1 Introduction

Electromagnetism is one of the fundamental interactions in nature. Its physical origin lies in a property possessed by elementary particles of matter—electrons and protons—called electric charge. The electromagnetic interaction also governs light and other forms of electromagnetic radiation. Electricity and magnetism can be observed in many natural phenomena. They are applied in many inventions that are used in technology and everyday life. The purpose of this article is to describe the basic science and applications of electromagnetism.

The scientific understanding of electricity and magnetism has developed over a period of centuries. Phenomena that we understand today as examples of electricity, magnetism, or electromagnetism have been observed since before the dawn of history: Light consists of electromagnetic waves. Lightning is an electric discharge in the atmosphere. Such common phenomena were often observed, but unexplained until the scientific revolution. Today we are able to describe and explain them accurately based on a precise theory of the electromagnetic interaction.

The first discoveries of the fundamental electric and magnetic forces were made by philosophers of ancient Greece. They observed that when amber is rubbed with animal fur, it acquires the ability to attract small bits of reed or feathers. This small effect was the first observation of static electricity. (The Greek word for amber, $\epsilon\lambda\epsilon\kappa\tau\rho o\nu$, is the origin of our word “electric.”) They also observed that a lodestone exerts a force on iron—the earliest example of magnetism. (The Greek province of Magnesia, where the magnetic ore magnetite occurs naturally, is the origin of our word ”magnetic.”) These early discoveries of weak and mysterious forces were the first steps toward our scientific understanding of electromagnetism. Today science has dispelled much of the mystery, and created technological power beyond the dreams of the ancient philosophers.

2 The science of electricity and magnetism

2.1 Electrostatics

Electric charge is a property of matter. At the most basic level, the constituents of atoms are charged particles—electrons with negative charge and protons with positive charge. An atom has equal numbers of electrons and protons; it is electrically neutral because the electron charge $-e$ is equal but opposite to the proton charge $+e$. However, a sample of matter becomes electrically charged if the balance of electrons and protons is unequal. For example, when amber is rubbed with fur, electrons are transferred from the amber to the fur; the amber then has net positive charge.

Like charges repel and unlike charge attract. That is, two samples of matter, both with net positive charge or both with net negative charge, exert equal but opposite repulsive forces on one another. If the samples have unlike charges, one positive and the other negative, then each exerts an attractive force on the other. The strength of the electric force **F** was measured accurately by Charles Augustin de Coulomb. The force is proportional to the product of the charges, q_1 and q_2, and inversely proportional to the square of the distance of separation r,

$$F = \frac{kq_1q_2}{r^2} \qquad \text{where} \qquad k = 8.99 \times 10^9 \ \mathrm{Nm^2/C^2}.$$

This simple mathematical formula has been tested to extreme precision. It forms one basis for the theory of electromagnetism.

2.2 Magnetostatics

Most people have observed magnets and their mysterious forces, which can be quite strong even in small magnets. A magnet has a special attraction to iron, and brought close there is a force. Two magnets repel if their north poles approach each other, repel if the south poles approach, but attract if north approaches south.

Science has identified the origins of magnetic forces. Technologies based on magnetic forces are used in everyday life throughout the world.

There is a very close connection between magnetism and the electric charge of subatomic particles. The most familiar example of a magnet is a ferromagnet—a piece of magnetized iron. (Cobalt and nickel are also ferromagnetic elements but iron is the most common example.) However, the ferromagnet is neither the only source of magnetism nor even the most basic. Electric currents also produce magnetic forces, and in some ways the magnetic field associated with electric current is the most basic form of magnetism.[1]

2.2.1 Electric current as a source of the magnetic field

An electric current is a stream of electrically charged particles moving in the same direction. The current may be constant in time (DC, or direct current), oscillating in time with a constant frequency (AC, or alternating current), or varying in time. Currents can exist in metals and in several other forms of

[1] The term "magnetic field" used here refers to any magnetic effect. The more technical meaning of the term is explained in Sec. 3.

conducting matter. In a metal, some electrons occupy states that extend over large distances and are not bound to a single atomic core. If an electric force is applied then these conduction electrons will move in response, creating an electric current. (Ohm's law, $V = IR$ where $V =$ potential difference in volts, $I =$ current in amps, and $R =$ resistance in ohms, expresses quantitatively the relation between the electric force and the current.) In metals the positively charged atomic nuclei are fixed in a crystalline lattice, so the electric current is due to motion of electrons.

The first laboratory observation of the magnetic field associated with an electric current was an accidental discovery by Hans Christian Oersted during a public lecture in 18xx. The current existed in a metal wire connected across a battery. Oersted noticed that a nearby compass needle deflected while the current was flowing. The strength of the magnetic field at a distance of 1 centimeter from a 1 ampere current is 2×10^{-5} tesla, comparable to the Earth's magnetic field of approximately 5×10^{-5} tesla.

Oersted's discovery was studied in further detail by Jean Marie Ampère. The theory of the magnetic field created by an electric current is called Ampère's law in physics. The magnetic field can be studied quantitatively by measuring the force on a pole of a magnet or the torque on a compass needle.

Ampère found that the field direction "curls around" the current (see Fig. 1). A compass needle points in the direction of the field vector at the position of the compass. Therefore a compass can be used to map the field directions. Figure 1 shows a segment of current-carrying wire and the associated magnetic field encircling the wire. The field directions follow from the right-hand rule: With the thumb of your right hand aligned along the wire pointing in the direction of the current, the fingers naturally curl in the direction of the magnetic field around the wire.

Electric currents can also exist in materials other than metals, such as plasmas and ionic solutions. A plasma is a hot gas in which the atoms or molecules are partially ionized so that free electrons and positive ions can move in response to electric forces. Plasmas occur in nature. Examples are stars, the ionosphere of the Earth, and the path of a lightning strike. The currents that occur in these natural plasmas produce interesting magnetic effects. The sun has a magnetic field, and magnetic storms or sunspots, due to electric currents in the solar material. The ionosphere reflects short-wave radio waves because of the electromagnetic interaction. The magnetic fields produced by a lightning strike can be measured to learn about the properties of lightning.

Electric current may occur in an ionic solution, such as an acid or salt solution. Positive and negative ions moving through the liquid form the current.

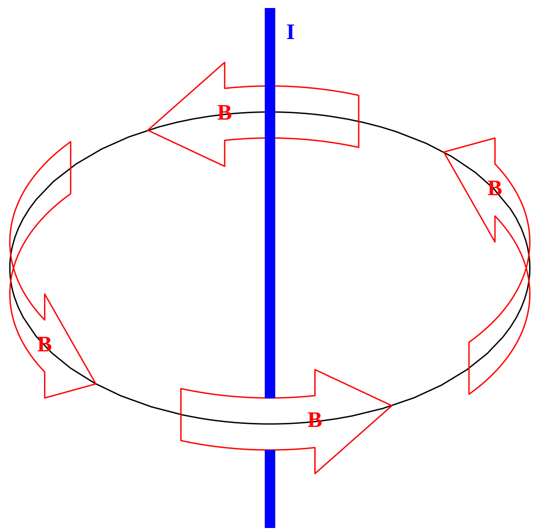

Figure 1: The magnetic field curls around the current. The dashed curve indicates an imaginary circle around the wire segment.

An everyday example is the current in a lead-acid car battery. A biomedical example is the electric current in a nerve cell; the associated magnetic fields are observed in magnetocardiology and magnetoencephalography.

The magnetic field of the Earth is another natural example of a field produced by an electric current. The core of the Earth is highly metallic and at high temperature and pressure. Currents flowing in this metal core create the Earth's magnetism, which we observe at the surface of the Earth.

Electromagnets Ampère's law is applied in electromagnets. The magnetic field of a straight length of current-carrying wire is weak. However, if the wire is wound around a cylinder, making a coil with many turns as illustrated in Fig. 2, then the field inside and near the ends of the cylinder may be strong for a practical current. The cylindrical coil is called a solenoid. The field strength can be increased by putting an iron core in the cylinder. Unlike a permanent magnet, the fields of an electromagnet can be turned off by cutting the electric current. Electromagnets are commonly used in electric motors and relay switches.

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Figure 2: An electromagnet.

2.2.2 The magnetic force on an electric current

A magnetic field—whether produced by a ferromagnet or by an electric current—exerts a force on any electric currents that are placed in the field. More fundamentally, there exists a magnetic force on any charged particle that moves across the magnetic field vectors. So the interaction between electric charges in motion and a magnetic field has two aspects: (i) an electric current creates a magnetic field (Ampère's law); (ii) a magnetic field exerts a force on an electric current.

The electric motor. The magnetic force on a current is the basis for all electric motors. The magnetic field may be produced by a permanent magnet (in small DC motors) or by an elecgtromagnet (in large DC and AC motors). The current flows in a rotating coil of wire, and may be produced by a battery or some other source of electromotive force. Many practical designs have been invented, with the common feature that a magnetic force acts on the current-carrying coil, in opposite directions on opposite sides of the coil, creating a torque that drives the rotation of the coil. A co-rotating shaft attached to the coil is then connected to a mechanical system to do practical work.

3 The field concept

Electric and magnetic forces are described by the effects of electric and magnetic fields. The theory of electromagnetic phenomena is entirely based on the field concept.

Any electrically charged particle q creates an associated electric field $\mathbf{E}$. The field extends throughout the space around q, varying with the position $\mathbf{x}$ in space. If the particle is at rest in some frame of reference, then its field in that reference frame is independent of time, $\mathbf{E}(\mathbf{x})$. In a system with many charges, the full electric field is the sum of the electric fields of the individual charges. Thus the electric field in an electrostatic system, denoted mathematically by $\mathbf{E}(\mathbf{x})$, varies with the position $\mathbf{x}$ and depends on the locations and charge strengths of all the charged particles.

The field exists throughout a volume of space and it is a function of position $\mathbf{x}$. It exerts a force on any charge in the space. A simple electrostatic system to illustrate the field concept has two charged particles, q_1 and q_2, located at positions $\mathbf{x}_1$ and $\mathbf{x}_2$, respectively. (See Fig. 3.) The electric field at an arbitrary point $\mathbf{x}$ is

$$\mathbf{E}(\mathbf{x}) = \frac{kq_1}{r_1^2}\mathbf{e}_1 + \frac{kq_2}{r_2^2}\mathbf{e}_2$$

where r_1 is the distance from q_1 to $\mathbf{x}$, $\mathbf{e}_1$ is the unit vector in the direction from q_1 to $\mathbf{x}$, and q_2 and $\mathbf{e}_2$ are the analogous quantities for q_2. A small test charge q placed at $\mathbf{x}$ will experience a force $\mathbf{F} = q\mathbf{E}(\mathbf{x})$. Since $\mathbf{E}(\mathbf{x})$ is the sum of the fields due to q_1 and q_2, the force $\mathbf{F}$ on the test charge is the sum of the two forces exerted on q. The charges q_1 and q_2 also experience forces due to the presence of hte other charge. For example, the force on q_2 due to q_1 is $q_2\mathbf{E}_1(\mathbf{x}_2)$ where $\mathbf{E}_1$ is the field due to q_1 alone. The field due to a charged particle is inversely proportional to the square of the distance from the particle, so the force obeys the inverse-square law observed by Coulomb.

In the field theory of static electricity, the force on a charged particle q is attributed to the field $\mathbf{E}(\mathbf{x})$ created by the other charges. The field concept is significantly different from "action at a distance." The field concept is that the force on the particle, equal to $q\mathbf{E}(\mathbf{x})$, is exerted by the field at the position of q, rather than by direct actions by the distant charges that created the field $\mathbf{E}(\mathbf{x})$. In other words, an electrostatic system consists of two physical entities: a set of charged particles and an electric field $\mathbf{E}(\mathbf{x})$. The field is just as real as the particles.

Figure 3 illustrates the electric field for a system of two charged particles with equal but opposite charges. The electric field is a vector at each point throughout the space around the charges. The curves in Fig. 1, called the electric field lines, represent the field. A positive test charge q in the field would experience a force in the direction of the field vector at the position of q. We visualize the field by drawing the field lines—curves that are everywhere tengent to the field vector directions.

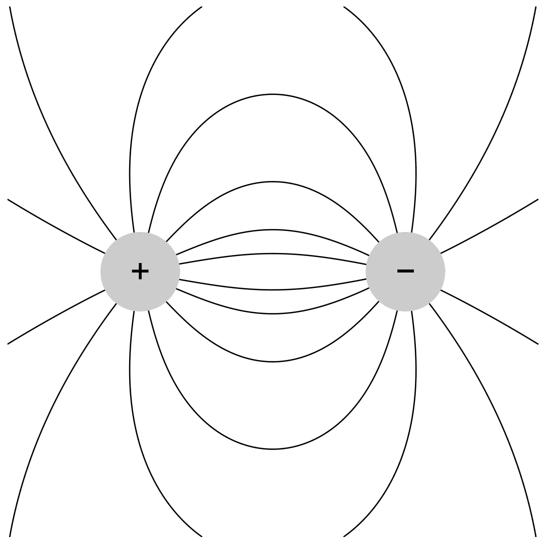

Figure 3: The electric field lines of a system of two charge with equal but opposite charge.

Electric charges at rest produce an electric field $\mathbf{E}(\mathbf{x})$. Ferromagnets and electric currents produce another field—the magnetic field $\mathbf{B}(\mathbf{x})$. Figure 4 illustrates the magnetic fields created by two current sources: a small current loop, and a long current-carrying wire. Both $\mathbf{E}$ and $\mathbf{B}$ are force fields, which extend throughout a volume of space. But they exert distinct and different forces. The electric field $\mathbf{E}$ exerts a force on a charge q in the direction of the field vector. The magnetic field $\mathbf{B}$ exerts a force on any moving charges or current-carrying wires. The direction of the magnetic force is perpendicular to both the current and the field, as illustrated in Fig. 5.

The magnetic field $\mathbf{B}$ also exerts forces on the poles of a ferromagnet. The direction of the force is parallel (for a north pole) or antiparallel (for a south pole) to the magnetic field vector. A compass needle aligns with the magnetic field because the equal but opposite forces on the two poles of the needle compose a torque that twists the needle toward alignment with the field.

The field concept was first stated by Michael Faraday. From years of experimental studies on electricity and magnetism, Faraday had formed the idea that a physical entity extends throughout the space outside charges or magnets, and exerts forces on other charges and magnets in the space. He referred to this extended entity as the "lines of force." The term *Electromag-*

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Figure 4: Magnetic field of (a) a small current loop and (b) a long straight wire segment.

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Figure 5: The magneitc force on a moving charge or current segment.

netic Field was coined by James Clerk Maxwell, the renowned theoretical physicist who developed a mathematical theory of electromagnetism based on the field concept.

4 Electromagnetic Induction

The previous sections were concerned with static systems of electric charge or current, i.e., in which the fields $\mathbf{E}(\mathbf{x})$ and $\mathbf{B}(\mathbf{x})$ do not change in time. Some relationships exist between electricity and magnetism: a steady electric current produces a magnetic field (Ampère's law); a magnetic field exerts a force on any electric charge moving across the field lines. However, for static fields electric and magnetic phenomena appear to be rather distinct.

Time-dependent fields will be described next: the electric field $\mathbf{E}(\mathbf{x}, t)$ and magnetic field $\mathbf{B}(\mathbf{x}, t)$ are functions of time t as well as position $\mathbf{x}$. In dynamic systems the two fields affect each other significantly. Therefore electric and magnetic phenomena are connected, and must be described by a unified theory. Electricity and magnetism are then combined into electromagnetism.

Electromagnetic induction is a phenomenon with important technological applications. It was discovered in 18xx independently by Michael Faraday in England and Joseph Henry in the United States. The effect is that when a magnetic field changes in time, there exists an induced electric field in directions that curl around the change of the magnetic field.

Electromagnetic induction may be observed directly in simple physical demonstrations. Figure 6 shows schematically a coil of conducting wire C connected to a galvanometer G. The galvanometer acts as a current indicator: when current flows around the coil C the needle points at an angle to the central (zero current) position. No current source (such as a battery) is connected to the wire coil. In Fig. 6, M is a magnet that can be moved toward or away from the coil C. When the magnet is at rest no current flows in the coil and the galvanometer needle points in the central direction.

If the magnet in Fig. 6 is moving toward the coil, the galvanometer needle will be deflected in one direction, indicating that a current is flowing in C. The current exists while M is moving. When the motion of M ceases, the needle will return to the central position indicating that the current has stopped. If the demonstration is repeated with the magnet moving away from the coil, the galvanometer needle will be deflected in the opposite direction while M moves. These demonstrations show directly that a change in the magnetic field through the coil produces a current in the coil.

The magnetic field in the demonstration might be varied in a different way. In the apparatus in Fig. 6 the bar magnet M could be replaced by an

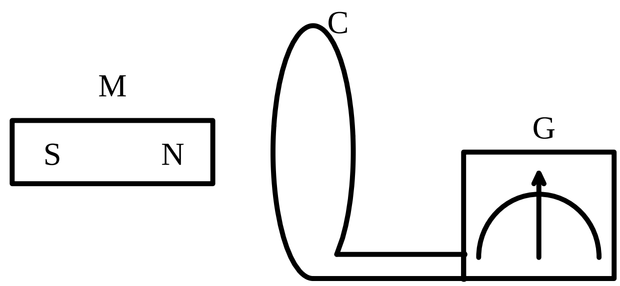

Figure 6: Schematic apparatus for demonstrations of electromagnetic induction.

electromagnet that does not move. Electric current $I(t)$ in the electromagnet solenoid produces a magnetic field $\mathbf{B}(\mathbf{x}, t)$ according to Ampère's law. If the current in the solenoid is constant, then $\mathbf{B}(\mathbf{x}, t)$ is constant in time and there is no current in the coil C. But if the electromagnet current is changing then the magnetic field changes. A deflection of the galvanometer will be observed while the magnetic field is varying in time. The current around the coil C is again produced by the changing magnetic field through the coil.

These demonstrations show when a magnetic field $\mathbf{B}(\mathbf{x}, t)$ changes in time, a current is induced in a conductor that is present in the field. However, the induced current is actually a secondary effect. The current in C is created by an induced electric field $\mathbf{E}(\mathbf{x}, t)$, and $\mathbf{E}$ is the primary effect. Electromagnetic induction is fundamentally a phenomenon of the fields, $\mathbf{B}(\mathbf{x}, t)$ and $\mathbf{E}(\mathbf{x}, t)$. If a magnetic field is changing in time in some volume of space, there is an induced electric field in directions curling around the change of the magnetic field. If there happens to be a conducting coil in the space, as for example C in Fig. 6, then the induced electric field drives an electric current around C. But the primary effect is the induction of an electric field. The induced electric field $\mathbf{E}(\mathbf{x}, t)$ only exists while the magnetic field $\mathbf{B}(\mathbf{x}, t)$ is varying in time.

The apparatus shown in Fig. 6 is only schematic. The induced current in C would be very small for an ordinary bar magnet M moving at reasonable velocities. A practical demonstration would require a coil C with many turns of wire and a sensitve galvanometer. The effect might be increased by putting an iron core inside the coil to enhance the magnetic field.

Faraday and Henry performed laboratory experiments similar to the demonstrations illustrated in Fig. 6 in their discoveries of electromagnetic induction.

Faraday described the results of his many detailed studies in terms of the lines for force—his concept of a magnetic field filling space around a magnet. In modern language, this statement summarizes his observations:

> Faraday's Law: When the flux of magnetic field through a loop changes in time, an electromotive force (emf) is induced around the loop.

This statement is Faraday's law of electromagnetic induction. In the form of an equation it is written $d\Phi/dt = -\mathcal{E}$, where Φ is the magnetic flux through the loop, $d\Phi/dt$ is the rate of change of the flux, and $\mathcal{E}$ is the electromotive force around the loop. The "loop" referred to in Faraday's law is a closed curve, e.g., a circle. The loop may be a conducting loop, in which case the induced emf drives a current; or it may just be an imaginary curve in space. In either case the emf is associated with an induced electric field curling around the change of the magnetic field.

Another, related demonstration may be carried out with the simple apparatus of Fig. 6. Instead of moving the magnet M and holding the coil C fixed, move the coil with the magnet held fixed. Again a current will be observed around C. The phenomenon in this case is called "motional emf"—an electromotive force is induced in a conductor that moves through a magnetic field. In the language of Faraday, when a conducting wire cuts through magnetic lines of force, an induced current flows in the wire. Evidently any change of the magnetic flux through a conducting loop will induce a current in the loop.

Yet another way to change the magnetic flux through the coil C in Fig. 6 is to change the orientation of the coil. In Fig. 6 the plane of the coil is shown perpendicular to the bar magnet. In this orientation the magnetic lines of force pass through the coil; the flux is the product of the magnetic field strength B and the area A of the loop, $\Phi = BA$. Now suppose the magnet M is held fixed, and the center position of the coil is also fixed, but the coil is rotated about a vertical axis. Then the flux of magnetic field through the coil changes as the plane of the loop is at a varying angle to the field vector. When the plane of the loop is parallel to the field lines the flux is zero because no field lines pass through the coil. As the coil rotates a deflection of the galvanometer needle will be observed, consistent with Faraday's law, because the flux changes. This is another example of motional emf: the conducting wire cuts through the magnetic field lines and there is an induced current.

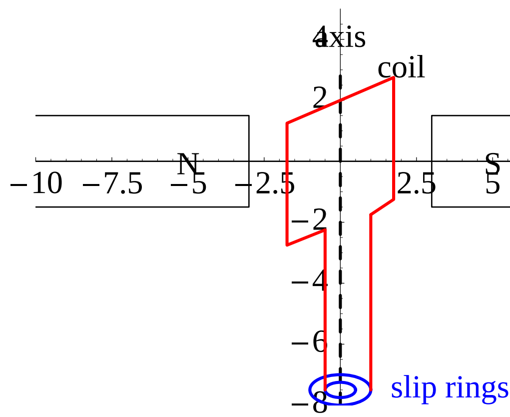

Figure 7: The basic principle of an electric generator.

5 Applications of Electromagnetic Induction

A number of important inventions are based on the phenomenon of electromagnetic induction. Two that have great technological significance will be described here.

5.1 The electric generator

5.2 The transformer

6 Maxwell's Equations of the Electromagnetic Field

The mathematical theory of electromagnetism was developed and published in 1864 by James Clerk Maxwell. He described the known electromagnetic effects in terms of four equations relating the electric and magnetic fields and their sources—charged particles and electric currents. The development of this theory was a supreme achievement in the history of science. Maxwell's theory is still used today by physicists and electrical engineers. The theory was further developed in the 20th century, to account for the quantum theory of light. But even in quantum electrodynamics Maxwell's equations remain valid although the interpretation is somewhat different from the classical theory. In any case, Maxwell's theory continues today to be an essential part of theoretical physics.

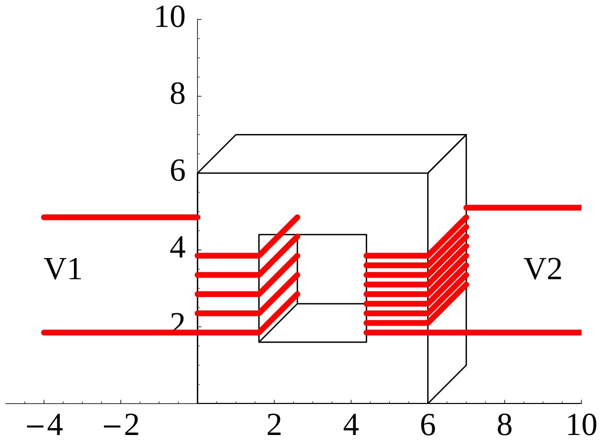

Figure 8: The basic principle of a transformer.

A knowledge of calculus and vectors is necessary for a full understanding of Maxwell's equations. However, the essential structure of the theory can be understood without going into the mathematical details. Each equation is expressed most powerfully as a partial differential equation relating variations of the electric and magnetic fields, with respect to variations of position or time, and the charge and current densities in space.

Gauss's law. Gauss's law, written as a field equation, is $\nabla \cdot \mathbf{E} = \rho/\epsilon_0$. Here $\rho(\mathbf{x}, t)$ is the charge per unit volume in the neighborhood of $\mathbf{x}$ at time t; ϵ_0 is a constant of nature equal to $8.85 \times 10^{-12} XXX$. Gauss's law relates the electric field $\mathbf{E}(\mathbf{x}, t)$ and the charge density. The solution for a charged particle q at rest is $\mathbf{E}(\mathbf{x}) = kq\mathbf{e}/r^2$ where r is the distance from the charge to $\mathbf{x}$, $\mathbf{e}$ is the direction vector, and $k = 1/(4\pi\epsilon_0)$; this is the familiar inverse square law of electrostatics. Electric field lines diverge at a point charge.

Gauss's law for magnetism. The analogous equation for the magnetic field $\mathbf{B}(\mathbf{x}, t)$ is $\nabla \cdot \mathbf{B} = 0$. There are no magnetic monopoles—particles that act as a point source of $\mathbf{B}(\mathbf{x}, t)$. Unlike the electric field lines, which may terminate on charges, the magnetic field lines always form closed curves because magnetic charges do not exist. There is no divergence of magnetic field lines.

Faraday's law. The field equation that describes Faraday's law of electromagnetic induction is $\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$. The quantity $\nabla \times \mathbf{E}$, called the curl of $\mathbf{E}(\mathbf{x}, t)$, determines the way that the vector field $\mathbf{E}$ curls around each direction in space. Also, $\partial \mathbf{B}/\partial t$ is the rate of change of the magnetic field. This field equation is equivalent to the statement that a magnetic field that varies in time implies an electric field that curls around the change of the magnetic field. Or, Faraday's statement that the rate of change of magnetic field flux through a surface S implies an electromotive force (emf) around the boundary curve of S is mathematically equivalent to the partial differential equation.

The Ampère-Maxwell law. In a system of steady electric currents the magnetic field is constant in time and curls around the current in directions defined by the right-hand rule. The field equation that expresses this form of $\mathbf{B}$ (Ampère's law) is $\boldsymbol{\nabla} \times \mathbf{B} = \mu_0 \mathbf{J}$, where $\mathbf{J}(\mathbf{x}, t)$ is the current per unit area at $\mathbf{x}$ and μ_0 is a constant equal to $4\pi \times 10^{-7} xxx$. But Ampère's law is incomplete; it does not apply to systems in which the currents and fields vary in time. Maxwell deduced from mathematical considerations a generalization of Ampère's law,

$$\boldsymbol{\nabla} \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t},$$

in which the second term on the right side is called the displacement current. The displacement current is a necessary term in order for the system of four partial differential equations to be self-consistent.

Analogy between displacement current and electromangitc induction.

Maxwell's theory of the displacement current was a daring theoretical prediction. At that time there was no experimental evidence for the existence of displacement current. Laboratory effects predicted by the displacement current are very small, and their observation was not possible with the apparatus available at that time. However, Maxwell's equations, including the displacement current, made a striking prediction—that light consists of electromagnetic waves. The fact that Masxwell's theory explained the properties of light, and later other forms of electromagnetic radiation, provided evidence for the existence of the displacement current.