**QUANTUM MECHANICS-P16PY22**

**UNIT-IV**

**ANGULAR MOMENTUM**

**ORBITAL ANGULAR MOMENTUM:**

The moment of linear momentum is known as angular momentum. For a given particle angular momentum is given by the cross product of its position vector with linear momentum.

L = r X p

 It is easily shown that

 Lx = ypz - zpy

Ly = zpx - xpz ........(1)

Lz = xpy - ypx

And

Where x, y, z are the components of r and px, py  and pz  are the components of linear momentum in three mutually perpendicular axis.

The quantum mechanical equivalent of set (1) equations are

 Lx = - i*ћ*

Ly = - i*ћ* …………..(2)

Lz = - i*ћ*

 The second set of equations can be written in polar co-ordinate form.

 Lx = - i*ћ*

Ly = - i*ћ* …………… (3)

Lz = - i*ћ*

 Using the second set of definitions it is easy to show that Lx , Ly and Lz satisfies the following commutation rules,

 [Lx , Ly] = iћLz

[Ly , Lz] = i*ћ*Lx

[Lz , Lx] = i*ћ*Ly

In compact form these can be quoted as

 L X L = i*ћ*L

**SPIN ANGULAR MOMENTUM:**

 The electron also possesses spin motion and hence contributes to the total angular momentum. It is denoted by S. It follows the same commutation relations as those of orbital angular momentum.

 [Sx , Sy] = i*ћ*Sz

[Sy , Sz] = i*ћ*Sx

[Sz , SX] = i*ћ*Sy

And

 [S2, Sx] = 0

[S2 , Sy] = 0

[S2, Sz] = 0

Where

 S2 = Sx2 + S2y + Sz2

**THE TOTAL ANGULAR MOMENTUM OPERATORS:**

 The total angular momentum which may include the spin contribution is conveniently denoted by

 J = (Jx  , Jy , Jz)

 The total angular momentum is defined as generalized angular momentum operator J as any Hermitian operator whose components, satisfy the commutation rule

 [Jx, Jy] = i*ћ*Jz

[Jy, Jz] = iћJx

[Jz, JX] = i*ћ*Jy

In compact form, we can write,

 J X J = i*ћ*J

**LADDER OPERATORS:**

 Two new operators J+and J-are known as ladder operators and can be defined as

 J+ = Jx + iJy

J- = Jx - iJy

 Ladder operators do not commute but J2 commute with both ladder operators

 i.e., [J2, J±] =0

 also [J2, J±]= ± ћ J±

 [J+,J-] = 2 ћJz

**EIGEN VALUES OF LZ:**

To find out the Eigen values of Lz we need to satisfy the relation

 Lz ѱ = kѱ

 -iћ = kѱ

 The general solution of this equation be

Ѱ=ikφ/ћ

 ( is a function of γ and ϴ)

 An increment of 2π in φ does not change the wave function

ikφ/ћ = ik(φ+2π)/ћ

 Or ik.2π/ћ = 1

 Or = 2mπ

 Or k=mћ

Where m is an integer. Thus Eigen values of Lz are given by

 Lz = mћ

And the Eigen function be

Ѱimφ