6.3.19

UNIT - V

16. Angle between the generating line in which a plane cuts a cone :

1. Determine the angle between the lines of intersection of the plane $x-3y+z=0$ and the cone $x^2-5y^2+z^2=0$

Soln:

Let $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$ be a line of intersection of the cone $x^2-5y^2+z^2=0 \longrightarrow (1)$

and $x-3y+z=0 \longrightarrow (2)$

Then, $l^2-5m^2+n^2=0 \longrightarrow (3)$

$l-3m+n=0 \longrightarrow (4)$

$(4) \Rightarrow n=-l+3m$

putting this value of n in (3) we get

$l^2-5m^2+(-l+3m)^2=0$

$l^2-5m^2+9m^2+l^2-6ml=0$

$2l^2+4m^2-6ml=0$

$l^2-3ml+2m^2=0$

$(l-m)(l-2m)=0$

$l=m,\ l=2m$

$\frac{l}{1}=\frac{m}{1},\ \frac{l}{2}=\frac{m}{1} \longrightarrow (5)$

$l=1, m=1$
$n=-l+3m$
$n=-1+3$
$n=2$
$\therefore \frac{l}{1}=\frac{m}{1}=\frac{n}{2}$

$l=2, m=1$
$n=-l+3m$
$n=-2+3$
$n=1$
$\therefore \frac{l}{2}=\frac{m}{1}=\frac{n}{1}$

2
-1 -2

On eliminating l between (3) and (4)

$$(3m-n)^2 - 5m^2 + n^2 = 0$$

$$9m^2 + n^2 - 6mn - 5m^2 + n^2 = 0$$

$$4m^2 + 2n^2 - 6mn = 0$$

$$2m^2 + n^2 - 3mn = 0$$

$$(2m-n)(m-n) = 0$$

$$2m = n \quad , \quad m = n$$

$$\frac{m}{1} = \frac{n}{2} \quad , \quad \frac{m}{1} = \frac{n}{1} \longrightarrow (6)$$

From (5) and (6) we get,

$$\frac{l}{1} = \frac{m}{1} = \frac{n}{2} \text{ and } \frac{l}{2} = \frac{m}{1} = \frac{n}{1}$$

Direction ratios of the line are $(1, 1, 2)$ and $(2, 1, 1)$. Let θ be the angle between the lines.

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\cos\theta = \frac{(1\times2)+(1\times1)+(2\times1)}{\sqrt{1^2+1^2+2^2}\sqrt{2^2+1^2+1^2}} = \frac{2+1+2}{\sqrt{6}\sqrt{6}}$$

$$\cos\theta = 5/6$$

$$\theta = \cos^{-1}(5/6)$$

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8. Show that the equ:- $\sqrt{fx} \pm \sqrt{gy} \pm \sqrt{hz} = 0$ represents a cone which touches the co-ordinate planes.

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Soln:-

$$\sqrt{fx} \pm \sqrt{gh} \pm \sqrt{hz} = 0$$

$$(\sqrt{fx} \pm \sqrt{gh})^2 = (-\sqrt{hz})^2$$

$fx + gy - hz = \pm 2\sqrt{fgxy}$

squaring

$(fx + gy - hz)^2 = 4fgxy$

$f^2x^2 + g^2y^2 + h^2z^2 - 2fgxy - 2hgyz - 2hfzx = 0$

This being a homogenous equ. of 2nd degree in (x, y, z) it represents a cone. When this cone meets the plane x=0 we get $(gy - hz)^2 = 0$. Hence the above cone is cut by the plane x=0 and hence x=0 toches the cone. Similarly y=0, z=0 also touch the cone.

Condition that the cone has 3 Mutually ⊥r generators:

The condition that the plane cut the cone in ⊥r generator. (i.e) θ = 90°

$(a+b+c)(u^2+v^2+w^2) = f(u, v, w)$

The 3rd generator is ⊥r to the two generators. Hence it is normal to the plane containing these ⊥r generators. If the normal to the plane $ux + vy + wz = 0$ lies on the cone we have $f(u, v, w) = 0$

$\therefore a + b + c = 0$.

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Central Quadric

If $P(x_1, y_1, z_1)$ lies on the surface and O be the origin is the midpoint of PQ. Hence all chords equ(i) which passes through O or bisected at O. ∴ eqn (i) is called a central quadrics, O is called ~~called~~ its centre and a chord through O is called a diameter.

The Intersection of a line and quadric:

Let the equ of the straight line be

$$\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n} = r$$ and the quadric

be $ax^2 + by^2 + cz^2 = 1$.

The co-ordinates of any point on the line $x_1 + lr,\ y_1 + mr,\ z_1 + nr$.

If this point lies on the quadric

$$a(x_1 + lr)^2 + b(y_1 + mr)^2 + c(z_1 + nr)^2 = 1$$

Tangent and Tangent plane:

Any line through $p(x_1, y_1, z_1)$ is of the form $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n} = r$ and this line meets the conicoid $ax^2 + by^2 + cz^2 = 1$

At the point $[(x_1 + lr), (y_1 + mr), (z_1 + nr)]$

$$a(x_1 + lr)^2 + b(y_1 + mr)^2 + c(z_1 + nr)^2 = 1$$

$$(al^2 + bm^2 + cn^2)r^2 + 2r(alx_1 + bmy_1 + cnz_1) + ax_1^2 + by_1^2 + cz_1^2 - 1 = 0$$

If (x_1, y_1, z_1) lies on the conicoid

$$ax_1^2 + by_1^2 + cz_1^2 = 1$$

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$+cz^2=1$
$\lambda_1+n\lambda$
$(z_1+n\lambda)]$
$+ax_1^2$

1\. Find the angle between the lines of section of the plane $3x+y+5z=0$ and the cone $6yz-2zx+5xy=0$

Soln:

Let the eqn of the line

$$\frac{x}{l}=\frac{y}{m}=\frac{z}{n} \longrightarrow (1)$$

$$3x+y+5z=0$$

$$3l+m+5n=0 \longrightarrow (2)$$

$$6yz-2zx+5xy=0$$

$$6mn-2nl+5lm=0 \longrightarrow (3)$$

from (2) $m=-3l-5n$

$m=-(3l+5n)$

$l+n=0$
$l=-n$
$\frac{l}{-1}=\frac{n}{1}$
$m=-3(-1)-5(1)$
$m=3-5$
$m=-2$
$\therefore \frac{l}{-1}=\frac{m}{-2}=\frac{n}{1}$
$\frac{l}{1}=\frac{m}{2}=\frac{n}{-1}$

sub in (3)

$$-6n(3l+5n)-2nl-5l(3l+5n)=0$$

$$-18nl-30n^2-2nl-15l^2-25nl=0$$

$$-15l^2-45nl-30n^2=0$$

$$15l^2+45nl+30n^2=0$$

$\div 15$ $$l^2+3nl+2n^2=0$$

$$(l+n)(l+2n)=0$$

$l+n=0$ $\quad$ $l+2n=0$

$l=-n$ $\quad$ $\frac{l}{-1}=\frac{n}{1}$ $\quad$ $l=-2n$ $\quad$ $\frac{l}{-2}=\frac{n}{1}$

solving $l+n=0$ and $3l+m+5n=0$

l	m	n
0	2	1	0
1	5	3	1

we get $\frac{l}{1}=\frac{m}{2}=\frac{n}{-1}$

$\Rightarrow \frac{l}{0-3}=\frac{m}{6-5}=\frac{n}{1-0}$

solving $l+2n$ and $(3l+m+5n)=0$

$-6+m+5$
$m=$

$$\frac{l}{-2} = \frac{m}{1} = \frac{n}{1}$$

$$\cos\theta = \frac{(1)(-2) + (2)(1) + (-1)(1)}{\sqrt{1^2+2^2+(-1)^2}\,\sqrt{(-2)^2+1^2+1^2}}$$

$$= \frac{-2+2-1}{\sqrt{6}\sqrt{6}} = -1/6$$

$$\theta = \cos^{-1}(-1/6)$$

Find the locus of the point of the intersection of 3 mutually ⊥r tangent plane to the central conicoid $ax^2+by^2+cz^2=1$

$ax^2+by^2+cz^2=1$

Soln:

The equ- of the 3 tangent planes are of the form

$$l_r x + m_r y + n_r z = \left(\frac{l_r^2}{a} + \frac{m_r^2}{b} + \frac{n_r^2}{c}\right)^{1/2}$$

$r = 1, 2, 3$

Let (x_1, y_1, z_1) be the co-ordinate of the point of intersection of these planes then

$r=1$

$$l_1 x + m_1 y + n_1 z = \left(l/a + \frac{m}{b} + n/c\right)^{1/2} \longrightarrow (1)$$

$$l_2 x + m_2 y + n_2 z = \left(\frac{l(2)^2}{a} + \frac{m(2)^2}{b} + \frac{n(2)^2}{c}\right)^{1/2}$$

$$l_2 x + m_2 y + n_2 z = \left[\frac{4l}{a} + \frac{4m}{b} + \frac{4n}{c}\right]^{1/2} \longrightarrow (2)$$

$l_3 x + m_3 y + n_3 z = \left(\frac{a l}{a} + \frac{a m}{b} + \frac{a n}{c} \right)^{1/2} \longrightarrow (3)$

The planes are mutually at right angles.

The direction cosines of the line

(l_1, m_1, n_1) & (l_2, m_2, n_2) & (l_3, m_3, n_3)

These lines are mutually at right angles.

$l_1^2 + l_2^2 + l_3^2 = 1, \quad m_1^2 + m_2^2 + m_3^2 = 1, \quad n_1^2 + n_2^2 + n_3^2 = 1$

Since OX, OY, OZ are mutually at right angles.

$l_1 m_1 + l_2 m_2 + l_3 m_3 = 0$

$m_1 n_1 + m_2 n_2 + m_3 n_3 = 0$

$n_1 l_1 + n_2 l_2 + n_3 l_3 = 0$

These squaring equ (1), (2), (3) and adding

$x_1^2 + y_1^2 + z_1^2 = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$

Hence the locus of the point (x_1, y_1, z_1)

is $x^2 + y^2 + z^2 = 1/a + 1/b + 1/c$

U.Q

3. Prove that the cones are $ax^2 + by^2 + cz^2 = 0$ and $\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c}$ are reciprocal.

5m

Soln:

The reciprocal cone of $ax^2 + by^2 + cz^2 = 0$ is

$Ax^2 + By^2 + Cz^2 + 2Fyz + 2Gzx + 2Hxy = 0 \longrightarrow (1)$

Where, $\Delta = \begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix}$

$\Delta = abc$

$A = \frac{\partial \Delta}{\partial a} = bc$

$$B = \frac{\partial A}{\partial b} = ac$$

$$C = \frac{\partial A}{\partial c} = ab$$

$$F = \frac{1}{2} \frac{\partial A}{\partial f} = 0$$

$$G = \frac{1}{2} \frac{\partial A}{\partial g} = 0$$

$$H = \frac{1}{2} \frac{\partial A}{\partial h} = 0$$

By putting those value (1) &

$$bcx^2 + cay^2 + abz^2 = 0$$

$\div abc$

$$\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c} = 0$$

S.T the cone whose vertex is at the origin and which passes through the circle of intersection of the sphere $x^2+y^2+z^2=3r^2$ (or) and any plane at a distance r from the origin has 3 mutually for generators.

Soln:

Any plane at a distance r from the origin is $lx+my+nz=r$, where l, m, n are distance cosines of normal to the plane.

The equ of the cone where vertex is origin $x^2+y^2+z^2 = 3r^2 \left(\frac{lx+my+nz}{r}\right)^2$

$$x^2+y^2+z^2 = 3(l^2x^2+m^2y^2+n^2z^2+2lmxy+2mnzy+2nzlx)$$

$$x^2+y^2+z^2-3l^2x^2-3m^2y^2-3n^2z^2-6lmxy-6mnzy-6nzlx=0$$

$$(1-3l^2)x^2+y^2(1-3m^2)+z^2(1-3n^2)-6lxmy-6mnzy-6nzlx=0$$

co-eff. of x^2 + co-eff of y^2 + co-eff. of z^2

$$= (1-3l^2)+(1-3m^2)+(1-3n^2)$$

$$= 3-3(l^2+m^2+n^2)$$

$$= 0$$

Hence plane ① cuts the cone in 3 mutually ⊥r generator.

Cylinder:-

A cylinder is a surface generator by a straight line which is always parallel to a fixed line and is surface to one more condition for instance it make intersect a given curve or touch a given surface.

Equation of a cylinder:-

To find the equation of the cylinder whose generator's intersect the conic $ax^2+2hxy+by^2+2gx+2fy+c=0$, $z=0$ and are ∥ to the line $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$

1. Find the equ of cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and whose guiding curve is the ellipse $x^2+2y^2=1$, $z=0$

Soln:

Let α, β, γ be any point on the cylinder then equ of a generator through (α, β, γ) are

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}, \quad \frac{x-\alpha}{1} = \frac{y-\beta}{-2} = \frac{z-\gamma}{3}$$

This meets the plane $z=0$ at the point given by

$$\frac{x-\alpha}{1} = \frac{y-\beta}{-2} = \frac{-\gamma}{3}$$

$$\frac{x-\alpha}{1} = \frac{-\gamma}{3} \qquad \frac{y-\beta}{-2} = \frac{-\gamma}{3}$$

$$x = \alpha - \gamma/3 \qquad y = \beta + \frac{2\gamma}{3}$$

$$\left(\alpha - \gamma/3, \ \beta + \frac{2\gamma}{3}, \ 0\right)$$

$$x^2+2y^2=1$$

$$\left(\alpha - \gamma/3\right)^2 + 2\left(\beta + \frac{2\gamma}{3}\right)^2 = 1$$

Hence the locus of α, β, γ is

$$\left(x - z/3\right)^2 + 2\left(y + 2z/3\right)^2 = 1$$

$$\left(\frac{3x-z}{9}\right)^2 + 2\left(\frac{3y+2z}{3}\right)^2 = 1$$

$$\frac{9x^2-6xz+z^2}{9} + \frac{2}{9}\left(9y^2+12yz+4z^2\right) = 1$$

$$9x^2-6xz+z^2+18y^2+24yz+8z^2=9$$

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$$9x^2+18y^2+9z^2+24yz-6xz-9=0$$

$\div$ by 3

$$3x^2+6y^2+3z^2+8yz-2xz-3=0$$

2. Find the equ of quadric cylinder with generator parallel to x-axis and passing through the curve $ax^2+by^2+cz^2=1$, $lx+my+nz=p$. → y axis

Soln:

The equ of required cylinder is obtained by eliminating x between the equations $ax^2+by^2+cz^2=1$ and $lx+my+nz=p$

$$lx = p-my-nz$$

$$x = \frac{p-my-nz}{l}$$

$$ax^2+by^2+cz^2=1$$

$$a\left(\frac{p-my-nz}{l}\right)^2+by^2+cz^2=1$$

$$\frac{a(p^2+m^2y^2+n^2z^2-2pmy+2myn z-2nzp)}{l^2}+by^2+cz^2=1$$

$$ap^2+am^2y^2+an^2z^2-2apmy+2amynz-2anzp+bl^2y^2+cl^2z^2=l^2$$

$$(am^2+bl^2)y^2+(cl^2+an^2)z^2+2amnyz-2apmy-2anzp+ap^2-l^2=0$$

The eqn to the cylinder whose generator touch the sphere and are parallel to the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$.

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1. Find the eqn of enveloping cylinder of the sphere $x^2+y^2+z^2=25$, whose generator are parallel to the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$

Soln:

Let (α, β, γ) be any point on the cylinder.

Eqn of generator

$$\frac{x-\alpha}{1} = \frac{y-\beta}{2} = \frac{z-\gamma}{3} = r$$

Any point on generator is $[r+\alpha, 2r+\beta, 3r+\gamma]$

$$x^2+y^2+z^2=25$$

$$(r+\alpha)^2+(2r+\beta)^2+(3r+\gamma)^2=25$$

$$r^2+\alpha^2+2r\alpha+4r^2+\beta^2+4r\beta+9r^2+\gamma^2+6r\gamma=25$$

$$14r^2+\overset{2\times 2}{2r}[\alpha+2\beta+3\gamma]-\overset{1\times 4}{(\alpha^2+\beta^2+\gamma^2-25)}=0$$

$$4(\alpha+2\beta+3\gamma)^2 = 4\cdot 14(\alpha^2+\beta^2+\gamma^2-25)$$

$$4(\alpha^2+4\beta^2+9\gamma^2+4\alpha\beta+12\beta\gamma+6\alpha\gamma) = 56\alpha^2+56\beta^2+56\gamma^2-1400$$

$$4\alpha^2+16\beta^2+36\gamma^2+16\alpha\beta+48\beta\gamma+24\alpha\gamma-56\alpha^2-56\beta^2-56\gamma^2+1400=0$$

$$-52\alpha^2-40\beta^2-20\gamma^2+16\alpha\beta+48\beta\gamma+24\gamma+1400=0$$

÷ by 4

$13\alpha^2 - 10\beta^2 + 5\gamma^2 - 4\alpha\beta - 6\gamma\alpha - 12\beta\gamma - 350 = 0$

Locus of (α, β, γ) is

$$13x^2 - 10y^2 + 5z^2 - 4xy - 6zx - 12yz - 350 = 0$$

Right circular cylinder

A right circular cylinder is a surface generated by a line which intersect a fixed circle called the guiding circle and is $\perp$r to its plane

Equation of right circular cylinder

To find the equ of right circular cylinder. whose axis is the line $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ and whose radius is r

Let x, y, z be a point on the cylinder Equating the $\perp$r distance of the point from the axis to the radius r we get

$$(x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2 - \left[\frac{l(x-\alpha) + m(y-\beta) + n(z-\gamma)}{l^2+m^2+n^2}\right]^2 = r^2$$

1. Find the equ of right circular cylinder whose guiding curve is $x^2+y^2+z^2=9$, $x - y + z = 3$

U.Q

Soln:

Radius of sphere = 3

length $\perp$r from the centre $O(0,0,0)$ to the given plane.

$$= \frac{-3}{\sqrt{1+1+1}} = \frac{-3}{\sqrt{3}} = -\sqrt{3}$$

Radius of circle $= \sqrt{3^2 - 3}$

$= \sqrt{6}$

The axis of cylinder passes through $(0,0,0)$ and is $\perp$r to the plane

$x - y + z = 3$

Its equ are $\frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$

The equ of circular cylinder is

formula

$$\left(\frac{1}{\sqrt{3}}\right)^2 \left\{ \begin{vmatrix} y & z \\ -1 & 1 \end{vmatrix}^2 + \begin{vmatrix} z & x \\ 1 & 1 \end{vmatrix}^2 + \begin{vmatrix} x & y \\ 1 & -1 \end{vmatrix}^2 \right\} = (\sqrt{6})^2$$

$$\frac{1}{3}\left\{(y+z)^2 + (z-x)^2 + (-x-y)^2\right\} = 6$$

$$(y+z)^2 + (z-x)^2 + (-x-y)^2 = 18$$

$$y^2 + z^2 + 2yz + z^2 + x^2 - 2xz + x^2 + y^2 + 2xy = 18$$

$$2x^2 + 2y^2 + 2z^2 - 2xz + 2yz + 2xy = 18$$

$$x^2 + y^2 + z^2 - xz + yz + xy - 9 = 0$$

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1. If the $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ represents one of a set of three mutually $\perp$r generators of the cone $5yz - 8zx - 3xy = 0$ find the equation of the other two.

Soln:

the given line is $\frac{x}{1} = \frac{y}{2} = \frac{z}{3} \longrightarrow (1)$

and the cone is $5yz - 8zx - 3xy = 0 \longrightarrow (2)$

① ⇒ $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$

$l + 2m + 3n = 0 \longrightarrow ③$

$5mn - 8nl - 3lm = 0 \longrightarrow ④$

Eliminating n between ③ & ④ we get,

$l + 2m + 3n = 0$

$3n = -(l + 2m)$

$n = -\frac{(l+2m)}{3}$ —— ⑤

$5mn - 8nl - 3lm = 0$

$(5m - 8l)n - 3lm = 0$

$(5m - 8l)\left(\frac{-l - 2m}{3}\right) - 3lm = 0$

$(5m - 8l)(-l - 2m) - 9lm = 0$

$-5ml - 10m^2 + 8l^2 + 16lm - 9lm = 0$

$8l^2 - 10m^2 + 2ml = 0$

÷ by ② $4l^2 - 5m^2 + ml = 0$

$(l - m)(4l + 5m) = 0$

$l - m = 0 \Rightarrow l = m \rightarrow ⑥ ⑦$

sub ⑥ eqn/-

$n = \frac{-m - 2m}{3} \Rightarrow \frac{n}{m} = \frac{-3}{3}$

$\frac{n}{m} = -1 \Rightarrow \frac{n}{-1} = \frac{m}{1}$

m = 1 in sub ⑤⑥ eqn/-

$l = m$

$l = 1$

$\therefore \frac{l}{1} = \frac{m}{1} = \frac{n}{-1}$

$4l + 5m = 0$

$5m = -4l$

$\frac{m}{-4} = \frac{l}{5}$

$n = \frac{-l - 2m}{3}$

$= \frac{-5 + 8}{3} = 3/3$

$n = 1$

$\frac{l}{5} = \frac{m}{-4} = \frac{n}{1}$

∴ The equation of the actual lines,

$\frac{x}{1} = \frac{y}{1} = \frac{z}{-1}$ and $\frac{x}{5} = \frac{y}{-4} = \frac{z}{1}$

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Q. Find the equation to the lines in which the plane $2x+y-z=0$ cuts the cone $4x^2-y^2+3z^2=0$

Soln:

Let $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$ be the equation of any of the two lines in which the given plane meets the given cone

We have,

$$2l+m-n=0 \; ; \; 4l^2-m^2+3n^2=0$$

$$-n=-2l-m$$

$$-n=0-(2l+m)$$

$$n=2l+m$$

Eliminating 'n' we get have

$$4l^2-m^2+3(2l+m)^2=0$$

$$4l^2-m^2+3(4l^2+4lm+m^2)=0$$

$$4l^2-m^2+12l^2+12lm+3m^2=0$$

$$16l^2+2m^2+12lm=0$$

$\div$ by 2

$$8l^2+6lm+m^2=0 \quad \div \text{ by } m^2 \quad 8\left(\frac{l}{m}\right)^2+6\left(\frac{l}{m}\right)+1=0$$

$$a=8, \; b=6, \; c=1$$

$$\frac{l}{m}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-6\pm\sqrt{36-32}}{16}$$

$$=\frac{-6\pm\sqrt{4}}{16}=\frac{-6\pm2}{16}$$

$$\frac{l}{m}=\frac{-6+2}{16}=\frac{-4}{16}=-\frac{1}{4} \quad (\text{or})$$

$\frac{l}{m} = \frac{-6-2}{16} = \frac{-8}{16} = -\frac{1}{2}$

$\frac{l}{m} = -\frac{1}{4}$ (or) $-\frac{1}{2}$

$2l + m - n = 0$

$\div m$

$\frac{2l}{m} + 1 - \frac{n}{m} = 0$

Put, $\frac{l}{m} = -\frac{1}{4}$

$2\left(-\frac{1}{4}\right) + 1 - \frac{n}{m} = 0$

$\frac{-2}{4} + 1 - \frac{n}{m} = 0 \Rightarrow -\frac{1}{2} + 1 - \frac{n}{m} = 0$

$\frac{1}{2} - \frac{n}{m} = 0 \Rightarrow -\frac{n}{m} = -\frac{1}{2}$

$\frac{n}{m} = \frac{1}{2}$

$\boxed{\frac{l}{m} = -\frac{1}{4}}$

$\frac{l}{-1} = \frac{m}{4}$

$\therefore 2(-1) + 4 - n = 0$

$-2 + 4 - n = 0$

$-n = +2$

$n = 2$

$\therefore \frac{l}{-1} = \frac{m}{4} = \frac{n}{2}$

$\boxed{\frac{l}{m} = -\frac{1}{2}}$

$\frac{l}{-1} = \frac{m}{2} \Rightarrow 2(-1) + 2 - n = 0$

$-2 + 2 - n = 0$

$n = 0$

$\therefore \frac{l}{-1} = \frac{m}{2} = \frac{n}{0}$

$\frac{l}{m} = -\frac{1}{2} \Rightarrow \frac{n}{m} = 0$

$\frac{l}{m} = -\frac{1}{4}, \frac{n}{m} = \frac{1}{2}$

$\Rightarrow \frac{l}{-1} = \frac{m}{4} = \frac{n}{2}$

$2l + m - n = 0$

$\div$ by m $\quad 2l/m + 1 - n/m = 0$

Put $(l/m) = -\frac{1}{2}$

$2\left(-\frac{1}{2}\right) + 1 - n/m = 0$

$-1 + 1 - n/m = 0$

$n/m = 0$

$\frac{l}{m} = -\frac{1}{2}, \frac{n}{m} = 0$

$\Rightarrow \frac{l}{-1} = \frac{m}{2}; n = 0$

The two required lines are

$\frac{x}{-1} = \frac{y}{4} = \frac{z}{2}; \quad \frac{x}{-1} = \frac{y}{2}; z = 0$

3. Find the equation of the tangent plane $x^2+y^2+4z^2=1$ which intersect in the line whose equations are $12x-3y-5=0$, $z=1$

Soln:-

Any plane which passes through the line is given by, $P+\lambda P'=0$

$$12x-3y-5+\lambda(z-1)=0$$

$$12x-3y+\lambda z-(\lambda+5)=0 \longrightarrow ①$$

The equations of the tangent plane at (x_1, y_1, z_1) is

$$x x_1+y y_1+4z z_1=1 \longrightarrow ②$$

Equations ① and ② represents the same plane.

$$\frac{x_1}{12}=\frac{y_1}{-3}=\frac{4z_1}{\lambda}=\frac{1}{\lambda+5}$$

$$x_1=\frac{12}{\lambda+5}\ ;\ y_1=\frac{-3}{\lambda+5}\ ;\ z_1=\frac{\lambda}{4(\lambda+5)}$$

Since (x, y, z) lies on the coincoid

$$x_1^2+y_1^2+4z_1^2=1$$

$$\left(\frac{12}{\lambda+5}\right)^2+\left(\frac{-3}{\lambda+5}\right)^2+4\left(\frac{\lambda}{4(\lambda+5)}\right)^2=1$$

$$\frac{(12)^2}{(\lambda+5)^2}+\frac{(-3)^2}{(\lambda+5)^2}+\frac{4\lambda^2}{16(\lambda+5)^2}=1$$

$$\frac{144(4)+9(4)+\lambda^2}{4(\lambda+5)^2}=1$$

$$\Rightarrow 576+36+\lambda^2=4(\lambda+5)^2$$

$$576+36+\lambda^2=4(\lambda^2+10\lambda+25)$$

$$\Rightarrow 576+36+\lambda^2-4\lambda^2-40\lambda-100=0$$

$\Rightarrow 3\lambda^2 + 40\lambda - 512 = 0$

$\lambda = 8$ (or) $\lambda = \frac{-64}{3}$

$-512/3$

$-8 \quad 64/3$

$12x - 3y - 5 + \lambda(z-1) = 0$

$12x - 3y - 5 + 8(z-1) = 0$ and

$12x - 3y - 5 - \frac{64}{3}(z-1) = 0$

$\Rightarrow 36x - 9y - 64z + 49 = 0 \rightarrow$ ⊛⊛

$12x - 3y - 5 + 8z - 8 = 0$

$12x - 3y + 8z - 13 = 0 \rightarrow$ ⊛⊛⊛

2.

UNIT - 5

08.3.19

1 The equation of the cylinder whose generators are parallel to the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ and whose guiding curve is $f(x, y, z) = 0$, $ax + by + cz + d = 0$

Soln:

Let (x_1, y_1, z_1) be any point on the cylinder. Then the equation of the generator passing through (x_1, y_1, z_1) is

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n} = k \longrightarrow (1)$$

Any point on this generator is $[x_1 + kl, y_1 + km, z_1 + kn]$

If this point lies on the guiding curve we get,

$f(x_1 + kl, y_1 + km, z_1 + kn) \longrightarrow (2)$

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and $a(x_1+kl)+b(y_1+km)+c[z_1+kn]$

Eliminate k from ② and ③ and suppose we get the equation $\phi(x_1, y_1, z_1)=0$. Hence the locus of (x_1, y_1, z_1) is $\phi(x, y, z)=0$ this is the equation of the required cylinder.

8. Find the equation of the cylinder whose generators are parallel to the z axis and the guiding curve is $ax^2+by^2=cz$, $lx+my+nz=p$.

Soln:

The equation of the z axis is

$$\frac{x}{0}=\frac{y}{0}=\frac{z}{1}$$

Let (x_1, y_1, z_1) be any point on the cylinder.

The equation of the generator passing through (x_1, y_1, z_1) is $\frac{x-x_1}{0}=\frac{y-y_1}{0}=\frac{z-z_1}{1}=k$

$\Rightarrow z=k+z_1$

Hence any point on this line is of the form (x_1, y_1, z_1)

In this point lies on the guiding curve

$$ax_1^2+by_1^2-c[z_1+k]=0 \longrightarrow ①$$

$$lx_1+my_1+n(z_1+k)=p \longrightarrow ②$$

Eliminate k from ① and ②

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from ②,

$$k = \frac{p - l x_1 - m y_1 - n z_1}{n}$$

Substituting the value of k in (1) we get,

$$a x_1^2 + b y_1^2 - c z_1 = \frac{c}{n}\left[p - l x_1 - m y_1 - n z_1\right]$$

(i.e) $n\left[a x_1^2 + b y_1^2\right] + c\left[l x_1 + m y_1 - p\right] = 0$

Hence the equation of the required cylinder is $n\left[ax^2 + by^2\right] + c(lx + my - p) = 0$

3. Find the equation of a right circular cylinder of radius 3 with axis

$$\frac{x+2}{3} = \frac{y-4}{6} = \frac{z-1}{2}$$ direction ratios: 3, 6, 2

Soln:

The axis passes through the point $Q(-2, 4, 1)$

Its direction cosines are $\frac{3}{7}, \frac{6}{7}, \frac{2}{7}$

Let $P(x_1, y_1, z_1)$ be any point on the cylinder and let the axis of the cylinder meet the plane ⊥r to the axis through the point $P(x_1, y_1, z_1)$ be M.

Then $MP = 3$

$$PQ^2 = (x_1+2)^2 + (y_1-4)^2 + (z_1-1)^2$$

QM = projection of QP on the axis

$$(x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2 - \frac{\left[l(x-\alpha) + m(y-\beta) + n(z-\gamma)\right]^2}{l^2+m^2+n^2} = r^2$$

$$= \frac{(x_1+2)3+(y_1-4)6+(z_1-1)\cdot 2}{7}$$

$$\boxed{PQ^2 = QN^2 + NP^2}$$

$$(x_1+2)^2+(y_1-4)^2+(z_1-1)^2 = \frac{[3(x_1+2)+(y_1-4)6+2(z_1-1)]^2}{49}$$

Simplifying we get,

$$40x^2+13y^2+45z^2+12yz+6zx+36xy+316x-152y-60z+192=0$$

4) The condition for the plane $lx+my+nz=p$ to touch the conicoid $ax^2+by^2+cz^2=1$

Soln :-

Let the plane touch the conicoid at (x_1, y_1, z_1).

The eqn:- of the tangent plane at (x_1, y_1, z_1) is

$$axx_1+byy_1+czz_1=1 \longrightarrow (1)$$

This plane is also represented by the eqn:- $lx+my+nz=p \longrightarrow (2)$

$$\therefore \frac{ax_1}{l} = \frac{by_1}{m} = \frac{cz_1}{n} = \frac{1}{p}$$

$$\text{(i.e)}\quad x_1 = \frac{l}{ap},\quad y_1 = \frac{m}{bp},\quad z_1 = \frac{n}{cp}$$

Since (x_1, y_1, z_1) lies on the conicoid

$$ax_1^2+by_1^2+cz_1^2=1$$

$$a\left(\frac{l}{ap}\right)^2 + b\left(\frac{m}{bp}\right)^2 + c\left(\frac{n}{cp}\right)^2 = 1$$

$$p^2 = \frac{l^2}{a} + \frac{m^2}{b} + \frac{n^2}{c}$$

Cor. 1

The equation of any tangent plane to the conicoid $ax^2 + by^2 + cz^2 = 1$ is of the form,

$$lx + my + nz = \pm\left(\frac{l^2}{a} + \frac{m^2}{b} + \frac{n^2}{c}\right)^{1/2}$$

Cor. 2

There are two tangent planes to a conicoid parallel to the plane. $lx + my + nz = 0$ and their equations are

$$lx + my + nz = \pm\left(\frac{l^2}{a} + \frac{m^2}{b} + \frac{n^2}{c}\right)^{1/2}$$

5. Find the locus of the point of intersection of three mutually perpendicular tangent planes to the central conicoid $ax^2 + by^2 + cz^2 = 1$

Soln:

The equation of three tangent plane are of the form.

$$l_r x + m_r y + n_r z = \left[\frac{l_r^2}{a} + \frac{m_r^2}{b} + \frac{n_r^2}{c}\right]^{1/2} \quad (r = 1, 2, 3)$$

Let (x_1, y_1, z_1) be the co-ordinates of the point of intersection of these planes.

Then

$r=1$

$$l_1 x_r + m_1 y_r + n_1 z_r = \left[\frac{l_1^2}{a} + \frac{m_1^2}{b} + \frac{n_1^2}{c}\right]^{1/2} \rightarrow ①$$

$r=2$

$$l_2 x_r + m_2 y_r + n_2 z_r = \left[\frac{l_2^2}{a} + \frac{m_2^2}{b} + \frac{n_2^2}{c}\right]^{1/2} \rightarrow ②$$

$r=3$

$$l_3 x_r + m_3 y_r + n_3 z_r = \left[\frac{l_3^2}{a} + \frac{m_3^2}{b} + \frac{n_3^2}{c}\right]^{1/2} \rightarrow ③$$

These planes are mutually at right angles. The direction cosines of the lines through C ⊥r to the planes are respectively

(l_1, m_1, n_1), (l_2, m_2, n_2) and (l_3, m_3, n_3)

These lines are mutually at right angles.

By considering these lines as co-ordinate axes, the co-ordinate axes OX, OY, OZ have the direction cosines.

~~(l_1, m_1, n_1), (~~

(l_1, l_2, l_3), (m_1, m_2, m_3) and (n_1, n_2, n_3)

$\therefore l_1^2 + l_2^2 + l_3^2 = 1$, $m_1^2 + m_2^2 + m_3^2 = 1$, $n_1^2 + n_2^2 + n_3^2 = 1$

Since OX, OY, OZ are mutually at right angles,

$$l_1 m_1 + l_2 m_2 + l_3 m_3 = 0$$
$$m_1 n_1 + m_2 n_2 + m_3 n_3 = 0$$
$$n_1 l_1 + n_2 l_2 + n_3 l_3 = 0$$

Squaring ①, ②, ③ and adding we get

$$x_1^2 + y_1^2 + z_1^2 = 1/a + 1/b + 1/c$$

Hence the locus of the point (x_1, y_1, z_1) is

$$x^2 + y^2 + z^2 = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$$

which is a ~~sphere~~ sphere concentric with the conicoid. This sphere is known as i.e. director sphere of the conicoid.

(6) Find the eqn. of the tangent planes to $x^2+y^2+4z^2=1$ which intersect in the line whose eqn. are $12x-3y-5=0$, $z=1$

Soln:

Any plane which passes through the line is given by,

$$12x-3y-5+\lambda(z-1)=0$$

(i.e) $12x-3y+\lambda z-(\lambda+5)=0 \longrightarrow (1)$

Let this plane touch the conicoid at (x_1,y_1,z_1)

The eqn of the tangent plane at (x_1,y_1,z_1) is

$$xx_1+yy_1+4zz_1=1 \longrightarrow (2)$$

Equations (1) and (2), represents the same plane

$$\therefore \frac{x_1}{12}=\frac{y_1}{-3}=\frac{4z_1}{\lambda}=\frac{1}{\lambda+5}$$

$$\therefore x_1=\frac{12}{\lambda+5},\quad y_1=\frac{-3}{\lambda+5},\quad z_1=\frac{\lambda}{4(\lambda+5)}$$

Since (x_1,y_1,z_1) lies on the conicoid

$$x_1^2+y_1^2+4z_1^2=1$$

$$\therefore \left(\frac{12}{\lambda+5}\right)^2+\left(\frac{-3}{\lambda+5}\right)^2+4\left(\frac{\lambda}{4(\lambda+5)}\right)^2=1$$

(i.e) $3\lambda^2+40\lambda-512=0$

$$\lambda=8 \text{ (or) } -64/3$$

Hence the eqn. of the tangent planes are

$$12x-3y-5+8(z-1)=0$$

and $15x - 3y - 5 - \frac{64}{3}(z-1) = 0$

(i.e) $15x - 3y + 2z - 13 = 0$ and

$36x - 9y - 64z + 49 = 0$

University Questions

1. write down the equation of tangent plane at (x_1, y_1, z_1) to the cone.

soln:

If $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ is a tangent to the cone at

$\therefore l(ax_1 + hy_1 + gz_1) + m(hx_1 + by_1 + fz_1)$
$+ n(gx_1 + fy_1 + cz_1) = 0$

important

2. write down the two tangent planes to a conicoid parallel to plane $lx + my + nz = 0$

soln:

The condition for the plane $lx + my + nz = 0$ to touch the conicoid $ax^2 + by^2 + cz^2 = 1$

Let the plane touch the conicoide at (x_1, y_1, z_1)

The equation of tangent plane at (x_1, y_1, z_1)

$ax x_1 + by y_1 + cz z_1 = 1 \longrightarrow (1)$

this plane is also respre/- by the eqn

$lx + my + nz = p \longrightarrow (2)$

$x_1 = \frac{l}{ap}, \quad y_1 = \frac{m}{bp}, \quad z_1 = \frac{n}{cp}$

3. write down the condition for a line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ in plane $ax + by + cz + d = 0$

$a(x-x_1)+b(y-y_1)+c(z-z_1)=0$

$al+bm+cn=0$

1) what is meant by unsymmetrical form of the equation of a line.

Soln:

To transform the equations $ax+by+cz+d=0$, $ax_1+by_1+cz_1+d_1=0$ of a line to the symmetrical form.

UNIT-3

1. Prove that the two spheres $x^2+y^2+z^2-2x+4y-4z=0$ and $x^2+y^2+z^2+10x+2z+10=0$ touch each other and find the co-ordinates of the point of contact.

Soln.

The given spheres are

$x^2+y^2+z^2-2x+4y-4z=0 \longrightarrow ①$

$x^2+y^2+z^2+10x+2z+10=0 \longrightarrow ②$

The centre and radius of equ:- ① are

$c_1=(1,-2,2)$

$r_1=\sqrt{1+4+4}=3$

The centre and radius of equ:- ② are

$c_2=(-5,0,-1)$

$r_2=\sqrt{25+1-10}=\sqrt{16}=4$

The distance b/w the centres

$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$

$$d = \sqrt{(1+5)^2+(-2-0)^2+(2+1)^2}$$

$$= \sqrt{6^2+2^2+3^2} = \sqrt{36+4+9} = \sqrt{49}$$

$d = 7$

To find

(∴ $\underline{d = r_1 + r_2}$. is satisfied the two sphere touch externally.) 2M

to find the point of contact.

The point of contact is the point which divides [the line joining the two points $c_1 = (1, -2, 2)$ and $c_2 = (-5, 0, -1)$ in the ratio 3 : 4

[∴ The co-ordinates of the point of contact are.]

$$= \left[\frac{3\times-5 + 4\times1}{3+4}, \frac{3\times0 + 4\times-2}{3+4}, \frac{3\times-1+4\times2}{3+4} \right]$$

$$= \frac{-15+4}{7}, \frac{0-8}{7}, \frac{-3+8}{7}$$

$$= \frac{-11}{7}, \frac{-8}{7}, \frac{5}{7}$$

25.7.19

UNIT - 4

UNIT-4

1. The plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, meets the co-ordinates axis in A, B, C. Prove that the equ to the cone generated by lines drawn from O to meet the circle A, B, C is $yz\left(b/c+\frac{c}{b}\right)+zx\left(\frac{c}{a}+\frac{a}{c}\right)+xy\left(\frac{a}{b}+\frac{b}{a}\right)=$

Soln:

Points A, B, C are $(a,0,0)$, $(0,b,0)$ and $(0,0,c)$ Equation of the sphere OABC is

$$x^2+y^2+z^2-ax-by-cz=0$$

$$(x^2+y^2+z^2)-(ax+by+cz)(1)=0$$

$$(x^2+y^2+z^2)-(ax+by+cz)\left(\frac{x}{a}+\frac{y}{b}+\frac{z}{c}\right)=0$$

$$(x^2+y^2+z^2)-\frac{ax^2}{a}-\frac{axy}{b}-\frac{axz}{c}-\frac{byx}{a}-\frac{by^2}{b}$$
$$-\frac{byz}{c}-\frac{czx}{a}-\frac{czy}{b}-\frac{cz^2}{c}=0$$

$$x^2+y^2+z^2-x^2-\frac{axy}{b}-\frac{axz}{c}-\frac{byx}{a}-y^2$$
$$-\frac{byz}{c}-\frac{czx}{a}-\frac{czy}{b}-z^2=0$$

$$-\frac{axy}{b}-\frac{axz}{c}-\frac{byx}{a}-\frac{byz}{c}-\frac{czx}{a}-\frac{czy}{b}=0$$

$$\frac{axy}{b}+\frac{axz}{c}+\frac{byx}{a}+\frac{byz}{c}+\frac{czx}{a}+\frac{czy}{b}=0$$

$$xy\left(\frac{a}{b}+\frac{b}{a}\right)+zx\left(\frac{a}{c}+\frac{c}{a}\right)+yz\left(\frac{b}{c}+\frac{c}{b}\right)=0$$

2. Prove that the $ax^2+by^2+cz^2+2ux+2vy+2wz+d=0$ represents a cone if $\frac{u^2}{a}+\frac{v^2}{b}+\frac{w^2}{c}=d$

Soln:

~~let $f(x, y, z, t) = ax^2+by^2+cz^2+2ux+2vy$~~

~~$2wz$~~

let $f(x, y, z, t) = ax^2+by^2+cz^2+2uxt+2vyt$
$+2wzt+dt^2=0$

$\frac{\partial F}{\partial x}=0$ for $t=1$

$\frac{\partial F}{\partial x}=2ax+2ut=0$

$2ax+2u=0$

$2ax=-2u$

$x=\frac{-2u}{2a}$

$x=\frac{-u}{a} \longrightarrow (1)$

$||^{ly}$

$\frac{\partial F}{\partial y}=0$ for $t=1$

$y=-\frac{v}{b} \longrightarrow (2)$

$\frac{\partial F}{\partial z}=0$ for $t=1$

$z=-\frac{w}{c} \longrightarrow (3)$

$\frac{\partial F}{\partial t}=0$ for $t=1$

$ux+vy+wz+d=0 \longrightarrow (4)$

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$+2wz+d=0$,

$=d$

$+2vy+$

Substituting the values of x, y, z in

$$ux+vy+wz+d=0$$

$$u\left(\frac{-u}{a}\right)+v\left(\frac{-v}{b}\right)+w\left(\frac{-w}{c}\right)+d=0$$

$$-\frac{u^2}{a}-\frac{v^2}{b}-\frac{w^2}{c}+d=0$$

$$\frac{u^2}{a}+\frac{v^2}{b}+\frac{w^2}{c}=d$$

U.Q 10m

(7) Planes through OX and OY include an angle 60°. Show that their line of intersection lies on the cone $z^2(x^2+y^2+z^2)=3x^2y^2$

Soln:

The equation of x axis are $y=0, z=0$

$\therefore$ plane passing through x axis

$$\Rightarrow y+k_1z=0 \longrightarrow (1)$$

The equation of y axis are $x=0, z=0$

$\therefore$ plane passing through y axis

$$\Rightarrow x+k_2z=0 \longrightarrow (2)$$

Let $P(x_0, y_0, z_0)$ be any point on the line of intersection of the planes (1), (2)

$$y_0+k_1z_0=0, \quad x_0+k_2z_0=0$$

$$k_1=\frac{-y_0}{z_0}, \quad k_2=\frac{-x_0}{z_0}$$

$(1) \Rightarrow y+k_1z=0$

$$y-\frac{y_0}{z_0}z=0$$

$$z_0y-y_0z=0$$

$(2) \Rightarrow x+k_2z=0$

$$x-\frac{x_0}{z_0}z=0$$

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$xz_0 - x_0 z = 0$

The direction ratios of the normals to the planes are $z_0 y - y_0 z = 0$, $xz_0 - x_0 z = 0$

$(0, z_0, -y_0)$; $(z_0, 0, -x_0)$

The angle between the normals is $60°$

$$\therefore \cos 60° = \frac{(0)(z_0) + (z_0)(0) + (-y_0)(-x_0)}{\sqrt{0^2 + (z_0)^2 + (-y_0)^2}\sqrt{(z_0)^2 + (0)^2 + (-x_0)^2}}$$

$$1/2 = \frac{x_0 y_0}{\sqrt{z_0^2 + y_0^2}\sqrt{z_0^2 + x_0^2}}$$

squaring on both sides,

$$1/4 = \frac{x_0^2 y_0^2}{(z_0^2 + y_0^2)(z_0^2 + x_0^2)} = \frac{x_0^2 y_0^2}{z_0^4 + z_0^2 x_0^2 + z_0^2 y_0^2 + y_0^2 x_0^2}$$

$$z^4 + z^2 x^2 + z^2 y^2 + y^2 x^2 = 4x^2 y^2$$

$$z^4 + z^2 x^2 + z^2 y^2 = 4x^2 y^2 - y^2 x^2$$

$$z^2(z^2 + x^2 + y^2) = 3x^2 y^2$$

4. Find the equation of the cone of the second degree which passes through the axes.

Soln:

The equation of the cone is a homogeneous equation of second degree in x, y and z

$$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$$

The cone passes through the x axis

$\therefore (1, 0, 0)$ is a point on the cone

$$a(1) + 0 = 0$$

$$a = 0$$

$$b = 0$$

$$c = 0$$

to these

(o)

$(-x_0)^2$

$+z_0^2y_0^2+y_0^2x_1^2$

econd

neous

x

one

The equation becomes.

$$2fyz+2gzx+2hxy=0$$

$$\Rightarrow fyz+gzx+hxy=0$$

H.W. 8 due: Find the equation of the cone with its vertex (1, -2, -3) with its base curve given by $x^2-2y^2+z^2=4$, $x-y+z=7$

Soln:

The given circle is

$$x^2-2y^2+z^2=4 \quad \rightarrow (1)$$

$$x-y+z=7 \quad \rightarrow (2)$$

The eqn of the line passing through (1, -2, -3)

$$\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}=r$$

$$\frac{x-1}{l}=\frac{y+2}{m}=\frac{z+3}{n}=r \longrightarrow (3)$$

Any point on this generator

$(lr+1, mr-2, nr-3)$ (1) and (2)

$$(lr+1)^2-2(mr-2)^2+(nr-3)^2=4$$

$$l^2r^2+2lr+1-2[m^2r^2-4mr+4]+n^2r^2-6nr+9=4$$

$$l^2r^2+2lr+1-2m^2r^2+8mr-8+n^2r^2-6nr+9-4=0$$

$$r^2[l^2-2m^2+n^2]+2r[l+4m-3n]-2=0$$

$$r^2[l^2-2m^2+n^2]+2r[l+4m-3n]-2=0 \rightarrow (4)$$

$$[lr+1]-[mr-2]+[nr-3]=7$$

$$lr+1-mr+2+nr-3-7=0$$

$$r[l-m+n]-7=0 \rightarrow r(l-m+n)=7$$

$$r=\frac{7}{l-m+n}$$

using the eqn (4) sub r value

$$\frac{49}{(l-m+n)^2}(l^2-2m^2+n^2)+2\left(\frac{7}{l-m+n}\right)(l+4m-3n)-2=0$$ 24.3.19

8.

$$\frac{49(l^2-2m^2+n^2]+14(l-m+n)(l+4m-3n)-2(l-m+n)^2=0}{(l-m+n)^2}$$

$$49[l^2-2m^2+n^2]+14[l-m+n][l+4m-3n)-2(l-m+n)^2=0$$

$$49l^2-98m^2+49n^2+\cancel{14l-14m+14n}\;14[l^2+4lm-3ln-lm$$
$$-4m^2+3mn+nl+4mn-3n^2]-2[l^2+m^2$$
$$+n^2-2lm-2mn+2nl]^2=0$$

$$49l^2-98m^2+49n^2+14l^2+56lm-42ln-56m^2+42mn$$
$$-14lm+14nl+56mn-42n^2-2l^2-2m^2-2n^2$$
$$+4lm+4mn-4nl=0$$