mean position Q.

22.03.17 The horizontal s with S.T a S.H.M of period $2\pi/n$ and amplitude a book of mass m resting on the self, will not leave it provided $n^2 \le g/a$ in the case where it leaves obtain the velocity then.

Sdn:



O : mean position S.H.M

A A' : Hightest & lowest points on path

j : unit vector vertically upward direction.

P : position of at book at time t

x : OP

$\bar{r}$: position vector.

The forces acting on a book earth ~~forces~~

So the eqn of motion $m\ddot{\vec{r}} = \vec{F}$. So the eqn of motion of the book is $m\ddot{x}\vec{J} = -mg\vec{J} + R\vec{J}$

$$m\ddot{x} = R - mg \longrightarrow (1)$$

The period of S.H.M of the self is $2\pi/n$

$$\ddot{x} = -n^2 x \longrightarrow (2)$$

Thus eliminating $\ddot{x}$ eqn (1) & (2)

$$m(-n^2 x) = R - mg$$
$$R = mg + m(-n^2 x)$$
$$R = m(g - n^2 x)$$

At this moment the value of x is given by $g - n^2 x = 0$

$$g = n^2 x$$
$$x = g/n^2$$

The velocity v of the book then is obtained from $v^2 = n^2(a^2 - x^2)$

$$v^2 = n^2\left(a^2 - g^2/n^4\right)$$
$$= n^2\left(\frac{n^4 a^2 - g^2}{n^4}\right)$$
$$v^2 = \frac{n^4 a^2 - g^2}{n^2} = \frac{1}{n^2}(n^4 a^2 - g^2)$$
$$v = \frac{1}{n}\sqrt{n^4 a^2 - g^2}$$

R does not vanish at all when the self move from a' to 0. because in this motion x is negative and hence R is positive -ve.

S.H.M along a Horizontal line:

Hooke's Law:

$$\text{Tension } \lambda = \frac{\text{Extension (or) compression}}{\text{Natural length.}}$$

Book work:

1. One end of a light spiral spring of length l. it's fixed to a fixed point O. on a smooth horizontal table and a have-y particle of mass m is attached to the other end of the particle is pulled through a distance a and the let go find its motion

Soln:

Let OA

Let P be the position of particle at time t. after the particle let it go.

Let the direction is go positive

The acceleration of the particle in this horizontal direction in $\ddot{x}$ and the tension on it's in the opposition direction with magnitude $T = \lambda . \frac{x}{l}$

So the corresponding motion eqn of

$m\ddot{x} = -\lambda \frac{x}{l}$ $\Rightarrow \ddot{x} = \lambda/lm (-x)$ if denoted the positive

Constant $\boxed{\ddot{x} = \frac{\lambda}{lm}(-x)}$ λ/lm by n^2,

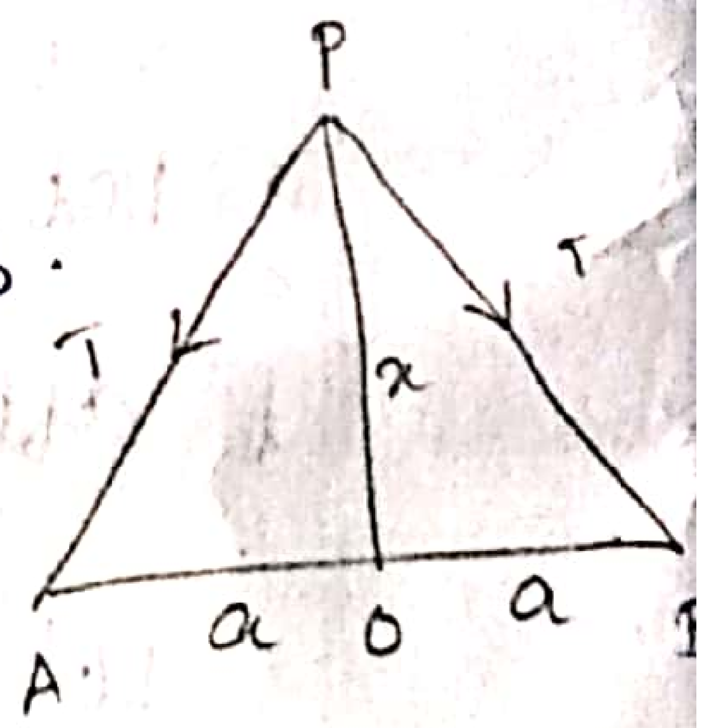
then $\ddot{x} = -n^2 x$ this shows that the particle excute a S.H.M such that

(i) $x = a\cos nt = a\cos\sqrt{\lambda/lm}$ $\left[\because n^2 = \lambda/lm \Rightarrow n = \sqrt{\lambda/lm}\right]$

(ii) $v^2 = n^2(a^2 - x^2) = \lambda/lm\,(a^2 - x^2)$

iii) $T = \frac{2\pi}{n} = 2\pi\sqrt{lm/\lambda}$

The enes of an elbastic string of natural length 'a' are fixed at points A & B at distance 2a apart, on a smooth horizontal table a particle of mass attached to the middle point of the string and displaced along the ⊥r to AB. S.T the period a small acceleration is $\pi\sqrt{2am/\lambda}$



soln: Let o be the mid point of AB. P be the position of the particle at time t. $OP = x$.

The force acting on m along PO and words O is $2T\cos APO$.

$\therefore m\ddot{x} = -2T\cos APO$

$$m\ddot{x} = -2\cdot\lambda\,\frac{\text{Extension}}{\text{Natural length}}\cdot\frac{x}{AP}$$

$$= -2\cdot\lambda\,\frac{2AP - a}{a}\cdot\frac{x}{AP}$$

$$= -\frac{2\cdot\lambda x}{a}\left(\frac{2AP - a}{AP}\right)$$

$$= -\frac{2\lambda x}{a}\left(2 - \frac{a}{AP}\right)$$

$AP^2 = OP^2 + OA^2$

$T^2 = x^2 + a^2$

$T = \sqrt{a^2 + x^2}$

$$m\ddot{x} = -\frac{2\lambda x}{a}\left(2 - \frac{a}{\sqrt{a^2+x^2}}\right)$$

For a small aslation $m\ddot{x}$ i's small, so

$2 \div \frac{a}{\sqrt{a^2 - x^2}} \simeq$

$\therefore m\ddot{x} = -\frac{2\lambda x}{a}$ $\quad \ddot{x} = -n^2 x$; $n^2 = 2\lambda/am$

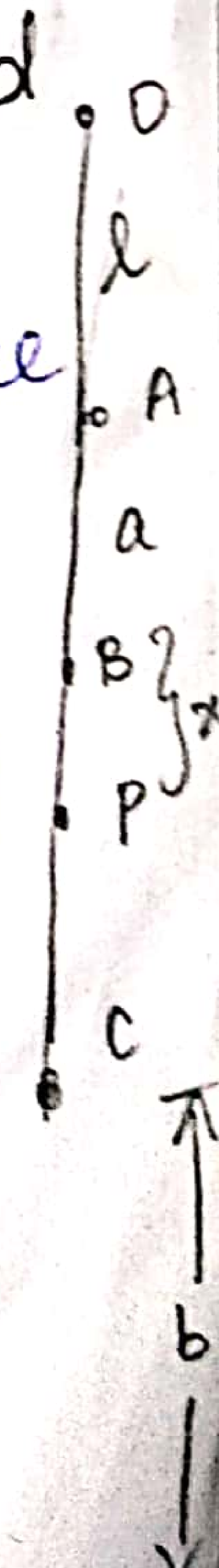
$\ddot{x} = -\frac{2\lambda x}{am} \Rightarrow \ddot{x} = \left(\frac{2\lambda}{am}\right)(-x) \Rightarrow n = \sqrt{2\lambda/am}$

$\therefore$ period $T = 2\pi = 2\pi\sqrt{am/2\lambda}$

$= \pi\sqrt{\frac{2am}{\lambda}}$ //

S.H.M along a vertical line :

A light sprial spring of length l hangs vertically in the position of OA where O is the point of suspension. Let OB be it's equilibrium position when a mass m is attached to its lower end and AB = a. If m is pulled vertically doownward through a distance b and from P to C, let go to find it's motion.



Soln: The forces on the particle in the equilibrium position are

(i) weight mg : vertically downtward

ii) Tension T of the String upwards

with their magnitudes equals.

$mg = T \longrightarrow (1)$

$mg = \lambda \frac{a}{l}$

let us choose the downwards the vertical direction as the +ive direction to measured the distace

let p be the position of the particle at time t. let the distance from the equilibrium position B to x Then the

The acceleration in the +ve direction is $\ddot{x}$ and the forces in the direction are

i) mg The weight

ii) The tension $-\lambda \frac{a+x}{l}$

$$m\ddot{x} = mg - \lambda \frac{a+x}{l}$$

$$= mg - \lambda \cdot \frac{a}{l} - \frac{\lambda x}{l}$$

$$= mg - mg - \frac{\lambda x}{l}$$

$$m\ddot{x} = -\frac{\lambda x}{l} = -(\lambda/l)x = -n^2 x$$

So the motion is S.H with B as the its mean position is amplitude is b because at C (BC = b)

It's velocity is 0. its period and maximum speed

$$T = \frac{2\pi}{n}$$

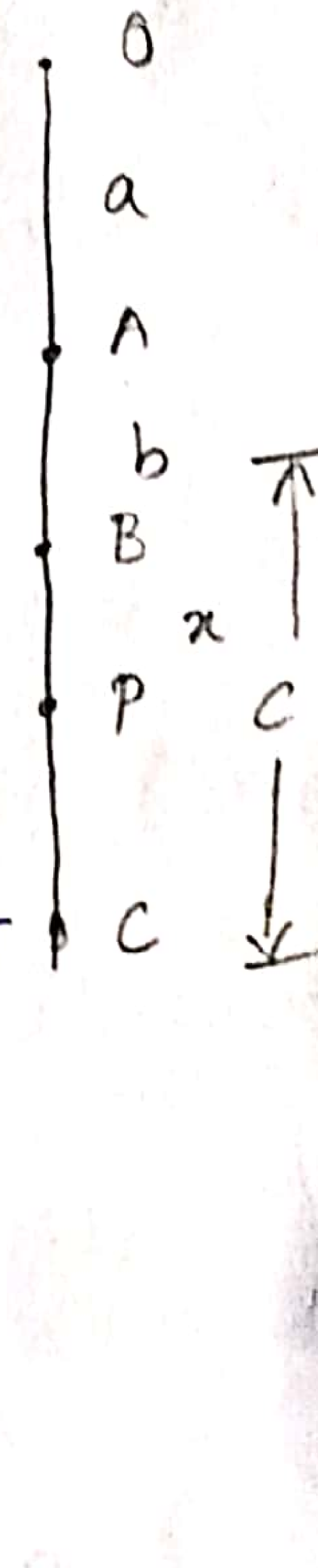
$$T = 2\pi\sqrt{\lambda/x}$$

$$= nb$$

$$= \sqrt{\lambda/l}\, b$$

Two bodies of masses m & m' attached to the lower end of an elastic string whose upper end its fixed & m have at rest m' fall off S.T the distance of m from the upper end of the String at time t is $a + b + c\cos\sqrt{g/b}\,t$

where a is unstretched length of string and b & c are the distances by which it would be stretched when supporting m & m' respectively



Soln:

Let OA denoted the natural length a of the String and B the equilibrium position of m so that $AB = b$.

When m is equilibrium the force an it in downward direction are mg, T there magnitudes are equal. $mg = T$

$$mg = \lambda \frac{b}{a} \longrightarrow (1)$$

ly for $m'g = \lambda \frac{c}{a} \longrightarrow (2)$

∴ From eqn (1) & (2)

$$(m + m')g = \lambda \frac{b+c}{a}$$

$AC = b + c$

i.e) If C is the equilibrium position of m & m', then $AC = b + c$.

Let P be the position of m at time t after m' falls off from C and $BP = x$,

then $m\ddot{x} = mg - \lambda \dfrac{b+x}{a}$

$$m\ddot{x} = mg - \frac{\lambda b}{a} - \frac{\lambda x}{a} = -\frac{\lambda x}{a}$$

$$\ddot{x} = -(\lambda/am)\,x$$

Thus the motion of m is S.H with B as the mean position the amplitude is $BC = c$ because the velocity at c is 0.

$$\therefore \quad x = c\cos nt$$

$$x = c\cos\sqrt{\lambda/am}\;t$$

$$OP = OA + AB + BP$$

a

$$= a + b + x$$

$$OP = a + b + c\cos\sqrt{\lambda/am}\;t.$$

$$mg = \lambda b/a \qquad OP = a+b+c\cos\sqrt{\frac{\frac{mga}{b}}{am}}\;t$$

$$\lambda = \frac{mga}{b} \qquad OP = a+b+c\cos\sqrt{g/b}$$

$$a+b+c\cos\sqrt{\lambda/am}\;t.$$

$$a+b+c\cos\ mga$$

23.03.19 Simple H.M of an a curve

If P is a position of a particle on a curve at time t. if the tangencial acceleration at P ~~where as~~ various as the arcual distance of P measured from a fixed point A on the curve and its directed A. Then the motion of P is said to be S.H.

We know that $\frac{d^2s}{dt^2}$ is the expression for tangential acceleration of a of a particle moving on curve.

Hence the differential eqn for S.H.M on a curve will be of the form $\frac{d^2s}{dt^2} = -\mu s$

s being arcual distance AP.

simple pendulum:

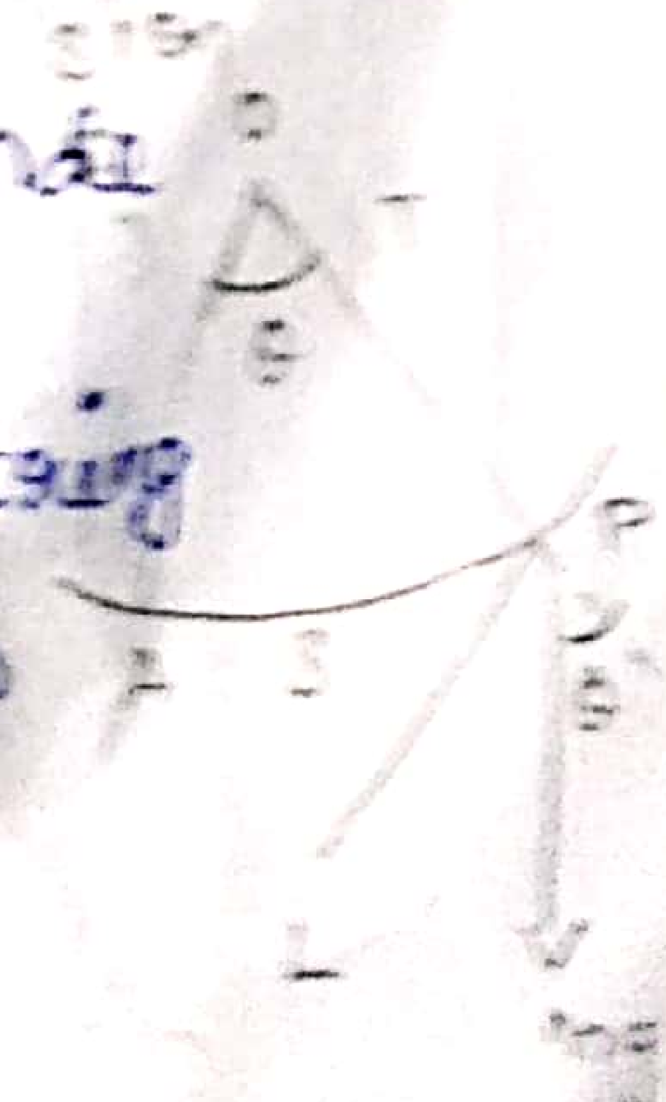
A simple pendulum consist of a small heavy particle are bob suspended from a fixed point by means of a light in inextensible string and oscillating in a vertical plane.

The time of ascilation depends on angle through with the swings on either sides of vertical if a angle of assilation is small.

We can So that the motion of the simple pendulum is simple harmonic.

period of assilation (ascilation) of a simple pendulum

Let op be the point of suspension



OA the vertical position of the string

l the length of the string and m

The length of the string and m the mass of the particle.

Let at time t the particle be at P, where Arc AP = s &

$\angle AOP = \theta$