Unit : 2

Curve fitting

Least square Curve fitting

Straight line $\Rightarrow y = ax+b$

Parabola $\Rightarrow y = a+bx+cx^2$

exponential $\Rightarrow ae^{bx}$

power $\Rightarrow y = ax^b$

Principle of least square :-

"The sum of squares of residues Should be minimum"

$E = d_1^2 + d_2^2 + d_3^2 + \dots + d_n^2$ (Should be minimum)

$d \rightarrow$ residual

= Real value - expected value

Straight line :-

Straight line Equ for the best fit } $y = ax+b$

Normal Equ for Straight line

$a\Sigma x + n.b = \Sigma y$

$$a \sum x^2 + b \sum x = \sum xy$$

$a, b \rightarrow$ are constant

$n \Rightarrow$ set of conservation

1. Fit a straight line using the method of least squares to the following data.

x	1	2	3	4	5
y	14	27	40	55	68

estimate the value of y when $x = 6$

Ans:

Straight line Equation for the best fit

$$y = ax + b \longrightarrow (1)$$

Normal Equation for straight line

$$a \sum x + n \cdot b = \sum y \longrightarrow (2)$$

$$a \sum x^2 + b \sum x = \sum xy \longrightarrow (3)$$

$n = 5$

x	y	xy	x^2
1	14	14	1
2	27	54	4
3	40	120	9
4	55	220	16
5	68	340	25
$\sum x = 15$	$\sum y = 204$	$\sum xy = 748$	$\sum x^2 = 55$

Sub $\Sigma x, \Sigma y, \Sigma xy, \Sigma x^2$ in Equ (2) & (3)

$$15a + 5b = 204 \longrightarrow (4)$$

$$55a + 15b = 748 \longrightarrow (5)$$

Equ (4) × 3 ⇒ $45a + 15b = 612$

$$55a + 15b = 748$$

$$(-) \quad (-) \quad (-)$$

$$-10a = -136$$

$$a = \frac{136}{10}$$

$$\boxed{a = 13.6}$$

Sub (a) Value in Equ (4)

$$15a + 5b = 204$$

$$15(13.6) + 5b = 204$$

$$204 + 5b = 204$$

$$5b = 204 - 204$$

$$\boxed{b = 0}$$

Sub (a) & (b) Value in Equ (1)

$$y = ax + b$$

$$y = 13.6x + 0$$

when $x = 6 \Rightarrow y = 13.6(6)$

$$\boxed{y = 81.6}$$

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Use the method of least squares to fit a straight line to the following data.

x	1	2	3	4	5
y	16	19	23	26	30

Ans:

Straight line Equation for the best fit

$$y = ax + b \longrightarrow (1)$$

Normal Equation for the straight line

$$a \Sigma x + n b = \Sigma y \longrightarrow (2)$$

$$a \Sigma x^2 + b \Sigma x = \Sigma xy \longrightarrow (3)$$

$n = 5$

x	y	xy	x^2
1	16	16	1
2	19	38	4
3	23	69	9
4	26	104	16
5	30	150	25

$\Sigma x = 15$ $\Sigma y = 114$ $\Sigma xy = 377$ $\Sigma x^2 = 55$

sub Σx, Σy, Σxy Σx^2 in Equ (2) & (3)

$$15a + 5b = 114 \longrightarrow (4)$$

$55a + 15b = 377 \longrightarrow (5)$

$\text{Equ }(4) \times 3 \Rightarrow 44a + \cancel{15b} = 342$

$55a + \cancel{15b} = 377$

$(-) \quad (-) \quad (-)$

$-11a = -35$

$a = \frac{-35}{-11}$

$\boxed{a = 3.1818}$

Sub (a) in Equ (4)

$15(3.1818) + 5b = 114$

$5b = 114 - 47.727$

$b = \frac{66.273}{5}$

$\boxed{b = 13.2546}$

Sub (a) & (b) in Equ (1)

$y = ax + b$

$\boxed{y = 3.1818x + 13.2546}$

2. Use the method of least square to fit a straight line to the following data

x	0	5	10	15	20
y	7	11	16	20	26

Estimate the value of y when $x = 25$

Ans:-

Straight line Equation for the best fit

$$y = ax + b \longrightarrow (1)$$

Normal Equation for the straight line

$$a\Sigma x + n.b = \Sigma y \longrightarrow (2)$$

$$a\Sigma x^2 + b\Sigma x = \Sigma xy \longrightarrow (3)$$

$$n = 5$$

x	y	xy	x^2
0	7	0	0
5	11	55	25
10	16	160	100
15	20	300	225
20	26	520	400
$\Sigma x = 50$	$\Sigma y = 80$	$\Sigma xy = 1035$	$\Sigma x^2 = 750$

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Sub Σx, Σy, Σxy, Σx^2 in Equ (2) & (3)

$$50a + 5b = 80 \longrightarrow (4)$$

$$750a + 50b = 1035 \longrightarrow (5)$$

$$(10) \times 4 \Rightarrow \quad 500a + \cancel{50b} = 800$$

$$750a + \cancel{50b} = 1035$$

$$(-) \quad (-) \quad (-)$$

$$\cancel{-}250a = \cancel{-}235$$

$$a = \frac{235}{250}$$

$$\boxed{a = 0.94}$$

Sub (a) in Equ (4)

$$50a + 5b = 80$$

$$50(0.94) + 5b = 80$$

$$47 + 5b = 80$$

$$5b = 80 - 47$$

$$5b = 33$$

$$b = 33/5$$

$$\boxed{b = 6.6}$$

a & b value sub in Equ (1)

$$y = ax + b$$

$$y = 0.94x + 6.6$$

when $x = 25$

$$y = 0.94(25) + 6.6$$

$$= 23.5 + 6.6$$

$$\boxed{y = 30.1}$$

3. Find the least square curve $y = ax + \frac{b}{x}$ for following data:

x	1	2	3	4
y	-1.5	0.99	3.88	7.66

Ans:

$$y = ax + \frac{b}{x}$$

$$y = \frac{ax^2 + b}{x}$$

$$xy = ax^2 + b$$

$$\boxed{Y = aX + b} \longrightarrow (1)$$

(Straight line Equation)

Normal Equation for Straight line

$$a \Sigma X + n.b = \Sigma Y$$

$$a \Sigma X^2 + b \Sigma X = \Sigma XY$$

$$n = 4$$

x	y	$X = x^2$	$Y = xy$	XY	X^2
1	-1.5	1	-1.5	-1.5	1
2	0.99	4	1.98	7.92	16
3	3.88	9	11.64	104.76	81
4	7.66	16	30.64	490.24	256
		$\Sigma X = 30$	$\Sigma Y = 42.76$	$\Sigma XY = 601.42$	$\Sigma X^2 = 354$

$$30a + 4b = 42.76 \longrightarrow (4)$$

$$354a + 30b = 601.42 \longrightarrow (5)$$

$$(4) \times 15 \Rightarrow \quad 450a + \cancel{60b} = 641.4$$

$$(5) \times 2 \Rightarrow \quad 708a + \cancel{60b} = 1202.84$$

$$(-) \qquad (-) \qquad (-)$$

$$\cancel{-}258a = \cancel{-}561.44$$

$$a = \frac{561.44}{258}$$

$$\boxed{a = 2.1761}$$

Sub (a) in Equ (4),

$$30a + 4b = 42.76$$

$$30(2.1761) + 4b = 42.76$$

$$65.283 + 4b = 42.76$$

$$4b = 42.76 - 65.283$$

$$b = \frac{-22.523}{4}$$

$$b = -5.6307$$

Sub a & b value in Equ (1)

$$Y = aX + b$$

$$Y = 2.1761X - 5.630$$

$$xy = 2.176x^2 - \frac{5.630}{x}$$

$$y = 2.176\frac{x^2}{x} - \frac{5.630}{x}$$

$$y = 2.176x - \frac{5.630}{x}$$

2 mark :-

Convert $y = \frac{ax+b}{x}$ into linear form !

$$y = \frac{ax+b}{x}$$

$$xy = ax + b$$

$$xy = Y \quad ; \quad X = x$$

$$\boxed{Y = aX + b}$$

4. The observed values of a function are respectively 168, 120, 72 and 73 at the four positions 3, 7, 9 and 10 of the independent variable what is the least estimate you can give to the value of function on the position 6 of the independent Variable ? (OR)

Finding the missing term the following data :

x	3	6	7	9	10
y	168	?	120	72	73

Ans: Straight line Equation for the best fit

$$y = ax + b \longrightarrow (1)$$

Normal Equation for the Straight line

$$a \Sigma x + n.b = \Sigma y \longrightarrow (2)$$

$$a \Sigma x^2 + b \Sigma x = \Sigma xy \longrightarrow (3)$$

$$\boxed{n = 4}$$

x	y	xy	x^2
3	168	504	9
7	120	840	49
9	72	648	81
10	73	730	100
$\Sigma x = 29$	$\Sigma y = 433$	$\Sigma xy = 2722$	$\Sigma x^2 = 239$

Sub Σx, Σy, Σxy, Σx^2 in Equ (2) & (3)

$$29a + 4b = 433 \longrightarrow (4)$$

$$239a + 29b = 2722 \longrightarrow (5)$$

$(4) \times 29 \Rightarrow \quad 841\,a + 116\,b = 12557$

$(5) \times 4 \Rightarrow \quad 956\,a + 116\,b = 10888$

$-115\,a = 1669$

$$a = \frac{1669}{-115}$$

$$\boxed{a = -14.513}$$

Sub. (a) in Equ (4)

$29(-14.513) + 4b = 433$

$-420.877 + 4b = 433$

$4b = 433 + 420.877$

$$b = \frac{853.877}{4}$$

$$\boxed{b = 213.469}$$

Sub (a) & (b) value Equ (1)

$y = ax + b$

$y = -14.513x + 213.469$

When $x = 6$

$y = -14.513(6) + 213.469$

$y = -87.078 + 213.469$

$$\boxed{y = 126.391}$$

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Fitting Exponential curve :

Let $y = ae^{bx} \longrightarrow (1)$

$y = f(x)$

a, b are constants

Taking log on bothsides

$\log_{10} y = \log_{10} (ae^{bx})$

$= \log_{10} a + bx \log_{10} e$

$\log A \cdot B = \log_{10} A + \log_{10} B$

$\log_{10} y = \log_{10} a + bx \longrightarrow (2)$ Equation of straight line

$Y = \log_{10} y$; $A = \log_{10} a$

$B = bx \log_{10}$ $X = x$

$$\boxed{Y = A + BX} \longrightarrow (3)$$

This is the Equation of the straight line in Y and X which can be easily fitted using the method of least squares.

The normal Equation

$B \Sigma X + n \cdot A = \Sigma Y \longrightarrow (4)$

$B \Sigma X^2 + A \Sigma X = \Sigma XY \longrightarrow (5)$

from Equ (4) & (5) we get A, B

To find a, b

$\log_{10} a = A$

Taking antilog on bothsides

$a = \text{Anti}\log A$

$b \log_{10} e = B$

$b \log_{10} e = B$

$b = \dfrac{B}{\log_{10} e}$ | $\log_{10} e = 0.4343$

$b = \dfrac{B}{0.4343}$

Sub a, b in Equ (1)

$\boxed{y = ae^{bx}}$ ⟹ Exponential curve

1. Fit a curve of the form $y = ae^{bx}$ to the following data

x	0	2	4
y	5.012	10	31.62

Ans:-

To fit a exponetial curve

$y = ae^{bx} \longrightarrow (1)$

Taking log on bothsides

$\log_{10} y = \log_{10} a + \log_{10} e^{bx}$

$\log_{10} y = \log_{10} a + bx \log_{10} e$

$Y = A + BX \longrightarrow (2)$

$Y = \log_{10} y$; $A = \log_{10} a$

$X = x$; $B = bx \log_{10} e$

The normal Equation for straight line

$$B \sum X + n \cdot A = \sum Y \longrightarrow (3)$$

$$B \sum X^2 + A \sum X = \sum XY \longrightarrow (4)$$

$\boxed{n = 3}$

x	y	$X = x$	$Y = \log_{10} y$	X^2	XY
0	5.012	0	0.7	0	0
2	10	2	1.0	4	2
4	31.62	4	1.5	16	6
		$\sum X = 6$	$\sum Y = 3.2$	$\sum X^2 = 20$	$\sum XY = 8$

Equ (3) & (4) becomes

$$6B + 3A = 3.2 \longrightarrow (5)$$

$$20B + 6A = 8 \longrightarrow (6)$$

Equ (5) × 2 ⇒ $12B + \cancel{6A} = 6.4$

$$20B + \cancel{6A} = 8$$

$$(-) \quad (-) \quad (-)$$

$$\cancel{-}8B = \cancel{-}1.6$$

$$B = \frac{1.6}{8}$$

$\boxed{B = 0.2}$

Sub B in Equ (5)

$\cancel{6(5) + 3A = 3.2}$

$\cancel{35 + 3A = 3.2}$

$\cancel{3A = 3.2 - 35}$

$6(0.2) + 3A = 3.2$

$1.2 + 3A = 3.2$

$3A = 3.2 - 1.2$

$3A = 2$

$A = 2/3$

$\boxed{A = 0.666}$

To find a, b

$\log_{10} a = \text{Anti} \log A$

$= \text{anti} \log (0.666)$

$a = $

$b = B$

$\boxed{b = 0.2}$

a & b value sub equ (1)

$y = ae^{bx}$

$y =$

Power fit:- (fitting the curve $y = ax^b$)

To fit the curve of the form $y = ax^b$ to the given data.

$y = ax^b \longrightarrow (1)$

Taking log on both sides	$\log AB = \log A + \log B$
$\log_{10} y = \log_{10} (ax^b)$	$\log x^n = n \log x$
$= \log_{10} a + \log_{10} x^b$	

$\log_{10} y = \log_{10} a + b \log_{10} x \longrightarrow (2)$

put $\log_{10} y = Y$ $\quad$ $\log_{10} a = A$

$\log_{10} x = X$ $\quad$ $b = B$

Equ (2) becomes

$$Y = A + BX \longrightarrow (3)$$

This is straight line Equation

* Straight line curve is easily fitted using least squares method.

Normal Equation for Equ (2)

$$B \Sigma X + n \cdot A = \Sigma X \longrightarrow (4)$$

$$B \Sigma X^2 + A \Sigma X = \Sigma XY \longrightarrow (5)$$

from Equ (4) & (5)

Constants A and B are determined

To find a, b

$\log_{10} a = A$

Take on anti log

$a = \text{Anti log } A$

$b = B$

$y = \text{Anti log } Y$

$\log_{10} x = X$

$x = \text{Anti log } X$

Sub Equ a, b, x y in Equ (1)

$\boxed{y = ax^b}$ curve is fitted

1. Fit a ~~curve~~ proper curve of the form $y = ax^b$ to the following data

x	61	26	7	2.6
y	350	400	500	600

Ans:-

The curve of the form $y = ax^b \longrightarrow (1)$

Taking log on both sides

$$\log_{10} y = \log_{10} (ax^b)$$

$$\log_{10} y = \log_{10} a + \log_{10} x^b$$

$$\log_{10} y = \log_{10} a + b \log_{10} x$$

Put $Y = \log_{10} y$

$X = \log_{10} x$

$A = \log_{10} A$

$B = b$

$$Y = A + BX \longrightarrow (2)$$

Normal Equation

$$B \Sigma X + n.A = \Sigma Y \longrightarrow (3)$$

$$B \Sigma X^2 + A \Sigma X = \Sigma XY \longrightarrow (4)$$

$$n = 4$$

x	y	$X = \log_{10} x$	$Y = \log_{10} y$	XY	X^2
61	350				
26	400				
7	500				
2.6	600				
		$\Sigma X = 4.460$	$\Sigma Y = 10.623$	$\Sigma XY = 11.656$	$\Sigma X^2 = 6.075$

Equ (3) & (4) becomes

$$4.4602\,B + 4A = 10.6230 \longrightarrow (5)$$

$$6.0753\,B + 4.4602\,A = 11.6567 \longrightarrow (6)$$

$$(5) \times 4.4602 \Rightarrow 19.8933\,B + \cancel{17.8408\,A} = 47.3807$$

$$(6) \times 4 \Rightarrow \underline{24.3012\,B + \cancel{17.8408\,A} = 46.6268}$$

$$(-) \qquad (-) \qquad (-)$$

$$-4.40796\,B = 0.7539$$

$$B = \frac{0.7539}{-4.40796}$$

$$\boxed{B = -0.1710}$$

Sub (B) in (5)

$$4.4602 \times -0.1710 + 4A = 10.6230$$

$$-0.7626 + 4A = 10.6230$$

$$4A = 10.6230 + 0.7626$$

$$A = \frac{11.3856}{4}$$

$$A = 2.8464$$

To find a, b

$$\log_{10} a = \text{anti log } A$$

$$= \text{anti log } (2.8464)$$

$$\boxed{a = 701.4552}$$

$$b = B$$

$$\boxed{b = -0.1710}$$

a & b value sub in Equ (1)

$$y = ax^b$$

$$y = 701.455\, x^{-0.1710}$$

2. Using the method of least square fit a curve of the form $y = ab^x$ to the following data

x	1	2	3	4
y	4	11	35	100

Ans:

The curve of the form

$$y = ab^x \longrightarrow (1)$$

Taking log on both sides

$$\log_{10} y = \log_{10} a + \log_{10} b^x$$

$$\log_{10} y = \log_{10} a + x \log_{10} b$$

$$Y = A + BX \longrightarrow (2)$$

$\log_{10} y = Y$ $\qquad$ $\log_{10} a = A$

$\log_{10} x = X$ $\qquad$ $b = B$

Normal Equation

$$B \Sigma X + n.A = \Sigma Y \longrightarrow (3)$$

$$B \Sigma X^2 + A \Sigma X = \Sigma XY \longrightarrow (4)$$

$n = 4$

x	y	$X = \log_{10} x$	$Y = \log_{10} y$	XY	X^2
1	4	0	0.602	0	0
2	11	0.301	1.041	0.313	0.097
3	35	0.477	1.544	0.736	0.541
4	100	0.602	2	1.204	1.449
		$\Sigma X = 1.38$	$\Sigma Y = 5.187$	$\Sigma XY = 2.253$	$\Sigma X^2 = 2.087$

Equ (3) & (4) becomes

$$1.38\,B + 4A = 5.187 \longrightarrow (5)$$

$$2.087\,B + 1.38\,B = 2.253 \longrightarrow (6)$$

$$\text{Equ (5)} \times 1.38 \Rightarrow 1.9044\,B + \cancel{5.52\,A} = 7.158$$

$$\text{Equ (6)} \times 4 \Rightarrow \underset{(-)}{8.348\,B} \underset{(-)}{+ \cancel{5.52\,A}} = \underset{(-)}{9.012}$$

$$\cancel{-}\,6.443\,B = \cancel{-}\,1.854$$

$$B = \frac{1.854}{6.443}$$

$$\boxed{B = 0.287}$$

Sub. B in Equ (5)

$$1.38 \times 0.287 + 4A = 5.187$$

$$0.396 + 4A = 5.187$$

$$4A = 5.187 - 0.396$$

$$A = \frac{4.791}{4}$$

$$\boxed{A = 1.197}$$

To find a, b

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$\log_{10} a = \text{anti}\log A$ $\quad\quad b = B$

$= \text{anti}\log(1.197)$ $\quad\quad b = 0.287$

$a = 15.739$

$a \& b$ value sub in Equ (1)

$$y = ab^x$$

$$y = (15.739)(0.287)^x$$

3. Using the principle of least squres fit an Equation of the form $y = ae^{bx}$ to given data.

x	1	2	3	4
y	1.65	2.70	4.50	7.35

Ans: The curve of the form

$$y = ae^{bx} \longrightarrow (1)$$

Taking log on both sides

$$\log_{10} y = \log_{10} a + \log_{10} e^{bx}$$

$$\log_{10} y = \log_{10} a + bx \log_{10} e$$

Put $\log_{10} y = Y$; $\log_{10} a = A$

$\log_{10} x = X$ $\quad\quad b = B$

$$Y = A + BX \longrightarrow (2)$$

Normal Equation

$$B \Sigma X + n \cdot B = \Sigma Y \longrightarrow (3)$$

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$B \Sigma X^2 + A \Sigma X = \Sigma XY \longrightarrow (4)$

$n = 4$

x	y	$X = \log_{10} x$	$Y = \log_{10} y$	XY	X^2
1	1.65	0	0.217	0	0
2	2.70	0.301	0.431	0.129	0.016
3	4.50	0.477	0.653	0.311	0.096
4	7.35	0.602	0.866	0.521	0.271
		$\Sigma X = 1.38$	$\Sigma Y = 2.167$	$\Sigma XY = 0.961$	$\Sigma X^2 = 0.383$

Equ (3) & (4) becomes

$1.38B + 4A = 2.167 \longrightarrow (5)$

$0.383B + 1.38A = 0.961 \longrightarrow (6)$

Equ (5) $\times$ 1.38 $\Rightarrow$ $1.9044B + 5.52B = 2.990$

Equ (6) $\times$ 4 $\Rightarrow$ $1.532B + 5.52B = 3.844$

$\quad (-) \quad (-) \quad (-)$

$0.372B = -0.854$

$B = \frac{-0.854}{0.372}$

$B = -2.295$

Sub (B) in Equ (5)

$1.38 \times -2.295 + 4A = 2.167$

$-3.1671 + 4A = 2.167$

$4A = 2.167 + 3.1671$

$A = \cancel{2.16}\ \frac{5.334}{4}$

$A = 1.333$

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To find a, b

$\log_{10} a = \text{anti} \log A$

$= \text{anti} \log (1.333)$

$a = 21.527$

$f(x) = (x-x_1)(x-x_2)(x-x$

Interpolation :

→ Lagrange's interpolation (Non neccessarily on Equal space).

→ Newton's forward interpolation } Equally Spaced

→ Newton's backward interpolation

Lagrange's Interpolation :

$$f(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} y_0 + \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)} y_1 + \vdots +$$

$$\frac{(x-x_0)(x-x_1)\ldots\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\ldots(x_n-x_{n-1})}\,y_n$$

1. Using Lagrange's interpolation formula find the value of y corresponding to $x=10$ from the following table

x	5	6	9	11
y	12	13	14	16

Ans:-

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)\cancel{(x-x_4)}}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}\,y_0 +$$

$$\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}\,y_1 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}\,y_2 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}\,y_3$$

$x_0 = 5$ $\quad$ $y_0 = 12$

$x_1 = 6$ $\quad$ $y_1 = 13$

$x_2 = 9$ $\quad$ $y_2 = 14$

$x_3 = 11$ $\quad$ $y_3 = 16$

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To find a, b

$\log_{10} a = \text{anti log } A$ $\qquad b = B$

$= \text{Anti log } (1.333)$ $\qquad b = -2.295$

$a = 21.527$

a & b value sub in Equ (1)

$y = ae^{bx}$

$y = 21.527 e^{-2.295x}$

$$f(x) = \frac{(x-6)(x-9)(x-11)}{(5-6)(5-9)(5-11)} \times 12 + \frac{(x-5)(x-9)(x-11)}{(6-5)(6-9)(6-11)} \times 13 + \frac{(x-5)(x-6)(x-11)}{(9-5)(9-6)(9-11)} \times 14 + \frac{(x-5)(x-6)(x-9)}{(11-5)(11-6)(11-9)} \times 16$$

$$f(x=10) = \frac{(10-6)(10-9)(10-11)}{(-1)(-4)(-6)} \times 12 + \frac{(10-5)(10-9)(10-11)}{(+1)(-3)(-5)} \times 13 + \frac{(10-5)(10-6)(10-11)}{(4)(3)(-2)} \times 14 + \frac{(10-5)(10-6)(10-9)}{(6)(5)(2)} \times 16$$

$$\Rightarrow \frac{4 \times 1 \times -1 \times 12}{-24} + \frac{5 \times 1 \times -1 \times 13}{15} + \frac{5 \times 4 \times -1 \times 14}{-24} + \frac{5 \times 4 \times 1 \times 16}{60}$$

$$\Rightarrow 2 - \frac{13}{3} + \frac{35}{3}$$

$$= 14.66$$

Inverse Interpolation formula :

The process of finding x for a given value of y is called inverse interpolation.

$$f(y) = \frac{(y-y_1)(y-y_2)(y-y_3)}{(y_0-y_1)(y_0-y_2)(y_0-y_3)} x_0 +$$

$$\frac{(y-y_0)(y-y_2)(y-y_3)}{(y_1-y_0)(y_1-y_2)(y_1-y_3)} x_1 +$$

$$\frac{(y-y_0)(y-y_1)(y-y_3)}{(y_2-y_0)(y_2-y_1)(y_2-y_3)} x_2 +$$

$$\frac{(y-y_0)(y-y_1)(y-y_2)}{(y_3-y_0)(y_3-y_1)(y_3-y_2)} x_3 .$$

x	1.2	2.1	2.8	4.1	4.9	6.2
y	4.2	6.8	9.8	13.4	15.5	19.6

find the value of x corresponding to $y = 12$ using Lagrange's state

Ans:-
$$f(y) = \frac{(y-y_1)(y-y_2)(y-y_3)(y-y_4)(y-y_5)}{(y_0-y_1)(y_0-y_2)(y_0-y_3)(y_0-y_4)(y_0-y_5)} x_0 +$$

$$\frac{(y-y_0)(y-y_2)(y-y_3)(y-y_4)(y-y_5)}{(y_1-y_0)(y_1-y_2)(y_1-y_3)(y_1-y_4)(y_1-y_5)} x_1 +$$

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Ans: 3.55

$$\frac{(y-y_0)(y-y_1)(y-y_3)(y-y_4)(y-y_5)}{(y_2-y_0)(y_2-y_1)(y_2-y_3)(y_2-y_4)(y_2-y_5)}x_2 +$$

$$\frac{(y-y_0)(y-y_1)(y-y_2)(y-y_4)(y-y_5)}{(y_3-y_0)(y_3-y_1)(y_3-y_2)(y_3-y_4)(y_3-y_5)}x_3 +$$

$$\frac{(y-y_0)(y-y_1)(y-y_2)(y-y_3)(y-y_5)}{(y_4-y_0)(y_4-y_1)(y_4-y_2)(y_4-y_3)(y_4-y_5)}x_4 +$$

$$\frac{(y-y_0)(y-y_1)(y-y_2)(y-y_3)(y-y_4)}{(y_5-y_0)(y_5-y_1)(y_5-y_2)(y_5-y_3)(y_5-y_4)}x_5$$

$y_0 = 4.2$ $\quad x_0 = 1.2$

$y_1 = 6.8$ $\quad x_1 = 2.1$

$y_2 = 9.8$ $\quad x_2 = 2.8$

$y_3 = 13.4$ $\quad x_3 = 4.1$

$y_4 = 15.5$ $\quad x_4 = 4.9$

$y_5 = 19.6$ $\quad x_5 = 6.2$

$$f(y) = \frac{(y-6.8)(y-9.8)(y-13.4)(y-15.5)(y-19.6)}{(4.2-6.8)(4.2-9.8)(4.2-13.4)(4.2-15.5)(4.2-19.6)}(1.2)$$

$$+ \frac{(y-4.2)(y-9.8)(y-13.4)(y-15.5)(y-19.6)}{(6.8-4.2)(6.8-9.8)(6.8-13.4)(6.8-15.5)(6.8-19.6)} \times 2.1$$

$$+ \frac{(y-4.2)(y-6.8)(y-13.4)(y-15.5)(y-19.6)}{(9.8-4.2)(9.8-6.8)(9.8-13.4)(9.8-15.5)(9.8-19.6)} \times 2.8$$

$$+\frac{(y-4.2)(y-6.8)(y-9.8)(y-15.5)(y-19.6)}{(13.4-4.2)(13.4-6.8)(13.4-9.8)(13.4-15.5)(13.4-19.6)}\times 4.1$$

$$+\frac{(y-4.2)(y-6.8)(y-9.8)(y-13.4)(y-19.6)}{(15.5-4.2)(15.5-6.8)(15.5-9.8)(15.5-13.4)(15.5-19.6)}\times 4.9$$

$$+\frac{(y-4.2)(y-6.8)(y-9.8)(y-13.4)(y-15.5)}{(19.6-4.2)(19.6-6.8)(19.6-9.8)(19.6-13.4)(19.6-15.5)}\times 6.2$$

$$f(y=12) \Rightarrow \frac{(12-6.8)(12-9.8)(12-13.4)(12-15.5)(12-19.6)}{(-2.6)\times -5.6\times -9.2\times -11.3\times -15.4}\times 1.2$$

$$+\frac{(12-4.2)(12-9.8)(12-13.4)(12-15.5)(12-19.6)}{2.6\times -3\times -6.6\times -8.7\times -12.8}\times 2.1$$

$$+\frac{(12-4.2)(12-6.8)(12-13.4)(12-15.5)(12-19.6}{5.6\times 3\times -3.8\times -5.7\times -9.8}\times 2.8$$

$$+\frac{(12-4.2)(12-6.8)(12-9.8)(12-15.5)(12-19.6)}{(9.2\times 6.6\times 3.6\times -2.1\times -6.2}\times 4.1$$

$$+\frac{(12-4.2)(12-6.8)(12-9.8)(12-13.4)(12-19.6)}{11.3\times 8.7\times 5.7\times 2.1\times -4.1}\times 4.9$$

$$+\frac{(12-4.2)(12-6.8)(12-9.8)(12-13.4)(12-15.5)}{15.4\times 12.8\times 9.8\times 6.2\times -4.1}\times 6.2$$

$$\Rightarrow \frac{5.2\times 2.2\times -1.4\times -3.5\times -7.6\times 1.2}{-23310.32704} + 7.8\times 2.2\times -1.4\times -3.5\times$$

$y_p = y_0 + P \frac{\Delta y_0}{1!}$

Newton's forward Interpolation formula:

$$y_P = y_0 + P\frac{\Delta y_0}{1!} + P(P-1)\frac{\Delta^2 y_0}{2!} +$$

$$P(P-1)(P-2) \frac{\Delta^3 y_0}{3!} + P \dots (P-\overline{n-1}) \frac{\Delta^n y_0}{n!}$$

Newton's backward Interpolation formula:

$$y_p = y_n + P \Delta y_n + \frac{P(P+1)}{2!} \nabla^2 y_n +$$

$$\frac{P(P+1)(P+2)}{3!} \nabla^3 y_n + \dots$$

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1. Construct newton's forward interpolation polynomial for the following data

	x_0	x_1	x_2	x_3
$x =$	4	6	8	10
y	1	3	8	16

use it to find the value of y for $x = 5$

Ans: $h = 2$

$$p = \frac{x - x_0}{h} = \frac{5-4}{2} = \frac{1}{2} = 0.5$$

$$p = 0.5$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
4	1			
		2		
6	3		3	
		5		0
8	8		3	
		8		
10	16			

$$y_p = y_0 + p\frac{\Delta y_0}{1!} + p(p-1)\frac{\Delta^2 y_0}{2!} + p(p-1)(p-2)\frac{\Delta^3 y_0}{3!} + p(p-1)\dots(p-\overline{n-1})\frac{\Delta^n y_n}{n!}$$

$$y_{(0.5)} = 1 + \frac{0.5(2)}{1} + \frac{0.5(0.5-1)(3)}{2!} + \frac{0.5(0.5-1)(0.5-2)(0)}{3!}$$

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$= 1 + 1 - \frac{0.75}{2} + 0$

$= 2 - 0.375$

$y_{0.5} = 1.625$

2. Find the Value of y from the following data at $x = 2.65$

x	-1	0	1	2	3
y	-21	6	15	12	3

Newton backward interpolation formula

$$y_p = y_n + p\Delta y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \cdots$$

where $p = \frac{x - x_n}{h}$ $\boxed{h=1}$

$p = \frac{2.65 - 3}{1} = -0.35$

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
-1	-21				
		27			
0	6		-18		
		9		6	
1	15		-12		0
		-3		6	
2	12		-6		
		-9			
3	3				

$$y_{(-0.35)} = 3 + (-0.35)(-9) + \frac{(-0.35)(-0.35+1)}{2!} \times -6$$
$$+ \frac{(-0.35)(-0.35+1)(-0.35+2)}{3!}(6)$$

$$= 3 + 3.15 + \frac{(-0.35)(0.65)(-6)}{2!} + \frac{(-0.35)(0.65)(1.65)(6)}{1 \times 2 \times 3}$$

$$= 3 + 3.15 + 0.6825 - 0.3753$$

$$y_{(-0.35)} = 6.4572$$

3. A function $f(x)$ is given by the following find $f(0.2)$ by suitable formulae:

Ans:- 177.8

x	0	1	2	3	4	5	6
y	176	185	194	203	212	220	229

Ans:- The difference table

x	y	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$	$\Delta^5 y_0$	$\Delta^6 y_0$
0	176						
		9					
1	185		0				
		9		0			
2	194		0		0		
		9		0		-0	
3	203		0		-1		5
		9		-1		4	
4	212		-1		3		
		8					
5	220		1				
		9					
6	229						

$x_0 = 0 \quad y_0 = 176$

h = interval size

$x_0 + ph = x$

$ph = x - x_0$

$\boxed{h = 1}$

$h = x_1 - x_0$

$P = \frac{x - x_0}{h}$

$P = \frac{x - x_0}{h} = \frac{0.2 - 0}{h}$

$\boxed{P = 0.2}$

$$f(x_0+ph) = y_0 + \frac{P\Delta y_0}{1!} + P(P-1)\frac{\Delta^2 y_0}{2!} + P(P-1)(P-2)\frac{\Delta^3 y_0}{3!} + \dots + \frac{P(P-1)(P-2)(P-3)(P-4)}{5!}\Delta^5 y_0$$

$$f(0+0.2) = 176 + \frac{(0.2)9}{1} + \frac{(0.2)(0.2-1)}{2!}0 + 0 + 0 + \frac{(0.2)(0.2-1)(0.2-2)(0.2-3)(0.2-4)}{5!}(-1) + \frac{(0.2)(0.2-1)(0.2-2)(0.2-3)(0.2-4)(0.2-5)}{6!}(5)$$

$$f(0.2) = 176 + 1.8 + 0 + 0 + 0 + \frac{(0.2)(-0.8)(-1.8)(-2.8)(-3.8)(-1)}{1\times2\times3\times4\times5} + \frac{(0.2)(-0.8)(-1.8)(-2.8)(-3.8)(-3)}{1\times2\times3\times4\times5\times6}(5)$$

$$f(0.2) = 176 + 1.8 + 0 + 0 + 0 - \frac{3.06432}{120} \quad \frac{9.19296}{720}$$

$$= 176 + 1.8 + 0 + 0 + 0 - 0.02553 - 0.01276$$

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$$\boxed{f(0.2) = 177.8}$$

Using Newton_ Gregory backward formula find $e^{-1.9}$ from the following data.

x	1.00	1.25	1.50	1.75	2.00
y	0.3679	0.2865	0.2231	0.1738	0.1353

Ans:-

Backward difference table

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1.00	0.3679				
		-0.0814			
1.25	0.2865		0.0180		
		-0.0634		-0.0039	
1.50	0.2231		0.0141		0.0006
		-0.04		-0.0033	
1.75	0.1738		0.0108		
		-0.0385			
2.00	0.1353				

Here $x_n = 2$, $y_n = 0.1353$ from the difference table.

$\nabla y_n = -0.0385$; $\nabla^2 y_n = 0.0108$;

$\nabla^3 y_n = -0.0033$, $\nabla^4 y_n = 0.0006$

Let $x = 1.9$: $p = \dfrac{x - x_n}{h} = -\dfrac{0.1}{0.25}$

$$\boxed{p = -0.4}$$

By newton's Backward ~~formula~~ interpolation formula

$$y(x_n+ph) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \dots$$

$$y(1.9) = 0.1353 + (-0.4)(0.0385) - \frac{(0.4)(0.6)}{2}(0.0108) + \frac{(0.4)(0.6)(1.6)}{6}(-0.0033) + \dots$$

$$\boxed{e^{-1.9} = 0.1496}$$

Unit - III

Solution of Equation

Newton's Method (or) Newton's Raphson method

Let $f(x)=0 \longrightarrow$ the Equation

Let x_0 be an approximate root

x_0+h be the exact root

where h is small correction

Let $x_1 = x_0+h$

Since x_0+h will satisfy the Equation

$$\boxed{f(x_0+h) = 0}$$

f (x_0+h) by Taylor's series ②

$$f(x_0+h) = f(x_0) + \frac{hf'(x_0)}{1!} + \frac{h^2 f''(x_0)}{2!} + \dots = 0$$

Since h is small

h^2 and higher power of h are neglected

$$f(x_0) + hf'(x_0) = 0$$

$$hf'(x_0) = -f(x_0)$$

$$h = \frac{-f(x_0)}{f'(x_0)} \longrightarrow (1) \qquad h = -\frac{f(x_0)}{f'(x_0)}$$

$h = -\frac{f(x_0)}{f'(x_0)}$ but $x_1 = x_0 + h$ $h = -\frac{f(x_0)}{f'}$ $h = -\frac{f(x_0)}{f'(x_0)}$

$$x_1 - x_0 = h$$

Equ (1) becomes Let $f(x)=0$

$$x_1 - x_0 = \frac{-f(x_0)}{f'(x_0)}$$

This may be the closer approximation to the root.

Similarly x_1, x_2, x_3 are calculated for better approximation.

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}.$$

The process of ~~the~~ iteration is repeated until the last two (or) three values of x are same or x converges.

3

In general

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}} \quad n = 0, 1, 2, \ldots$$

Newton's iteration formula (or) Newton's Raphson's formula

Note :- If a, b are two value of x

$f(a) < f(b)$

a is taken as first approximation.

1. Find the smallest positive root of the Equation

$x^3 - 2x + 0.5 = 0$.

Ans: Let $f(x) = x^3 - 2x + 0.5 = 0$

$$f'(x) = x^3 - 2$$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$f(a) \Rightarrow f(0) = 0 - 2(0) + 0.5 = 0.5$ (+ve)

$f(b) \Rightarrow f(1) = 1^3 - 2 + 0.5 = -0.5$ (-ve)

Hence root lies between 0 and 1

Initial guess $x_0 = 0$ $\quad |f(a)| < |f(b)|$

Newton's Rapshon formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

I^{st} approximation

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$f(x_0) = f(0) = 0.5$

$f'(x_0) = f'(0) = -2$

$$= 0 - \left(\frac{0.5}{-2}\right)$$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$$x_1 = 0.25$$

$x(1) = x(0) - \frac{f(x_0)}{f'(x_0)}$

2^{nd} approximation

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$f(2) = x(1) = \frac{f(x_1)}{f'(x_1)}$

$$= 0.25 - \frac{\left((0.25)^3 - 2\right) \times 0.25 + 0.5}{3(0.25)^2 - 2}$$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$$= 0.25 - \frac{0.015625 - 2 \times 0.25 + 0.5}{(3 \times 0.0625) - 2}$$

$$= 0.25 - \frac{0.015625}{0.1875 - 2}$$

$x_{n+1} = x_n$

$$= 0.25 - \frac{0.015625}{-1.8125}$$

$$= 0.25 + 0.008620$$

$$= 0.2586$$

x_n

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

3^{rd} approximation

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = 0.2586 - \frac{\left[(0.2586)^3 - 2(0.2586) + 0.5\right]}{3(0.2586)^2 - 2}$$

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$= 0.2586$

$x_2 = x_3$

Hence the smallest possible root $= 0.2586$

2. Evaluate $\sqrt{12}$ to four decimal place by newton's Raphson method.

Ans: Let $x = \sqrt{12}$

$x^2 = 12$

$x^2 - 12 = 0$

Let $f(x) = x^2 - 12 = 0$

$f'(x) = 2x$

Initial guess $x_0 = 3$

$f(3) = (3)^2 - 12 = 9 - 12 = -3$ (-ve)

$f(4) = (4)^2 - 12 = 16 - 12 = 4$ (+ve)

The root lies between 3 and 4

Also $|f(3)| < |f(4)|$

1st approximation $x_0 = 3$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 3 - \frac{(3^2 - 12)}{2 \times 3} \Rightarrow 3 - \frac{(9-12)}{6}$$

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$= 3 + \frac{3}{6} \Rightarrow 3 + 0.5$

$x_1 = 3.5$

2^{nd} approximation

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 3.5 - \frac{[(3.5)^2 - 12]}{2 \times 3.5}$$

$$= 3.5 - \frac{42.875 - 12}{7}$$

4.4107

$$= 3.5 + \frac{\cancel{54.875}\ 30.875}{7}$$

$= \cancel{3.5 + 7.8392}$

$= 3.4642$

3^{rd} approximation

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 3.4642 - \frac{[(3.4642)^2 - 12]}{2 \times 3.4642}$$

=

=

$= 3.4641$

4 th approximination

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 3.4641 - \frac{[(3.4641)^2 - 12]}{2 \times 3.4641}$$

$$= 3.4641 - \frac{11.9999 - 12}{6.9282}$$

$$= 3.4641 - \frac{-0.0001}{6.9282}$$

$$= 3.4641 + 0.00014$$

$$= 3.4641$$

$x_4 = x_3$ Hence the smallest possible root $= 3.4641$.

3. Establish an iteraction formulae to find the reciprogal of a positive number N by Newton's Raphson method:

$$x = \frac{1}{N}$$

$$\frac{1}{x} = N$$

$$\frac{1}{x} - N = 0$$

Let $f(x) = \frac{1}{x} - N = 0$

$$f'(x) = -1/x^2$$

The newton's formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{(1/x_n - N)}{(-1/x_n^2)}$$

2.141

$$= x_n + x_n^2 (1/x_n - N)$$

$$= x_n + x_n - N x_n^2$$

$$= x_n (2 - N x_n^2)$$

4.

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Gauss Elimination Method :

1. Solve the Equation $2x + y + 4z = 12$

$$8x - 3y + 2z = 20$$

$$4x + 11y - z = 33$$

by gauss Elimination method.

Ans:

The given system is Equivalent to $AX = B$

$$A = \begin{pmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{pmatrix}, \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad B = \begin{pmatrix} 12 \\ 20 \\ 33 \end{pmatrix}$$

Principle : Reduce (A/B) to (U/K)

$$(A/B) \xrightarrow{\text{Gauss Elimination}} (U/K)$$

$$(A/B) = \left(\begin{array}{ccc|c} 2 & 1 & 4 & 12 \\ 8 & -3 & 2 & 20 \\ 4 & 11 & -1 & 33 \end{array}\right)$$

$R_2 \to R_2 - 4R_1 \Rightarrow$

$$\begin{array}{cccc} 8 & -3 & 2 & 20 \\ 8 & 4 & 16 & 4 \\ (-) & (-) & (-) & (-) \\ \hline 0 & -7 & -14 & - \end{array}$$

$$\sim \left(\begin{array}{ccc|c} 2 & 1 & 4 & 12 \\ 0 & -7 & -14 & -28 \\ 0 & 9 & -9 & 9 \end{array}\right) \quad \begin{array}{l} R_2 \to R_2 - 4R_1 \\ R_3 \to R_3 - 2R_1 \end{array}$$

$$\sim \left(\begin{array}{ccc|c} 2 & 1 & 4 & 12 \\ 0 & -7 & -14 & -28 \\ 0 & 0 & -27 & -27 \end{array}\right) \quad R_3 \to R_3 + 9/7 \cdot R_2$$

Upper triangular system of Equation is

$$2x + y + 4z = 12 \longrightarrow (1)$$

$$-7y - 14z = -28 \longrightarrow (2)$$

$$-27z = -27 \longrightarrow (3)$$

$$z = \cancel{-27}/\cancel{-27}$$

$$\boxed{z = 1}$$

By back substitution

sub $z=1$ in Equ (2)

$$-7y - 14(1) = -28$$

$$-7y = -28 + 14$$

$$y = \cancel{-14}^{2}/\cancel{-7}$$

$$\boxed{y = 2}$$

sub $y = 2$, $z=1$ in Equ (1)

$$2x + 2 + 4(1) = 12$$

$$2x = 12 - 2 - 4$$

$$2x = 6$$

$$x = \cancel{6}^{3}/\cancel{2}$$

$$\boxed{x = 3}$$

Solutions $x = 3$

$y = 2$

$z = 1$

Gauss - Jordan Method :

1. Solve the Equation $2x+y+4z=12$, $8x-3y+2z=20$, $4x+11y-z=33$ by Gauss - Jordan method.

Ans: Principle : Reduce (A/B) to (D/K) (or) (I/K)

$$(A/B) \xrightarrow[\text{method}]{\text{Gauss Jordan}} (D/K) \text{ (or) } (I/K)$$

The given system is equivalent to $AX=B$

$$A=\begin{pmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{pmatrix}, \quad X=\begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad B=\begin{pmatrix} 12 \\ 20 \\ 33 \end{pmatrix}$$

$$(A/B)=\left(\begin{array}{ccc|c} 2 & 1 & 4 & 12 \\ 8 & -3 & 2 & 20 \\ 4 & 11 & -1 & 33 \end{array}\right)$$

$$\begin{array}{cccc} 8 & -3 & 2 & 20 \\ 8 & 4 & 16 & 48 \\ (-) & & & (-) \\ \hline 0 & -7 & -14 & -28 \end{array}$$

$$\sim\left(\begin{array}{ccc|c} 2 & 1 & 4 & 12 \\ 0 & -7 & -14 & 28 \\ 0 & 9 & -9 & 9 \end{array}\right) \quad \begin{array}{l} R_2 \to R_2 - 4R_1 \\ R_3 \to R_3 - 2R_1 \end{array}$$

$$\sim\left(\begin{array}{ccc|c} 2 & 1 & 4 & 12 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & -1 & 1 \end{array}\right) \quad \begin{array}{l} R_2 \to R_2 \times -1/7 \\ R_3 \to R_3 \times 1/9 \end{array}$$

$$\sim\left(\begin{array}{ccc|c} 2 & 0 & 2 & 8 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -3 & 3 \end{array}\right) \quad \begin{array}{l} R_1 \to R_1 - R_2 \\ R_3 \to R_3 - R_2 \end{array}$$

$$\sim \left(\begin{array}{ccc|c} 2 & 0 & 2 & 8 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 1 \end{array}\right) \quad R_3 \to R_3 \times -1/3$$

$$\sim \left(\begin{array}{ccc|c} 2 & 0 & 0 & 6 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right) \quad \begin{array}{l} R_1 \to R_1 - 2R_3 \\ R_2 \to R_2 - 2R_3 \end{array}$$

from then $2x = 6 \Rightarrow x = 6/2 = x = 3$

$y = 2$

$z = 1$

Solution:- $x = 3$, $y = 2$, $z = 1$

HW. Gauss Elimination: method

$5x_1 + x_2 + x_3 + x_4 = 4$; $x_1 + 7x_2 + x_3 + x_4 = 12$

$x_1 + x_2 + 6x_3 + x_4 = -5$; $x_1 + x_2 + x_3 + 4x_4 = -6$.

Ans: The given system is Equivalent to $AX = B$

$$A = \begin{pmatrix} 5 & 1 & 1 & 1 \\ 1 & 7 & 1 & 1 \\ 1 & 1 & 6 & 1 \\ 1 & 1 & 1 & 4 \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 \\ 12 \\ -5 \\ -6 \end{pmatrix}$$

Principle:- Reduce (A/B) to (U/K)

$$(A/B) \xrightarrow{\text{Gauss Elimination}} (U/K)$$

$$(A/B) = \left(\begin{array}{cccc|c} 5 & 1 & 1 & 1 & 4 \\ 1 & 7 & 1 & 1 & 12 \\ 1 & 1 & 6 & 1 & -5 \\ 1 & 1 & 1 & 4 & -6 \end{array}\right)$$

Inverse - Gauss Jordan

Step - 1 : Consider the augmented matrix

Step - 2 : Reduce the matrix A in (A/I) to identy matrix by doing transformation

Transformation I to A^{-1}

Inverse matrix by Gauss Jordan method.

Principle :- $(A/I) \xrightarrow{\text{Gauss Jordan}} (I/A^{-1})$

1. Find the inverse of the given matrix $\begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix}$

Let $A = \begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix}$ $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Principle :- $(A/I) \xrightarrow[\text{Jordan}]{\text{Gauss}} (I/A^{-1})$

augmented $(A/I) = \left(\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{array}\right)$

$= \left(\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 2-2\times1 & 7-2\times3 & 0-2\times1 & 1-2\times0 \end{array}\right)$ $R_2 \to R_2 - 2R_1$

$= \left(\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{array}\right)$

$= \left(\begin{array}{cc|cc} 1 & 3-3\times1 & 1-(3\times2) & 0-3(1) \\ 0 & 1 & -2 & 1 \end{array}\right)$ $R_1 \to R_1 - 3R_2$

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$$= \left(\begin{array}{cc|cc} 1 & 0 & 7 & -3 \\ 0 & 1 & -2 & 1 \end{array}\right) \qquad \therefore A^{-1} = \begin{pmatrix} 7 & 0 \\ -2 & 1 \end{pmatrix}$$

$$\Rightarrow (I / A^{-1})$$

If A is not invertible then a zero row will show upon the left side row.

2. Find A^{-1} for $A = \begin{pmatrix} 1 & -3 \\ -2 & 6 \end{pmatrix}$

Principle :- $(A/I) \xrightarrow[\text{Jordan}]{\text{Gauss}} (I/A^{-1})$

Consider augmented form $A = \begin{pmatrix} 1 & -3 \\ -2 & 6 \end{pmatrix}$

$$A = \left(\begin{array}{cc|cc} 1 & -3 & 1 & 0 \\ -2 & 6 & 0 & 1 \end{array}\right) \qquad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \left(\begin{array}{cc|cc} 1 & -3 & 1 & 0 \\ -2+2\times1 & 6+(2\times-3) & 0+(2\times1) & 1+0 \end{array}\right) \quad R_2 \to R_2 + 2R_1$$

$$= \left(\begin{array}{cc|cc} 1 & -3 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{array}\right)$$

Left side has zero row.

$\therefore$ The given matrix is not invertible

(or) does not have A^{-1}.

Non - Singular matrix

A square matrix whose determinant Value is not Equal is Called non-singular matrix

Every Non - singular matrix has an inverse matrix

Singular matrix:

A square matrix whose determinant Value is Equal to Zero is Called Singular matrix

Singular matrix does not have inverse matrix.

3. Find A^{-1} for $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix}$

Principle: $(A/I) \xrightarrow[\text{Jordan}]{\text{Gauss}} (I/A^{-1})$

$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix}$

~~Ad~~ Augment matrix form

$(A/I) = \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right)$ $R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - R_1$

$$= \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2-(2\times1) & 5-(2\times2) & 3-(2\times3) & 0-2\times1 & 1-2\times0 & 0-2\times0 \\ 1-1 & 0-2 & 8-3 & 0-1 & 0-0 & 1-0 \end{array}\right)$$

$$= \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array}\right) R_3 \to R_3 + 2R_2$$

$$= \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -2+(1\times2) & 5+(2\times-3) & -1+(2\times-2) & 0+2\times1 & 1+2\times0 \end{array}\right)$$

$R_3 \to -R_3$

$R_1 \to R_1 - 3R_3$

$R_2 \to R_2 + 3R_3$

$R_1 \to R_1 - 2R_2$

$$= \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & -1 & -5 & 2 & 1 \end{array}\right) R_3 \to -R_3 \quad A^{-1} = \begin{pmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{pmatrix}$$

$$= \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right) R_1 \to R_1 - 3R_3$$

$$= \left[\begin{array}{ccc|ccc} 1-(3\times0) & 2-(3\times0) & 3-(3\times1) & 1-(3\times5) & 0-(3\times-2) & 0-(3\times-1) \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right]$$

$$= \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & -14 & 6 & 3 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right) R_2 \to R_2 + 3R_3$$

$$= \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & -14 & 6 & 3 \\ 0+3\times0 & 1+3\times0 & -3+3\times1 & -2+(3\times5) & 1+(3\times-2) & 0+(3\times-1) \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right)$$

$$= \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & -14 & 6 & 3 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right) \quad R_1 \rightarrow R_1 - 2R_2$$

$$= \left(\begin{array}{ccc|ccc} 1-2\times0 & 2-(2\times1) & 0-(2\times0) & -14-(2\times13) & 6-(2\times-5) & 3-(2\times-3) \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right)$$

$$= \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right)$$

$$\Rightarrow (I/A^{-1}) = A^{-1} = \begin{pmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{pmatrix} //.$$

Using Newton's method find the root between 0 and 1 of $x^3 - 6x + 4 = 0$ correct to 5 (five) decimal places.

Ans: $f(x) = x^3 - 6x + 4 = 0$

$f'(x) = 3x^2 - 6$

Initial guess = 0.5

Newton's Raphson = $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$n = 0, 1, 2, \ldots$

Ist approximation.

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 0.5 - \frac{(0.5)^3 - 6(0.5) + 4}{3(0.5)^2 - 6}$$

$$= 0.5 - \frac{0.125 - 3 + 4}{3 \times 0.25 - 6}$$

$$= 0.5 - \frac{0.125 - 3 + 4}{0.75 - 6}$$

$$= 0.5 - \frac{1.125}{-5.25}$$

$$= 0.5 + 0.21428$$

$$x_1 = 0.71429$$

IInd approximation

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.71429 - \frac{(0.71429)^3 - 6(0.71429) + 4}{3(0.71429)^2 - 6}$$

$$= 0.71429 - \frac{0.36443 - 4.28574 + 4}{3 \times 0.51021 - 6}$$

$$= 0.71429 - \frac{0.07869}{1.53063 - 6}$$

$$= 0.71429 - \frac{0.07869}{-4.46937}$$

$$= 0.71429 + 56.7971$$

$$x_2 = 0.7319$$

Third approximation:

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.7319 - \frac{(0.7319)^3 - 6(0.7319) + 4}{3(0.7319)^2 - 6}$$

$$= 0.7319 - \frac{0.39206 - 4.3914 + 4}{3 \times 0.53567 - 6}$$

$$= 0.7319 - \frac{0.00066}{1.60701 - 6}$$

$$= 0.7319 - \frac{0.00066}{-4.39299}$$

$$= 0.7319 + 0.00015$$

$$= 0.73205$$

fourth approximation:

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 0.73205 - \frac{(0.73205)^3 - 6(0.73205) + 4}{3(0.73205)^2 - 6}$$

$$= 0.73205 - \frac{0.39230 - 4.3923 + 4}{3 \times 0.53589 - 6}$$

$$= 0.73205 - \frac{0.0000}{}$$

$$= 0.73205$$

Find the Newton's method, the real positive root $x = \cos x$ correct to 3 decimal places

Let $x = \cos x$

Let $f(x) = \cancel{\cos x < x < 0}\; x - \cos x = 0$

$f'(x) = 1 + \sin x$

Newton's raphson $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$f(0) = \cancel{\cos \cos x 0} = 0 - \cos(0) = -1$

$f'(1) = \cdot 1$

Using Lagrange's interpolation formula fit the Polynomial of the following data $f(0) = -12$

$f(1) = 0$; $f(3) = 6$; $f(4) = 12$ (1)

Given data

$x_0 = 0$ $x_1 = 1$ $x_2 = 3$ $x_3 = 4$

$y_0 = -12$ $y_1 = 0$ $y_2 = 6$ $y_3 = 12$

Let $y = f(x)$

By Lagrangen interpolation formula

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$x_0 = 0$ $y_0 = -12$

$$\Rightarrow \frac{(x-1)(x-3)(x-4)}{(0-1)(0-3)(0-4)}(-12) + \frac{(x-0)(x-3)(x-4)}{(1-0)(1-3)(1-4)}(0) +$$

$$\frac{(x-0)(x-1)(x-4)}{(3-0)(3-1)(3-4)}(6) + \frac{(x-0)(x-1)(x-3)}{(4-0)(4-1)(4-3)}(12)$$

$$\Rightarrow \frac{(x^2-3x-x+3)(x-4)}{(-12)}(\cancel{-12}) + \frac{(x^2-3x)(x-4)}{(1)(-2)(-3)}(0) +$$

$\frac{(x^2 - x)(x-4)}{(3)(2)(-1)}(6) + \frac{(x^2 - x)(x-3)}{(4)(3)(1)}(12)$

(2)

$\Rightarrow \frac{(x^3 - 3x^2 - x^2 + 3x - 4x^2 + 12x + 4x - 12)}{(-\cancel{12})}(-\cancel{12}) + 0 +$

$\frac{(x^3 - 4x^2 - x^2 + 4x)}{(-\cancel{6})}(\cancel{6}) + \frac{(x^3 - 3x^2 - x^2 + 3x)}{1\cancel{2}}(\cancel{12})$

$\Rightarrow x^3 - 3x^2 - x^2 + 3x - 4x^2 + 12x + 4x - 12 - x^3 + 4x^2 + x^2 - 4x$

$+ x^3 - 3x^2 - x^2 + 3x$

$\Rightarrow x^3 - 7x^2 + 18x - 12$

This required degree polynomial 3

Given $u_0 = 6$; $u_1 = 9$; $u_3 = 33$; $u_7 = -15$

find u_2

$y = f(x) = (x - x_1)(x - x_2)y_0 / \ldots$

$y = u(x)$

$x_0 = 0 \quad x_1 = 1 \quad x_2 = 3 \quad x_3 = 7$

$y_0 = 6 \quad y_0 = 9 \quad y_2 = 33 \quad y_3 = -15$

$\Rightarrow \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}(y_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}(y_1) +$

$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}(y_2) + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}(y_3)$

$$\Rightarrow \frac{(x-1)(x-3)(x-7)}{(0-1)(0-3)(0-7)}(6) + \frac{(x-0)(x-3)(x-7)}{(1-0)(1-3)(1-7)}(9) + \frac{(x-0)(x-1)(x-7)}{(3-0)(3-1)(3-7)}(3) + \frac{(x-0)(x-1)(x-3)}{(7-0)(7-1)(7-3)}$$

$$\Rightarrow \frac{(2-1)(2-3)(2-7)}{(-1)(-3)(-7)}(6) + \frac{(2-0)(2-3)(2-7)}{(1)(-2)(-6)}(9) + \frac{(2-0)(2-1)(2-7)}{(3)(2)(-4)}(33) + \frac{(2-0)(2-1)(2-3)}{(7)(6)(4)}(3)$$

$$\Rightarrow \frac{(1)(-1)(-5)}{-21}(6) + \frac{(2)(-1)(-5)}{12}(9) + \frac{(2)(1)(-5)}{-24}(33) + \frac{(2)(1)(-1)}{168}(3)$$

$$\Rightarrow \frac{30}{-21} + \frac{90}{12} + \frac{330}{24} + \frac{30}{168}$$

3

$$\Rightarrow -1.4285 + 7.5 + 13.75 + 0.1785$$

$u_2 = 20$ //.

$x = f(y) = f(y)$ $(x-x_1)(x-x_2)(x-x_4)$

Apply Lagrange's formula inversly to obtain the root the Equation $f(x) = 0$ given that $f(0) = -4$; $f(1) = 1$; $f(3) = 29$ and $f(4) = 52$.

$x_0 = 0$ $\quad x_1 = 1$ $\quad x_2 = 3$ $\quad x_3 = 4$

$y_0 = -4$ $\quad y_1 = 1$ $\quad y_2 = 29$ $\quad y_3 = 52$

Let $y = f(x)$

By Lagrangen inverse interpolation formula

$$x = f(y) = \frac{(y-y_1)(y-y_2)(y-y_3)\cancel{(y-y_4)}}{(y_0-y_1)(y_0-y_2)(y_0-y_3)\cancel{(y_0-y_4)}}x_0 +$$

$$\frac{(y-y_0)(y-y_2)(y-y_3)\cancel{(y-y_4)}}{(y_1-y_0)(y_1-y_2)(y_1-y_3)\cancel{(y_1-y_4)}}x_1 + \frac{(y-y_0)(y-y_1)(y-y_3)\cancel{(y-y_4)}}{(y_2-y_0)(y_2-y_1)(y_2-y_3)\cancel{(y_2-y_4)}}x_2$$

0.8225

$$+\frac{(y-y_0)(y-y_1)(y-y_2)\cancel{(y-y_4)}}{(y_3-y_0)(y_3-y_1)(y_3-y_2)\cancel{(y_3-y_4)}}x_3 + \cancel{\frac{(y-y_0)(y-y_1)(y-y_2)(y-y_3)}{(y_4-y_0)(y_4-y}}$$

$$\Rightarrow \frac{(y-1)(y-29)(y-52)}{(-4-1)(-4-29)(-4-52)}(0) + \frac{(y-(-4))(y-29)(y-52)}{(1-(-4))(1-29)(1-52)}(1) +$$

$$\frac{(y+4)(y-1)(y-52)}{(29+4)(29-1)(29-52)}(3) + \frac{(y-(-4))(y-1)(y-29)}{(52+4)(52-1)(52-29)}(4)$$

$$\Rightarrow \frac{(y-1)(y-29)(y-52)}{(-5)(-33)(-56)}(0) + \frac{(y+4)(y-29)(y-52)}{(5)(-28)(-51)}(1) +$$

$$\frac{(y+4)(y-1)(y-52)}{(33)(28)(-23)}(3) + \frac{(y+4)(y-1)(y-29)}{(56)(51)(23)}(4)$$

$$\Rightarrow 0 + \frac{(0+4)(0-3)(0-4)}{+30}(1) + \frac{(0+4)(0-1)(0-52)}{-21252}(3) +$$

$$\frac{(0+4)(0-1)(0-29)}{65688}(4)$$

$$\Rightarrow 0 + \frac{6032}{\cancel{+30}\ 7140} - \frac{624}{21252} + \frac{464}{65688}$$

$$\Rightarrow \cancel{0 + 1.8 + 0.02936 + 0.0070063}$$

$$\Rightarrow 0 + 0.8448 - 0.02936 + 0.007063$$

$$\Rightarrow 0.8225$$

Find the polynomial Equation $y = f(x)$ Passing through $(-1, 3)$, $(0, -6)$, $(3, 39)$ $(6, 822)$ and $(7, 1611)$ ~~Divided difference~~

Ans: The given data

x	-1 (x_0)	0 (x_1)	3 (x_2)	6 (x_3)	7 (x_n)
y	3 (y_0)	-6 (y_1)	39 (y_2)	822 (y_3)	1611 (y_n)

Divided difference table

x	$y = f(x)$	∇y	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
-1	3				
		-9			
0	-6		54		
		45		684	
			738		-1416
3	39				
		783		-732	
6	822		6		
		789			
7	1611				

~~Here $x_n = 7$, $y_n = 1611$ from the difference table~~

~~$\Delta y_n = 789$ $\quad \Delta^2 y_n = 6$ $\quad \Delta^3 y_n = -732$ $\quad \Delta^4 y_n = -1416$~~

~~Newton's forward interpolation formula~~

$$y_p = y_0 + P \frac{\Delta y_0}{1!} + P(P-1) \frac{\Delta^2 y_0}{2!} +$$

$$P(P-1)(P-2) \frac{\Delta^3 y_0}{3!} + P \dots (P-\overline{n-1}) \frac{\Delta^n y_0}{n!}$$

Divided difference table formula:

x	y	$\Delta_1 f(x)$	$\Delta_2 f(x)$
x_0	$f(x_0)=y_0$		
		$f(x_0,x_1)=\frac{\overset{y_1-y_0}{f(x_1)-f(x_0)}}{x_1-x_0}$	
x_1	$f(x_1)=y_1$		$f(x_0,x_1,x_2)=\frac{f(x_2-x_1)-f(x_0-x_1)}{x_2-x_0}$
		$f(x_2,x_1)=\frac{\overset{y_2\ y_1}{f(x_2-f(x_1))}}{x_2-x_1}$	
x_2	$f(x_2)=y_2$		

Newton's divided difference formula:

$$f(x) = f(x_0) + (x-x_0)\,f(x_0-x_1)\,(x-x_0)(x-x_1)$$

$$f(x_0 x_1 x_2) + \ldots + (x-x_0)(x-x_1)(x-x_2)\ldots$$

$$(x-x_{n-1})\,f(x_0, x_1\, x_2 \ldots x_n)$$

Using newton divided differente formula Find the value of $f(8)$, $f(5)$ given the following data

	x_0	x_1	x_2	x_3	x_4	x_5
x	4	5	7	10	11	13
	y_0	y_1	y_2	y_3	y_4	y_5
$f(x)$	48	100	294	900	1210	2028

Newton's divided difference formula

$$f(x) = f(x_0) + (x-x_0)\,(x$$

x $f(x)$ Δy $\Delta^2 y$ $\Delta^3 y$ $\Delta^4 y$

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Divided difference table

x	$y=f(x)$	$\Delta_1 f(x)$	$\Delta_1^2 f(x)$	$\Delta_1^3 f(x)$
4	48			
		$\frac{100-48}{5-4} = 52$		
5	100		$\frac{97-52}{7-4} = 15$	
		$\frac{294-100}{7-5} = 97$		$\frac{21-15}{10-4} = 1$
7	294		$\frac{202-97}{10-5} = 21$	
		$\frac{900-294}{10-7} = 202$		$\frac{27-21}{11-4} = 1$
10	900		$\frac{310-202}{11-7} = 27$	
		$\frac{1210-900}{11-10} = 310$		$\frac{33-27}{13-10} = 1$
11	1210		$\frac{409-310}{13-10} = 33$	
		$\frac{2028-1210}{13-11} = 409$		
13	2028			

$\Delta_1^4 f(x)$ $\Delta_1^5 f(x)$

0

0

0

$x_0 = 4$, $x_1 = 5$, $x_2 = ?$ $x_3 = 10$

$x_4 = 11$, $x_5 = 13$ $y_0 = f(x_0) = 48$

$f(x) = 48 + (x-4)52 + (x-4)(x-5) + (x-4)$
$(x-5)(x-3)(1+0+0)$

$x = 8$

$f(8) = 48 + (8-4)52 + (8-4)(8-5) + (8-4)(8-5)$
$(8-3)(1+0+0)$

$= 48 + 4(52) + (4)(3)(4)(3)(5)(1)$

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$= 48 + 208 + 12 + 60 = 448$

$f(5) = 48 + (5-4)52 + (5-4)(5-5) + (5-4)(5-5)(5-3)$

$= 48 + 52$

$= 100$

Numerical Integration:

Trapezoidal Rule:

$$\int_a^b y\,dx = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + y_3 + \dots + y_{n-1})\right]$$

$$= \frac{h}{2}\left[\text{sum of extreme ordinates} + 2\left[\text{sum of remaining ordinates}\right]\right]$$

Error:

Error is order $= h^2$.

Total Error $E = \frac{-(b-a)}{12} h^2 y''(\bar{x})$

$y''(\bar{x})$ is the largest of $y_0'', y_1'', y_2'', \dots y_{n-1}''$

Simpson's 1/3 Rule:

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{3}\left[(y_0 + y_n) + 4(y_1 + y_3 + y_5 + \dots + y_{n-1}) + 2\left[y_2 + y_4 + y_6 + \dots + y_{2n-2}\right]\right]$$

~~Error~~ Interval must be divided into even number of sub interval of width 'h'.

Error:

Error order $= h^4$

Total Error $E = \frac{-(b-a)}{180} h^4 y^{iv}(\bar{x})$

y^{iv} is largest of 4th derivative

1. Evaluate $\int_4^{5.2} \log_e x \, dx$ using ①. Trapezoidal and ②. Simphson 1/3 rule.

	x_0	x_1	x_2	x_3	x_4	x_5	x_6
x	4	4.2	4.4	4.6	4.8	5	5.2
y	1.386	1.435	1.482	1.526	1.569	1.609	1.649
		y_1	y_2	y_3	y_4	y_5	y_6

Ans:

1. Trapezoidal rule

$$\int_a^b y \, dx = \frac{h}{2} \left[(y_0 + y_n) + 2(y_1 + y_2 + y_3 + \dots y_{n-1}) \right]$$

$h = 0.2$

$$\int_4^{5.2} \cancel{\log} \, \cancel{y/dx} = \frac{\cancel{0.2}}{2} \cancel{[(1.386 +}$$

$$\int_4^{5.2} \log_e x = \frac{h}{0.2} \left[(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5) \right]$$

$$= \frac{0.2}{2} \left[(1.386 + 1.649) + 2(1.435 + 1.482 + 1.526 + 1.569 + 1.609) \right]$$

$= 0.1\,[3.035 + 2(7.621)]$

$= 0.1\,[3.035 + 15.242]$

$= 0.1\,(18.277)$

$= 1.8277$

2

2. Simphson 1/3 rule:

$$\int_4^{5.2} \log_e x = \frac{h}{3}\left[(1.386 + 1.649) + 4(1.435 + 1.482 + 1.526 + 1.569 + 1.609)\right] +$$

$$= \frac{0.2}{3}\left[3.035 + 4(7.621)\right]$$

$$= 0.06\,[3.035 + 30.484]$$

$$= 0.06\,(33.519)$$

$$\int_4^{5.2} \log_e x = \frac{h}{3}\left[(1.386 + 1.649) + 4(1.435 + 1.526 + 1.609) + 2(1.482 + 1.569)\right]$$

$$= \frac{0.2}{3}\left[3.035 + 4(4.03) + 2(3.051)\right]$$

$$= 0.06\,[3.035 + 16.12 + 6.102]$$

$$= 0.06\,[25.257]$$

$= 1.$

2. Evaluate $\int_0^{\pi} \sin x \, dx$ by dividing the interval into 8 strips using Trapezoidal and Simphson 1/3 rule :-

Ans:- $y = \sin x$

$a = 0 \quad b = \pi \quad \boxed{n = 8}$

3

$h = \frac{b-a}{n} = \frac{\pi - 0}{8} = \frac{\pi}{8}.$

$x = 0 \quad \pi/8 \quad 2\pi/8 \quad 3\pi/8 \quad 4\pi/8 \quad 5\pi/8 \quad 6\pi/8 \quad 7\pi/8 \quad 8\pi/8$

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
x	0	$\pi/8$	$\pi/4$	$3\pi/8$	$2\pi/4$	$5\pi/8$	$3\pi/4$	$9\pi/8$	π
y	0	0.3827	0.7071	0.9239	1.0000	0.9239	0.7071	-0.3827	0.0000
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8

(i) Trapezoidal rule:

$$\int_a^b y \, dx = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + y_3 + y_4 + y_5 + \dots y_{n-1})\right]$$

$$\int_0^{\pi} \sin x = \frac{\pi/8}{2}\left[(y_0 + y_8) + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7)\right]$$

$= 0.1963 \left[(0 + 0.000) + 2(0.3827 + 0.7071 + 0.9239 + 1.0000 + 0.9239 + 0.7071 - 0.3827)\right]$

$= 0.1963 \left[0 + 2(\cancel{3.5549}\ 5.0284)\right]$ ~~4.2640~~

$= 0.1963 \left[0 + \cancel{7.1098}\ 10.0568\right)$

$= \cancel{1.3957}\ 1.9741$

(ii) Simphson 1/3 rule

$$\int_a^b y\,dx = \frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+y_5+\cdots y_{n-1})+2(y_2+y_4+y_6+\cdots y_{n-2})\right]$$

$$= \frac{\pi/8}{3}\left[(0+0.000)+4(0.3827+0.9239+0.9239+0.3827)+2(0.7071+1.0000+0.7071)\right]$$

$$= \frac{0.3927}{3}\left[0+4(2.6132)+2(2.4142)\right]$$

$$= 0.1309\left[10.4528+4.8284\right]$$

4

$$= 0.1309\,(15.2812)$$

$$\int_0^{\pi} \sin x\,dx = 2.0003$$

$$\int_a^b y\,dx = h/2$$

3. Find integral $\int_0^1 dx/1+x^2$ by using simphson's 1/3 rule. Hence obtain the approximate value of π in each case.

Ans: 0.7854

Ans:-

$$y = \frac{1}{1+x^2}$$

$$n = 6$$

$$h = \frac{b-a}{n} = \frac{1-0}{6} = 1/6$$

x	0	1/6	2/6	3/6	4/6	5/6	6/6
y							

x	0	$1/6$	$1/3$	$1/2$	$2/3$	$5/6$	1
y	0	0.9729	0.9000	0.8	0.6923	0.5901	1

$y_0 = \frac{1}{1+(0)^2} = 0$; $y_1 = \frac{1}{1+(1/6)^2} = \frac{1}{1.0277} = 0.9729$

$y_2 = \frac{1}{1+(1/3)^2} = \frac{1}{1+0.1111} = \frac{1}{1.1111} = 0.90000$

$y_3 = \frac{1}{1+(1/2)^2} = \frac{1}{1+0.25} = \frac{1}{1.25} = 0.8$

$y_4 = 0.$

Simpson's 1/3 rule:-

$$\int_a^b y\,dx = \frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3+y_5+\dots+y_{(n-1)}) + 2(y_2+y_4+\dots y_{2n-2})\right]$$

$$= \frac{h}{3}\left[(y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$$

$$= \frac{0.16666}{3}\left[(0+1) + 4(0.9729+0.8+0.5901) + 2(0.9000+0.6923)\right]$$

$$= 0.05553\left[1 + 4(2.363) + 2(1.5923)\right]$$

$$= 0.05553\left[1 + 9.452 + 3.1846\right]$$

$$= 0.05553\left[13.6366\right]$$

$$= 0.7854$$

$I = \int_0^1 \frac{dx}{1+x}$ correct to 4 decimal

$h = 0.5,\ 0.25,\ 0.125$

$\boxed{h = 0.25}$

x	0	0.5	1
y	1	0.667	0.5

Trapeziodal rule :

$$= \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+\cdots y_{n-1})\right]$$

$$= \frac{h}{2}\left[(y_0+y_2)+2(y_1)\right]$$

0.6944

$$= \frac{0.5}{2}\left[(1+0.5)+2(0.667)\right]$$

$$= 0.25\left[1.5+1.334\right]$$

$$= 0.25\left[2.834\right]$$

$I_1 = 0.7085$ //.

$\boxed{h = 0.25}$

x	0	0.25	0.5	0.75	1
y	1	0.8	0.667	0.5714	0.5

Trapezoidal rule $= \frac{h}{2}\left[(y_0+y_4)+2(y_1+y_2+y_3)\right]$

$= \frac{0.25}{2}\left[(1+0.5)+2(0.8+0.667+0.574)\right]$

$= 0.125\left[1.5+2(2.041)\right]$

$= 0.125\left[1.5+4.082\right]$

$= 0.125(5.582)$

$I_2 = 0.6970$ //.

when $h = 0.125$

x	0	0.125	0.25	0.375	0.5	0.625	0.750	0.875	1
y	1	0.8889	0.8	0.7273	0.667	0.615	0.5714	0.5333	0.5

Trapezoidal rule :

$= \frac{h}{2}\left[(y_0+y_8)+2(y_1+y_2+y_3+y_4+y_5+y_6+y_7)\right]$

$= \frac{0.125}{2}\left[(1+0.5)+2(0.8889+0.8+0.7273+0.667+0.615+0.5714+0.5333)\right]$

$= 0.0625\left[(1.5)+2(4.8029)\right]$

$= 0.0625\left[1.5+9.6058\right]$

$= 0.0625\left[11.1058\right]$

$I_3 = 0.6941$ //.

Romberg's method.

$$I = I_2 + \frac{I_2 - I_1}{2^n - 1} \quad \text{when } I\,(0.5,\ 0.25)$$

$$I = 0.6970 + \frac{0.6970 - 0.7085}{3}$$

$$= 0.6970 - 0.0038$$

$$= 0.6932 \,//.$$

0.0001

0.00061

when $I\,(0.25,\ 0.125)$

$$I = 0.6941 + \frac{0.6941 - 0.6970}{3}$$

$$= 0.6941 - 0.0029$$

$$= 0.6932$$

$I\,(0.5,\ 0.25,\ 0.125)$ $\quad I = \frac{I_1 h_2^2 - I_2 h_1^2}{h_2^2 - h_1^2}$

$$\Rightarrow I_2 + \frac{I_2 - I_1}{2^3 - 1}$$

$$= 0.6970 + 0.6970 - 0.7085$$

Gauss 2-point formula:

$$\int_{-1}^{1} f(x)\,dx = \left[f\left(-\sqrt{1/3}\right) + f\left(\sqrt{1/\sqrt{3}}\right) \right]$$

→ Exact for polynomial of degree upto 3

Gauss 3-point formula:

$$\int_{-1}^{1} f(x)\,dx = \frac{8}{9} f(0) + \frac{5}{9}\left[f\left(-\sqrt{3/5}\right) + f\left(\sqrt{\frac{3}{5}}\right) \right]$$

⇒ Exact for polynomial of degree upto 5

1. $\int_{-1}^{1} dx/1+x^2$ by 2 point and 3 point Gauss formula.

$$f(x) = \frac{1}{1+x^2}$$

1. By 2-point Gauss formula

$$\int_{-1}^{1} f(x)\,dx = \left[f\left(-\sqrt{\frac{1}{3}}\right) + f\left(\sqrt{1/3}\right) \right]$$

$$\int_{-1}^{1} f(x)\,dx = \left[\frac{1}{1+\left(-\sqrt{\frac{1}{3}}\right)^2} + \frac{1}{1+\left(\sqrt{1/3}\right)^2} \right]$$

$$= \left[\frac{1}{1+1/3} + \frac{1}{1+1/3} \right]$$

$$= \frac{1}{\frac{3+1}{3}} + \frac{1}{\frac{3+1}{3}} = \frac{3}{4} + \frac{3}{4} = 1.5$$

By 3 point Gauss formula :

$$\int_{-1}^{1} f(x)\,dx = \frac{8}{5} f(0) + \frac{5}{9}\left[f\left(-\sqrt{3/5}\right) + f\left(\sqrt{3/5}\right)\right]$$

$$= \frac{8}{9}\left[\frac{1}{1+0^2}\right] + \frac{5}{9}\left[\frac{1}{1+\left(-\sqrt{3/5}\right)^2} + \frac{1}{1+\left(\sqrt{3/5}\right)^2}\right]$$

$$= \frac{8}{9}\left[\frac{1}{1}\right] + \frac{5}{9}\left[\frac{1}{1+3/5} + \frac{1}{1+3/5}\right]$$

$$= \frac{8}{9} + \frac{5}{9}\left[\frac{5}{8} + \frac{5}{8}\right] \Rightarrow \frac{8}{9} + \frac{5}{9}\left[\frac{\cancel{10}^{5}}{\cancel{8}_{4}}\right]$$

$$= \frac{8}{9} + \frac{25}{36} \Rightarrow 0.88 + 0.694$$

$$= 1.574 \,//.$$

08.02.19 Friday

Unit - V

Euler's Method :- (Differential Equation)

Let $y_1 = y(x_1)$ where $x_1 = x_0 + h$

Then $y_1 = y(x_0 + h)$

By Taylor's series

$$y_1 = y_0(x_0) + \frac{h}{1!} y'(x_0) + \frac{h^2}{2!} y''(x_0) + \cdots$$

neglect h^2 and highest of h

Equ (1) become $y_1 = y_0 + \Delta y$

$$y_1 = y_0 + hy'(x_0)$$

$$y_1 = y_0 + hf(x_0, y_0) \longrightarrow (2)$$

$f(x_0, y_0) = y'(x_0)$

Equ (2) gives approximate value of

y at $x_1 = x_0 + h$

IIly $\quad y_2 = y_1 + hf(x_1, y_1)$

$\quad\quad y_3 = y_2 + hf(x_2, y_2)$ $\quad y_{n+1} = y_n + hf(x_n, y_n)$

In general $\quad y_{n+1} = \quad y_{n+1} = y_n + h($

$$\boxed{y_{n+1} = y_n + hf(x_n, y_n)} \longrightarrow (3)$$

$n = 0, 1, 2, \dots$

Equ (3) is Euler's method

$\Rightarrow$ to compute successive $y_1, y_2, y_3, \dots$

$\Rightarrow$ Error : $O^{\text{order}}(h^2)$

1\. Question:- (i) Using Euler's method, compute y in the range $0 \le x \le 0.5$ if y satisfy

$$\frac{dy}{dx} = 3x + y^2, \quad y(0) = 1$$

Ans:- $\quad f(x, y) = 3x + y^2$

$y(0) = 1$

$x_0 = 0 \quad y_0 = 1$

By Euler's method

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$y_{n+1} = y_n + hf(x_n, y_n)$

$y_{n+1} = y_n + hf$

$y_{n+1} = y_n = h(x_n, y_n)$

$y_{n+1} = y_n + hf(x_n, y_n)$

$y_{n+1} = y_n + hf(x_n$

$y_{n+1} = h + f(y_n, x_n, y_n)$

$x_1 = 0.1$ $\quad x_1 = x_0 + h$

$h = x_1 - x_0$

$= 0.1 - 0 = h \Rightarrow 0.1$

$\underline{x_1 = 0.1}$ $\quad y_1 = y_0 + hf\,(x_0, y_0)$

$y(0.1) = 1 + 0.1\,[3(0) + (1)^2]$

$= 1 + 0.1\,[1]$

$\boxed{y_1 = 1.1}$ //.

$y_{n+1} = y_n + hf\,(x_n, y_n)$

$y_{n+1} = y_n + hf\,(x_n, y_n)$

②

$x_2 = 0.2$ $\quad y_2 = y_1 + hf\,(x_1, y_1)$

$y(0.2) = 1.1 + 0.1\,[3(0.1) + (1.1)^2]$

$= 1.1 + 0.1\,[0.3 + 1.21]$

$= 1.1 + 0.151$

$\boxed{y_2 = 1.251}$ //.

$y_3 = 1.4675$

$x_3 = 0.3$ $\quad y_3 = y_2 + hf\,(x_2, y_2)$ $\quad y_4 = 1.7728$

$y(0.3) = 1.251 + 0.1\,[3(0.2) + (1.251)^2]$ $\quad 2.2071$

$= 1.251 + 0.1\,[0.6 + 1.5650]$

$= 1.251 + 0.1\,[2.165]$

$= 1.251 + 0.2165$

$y_1 = y_0 + hf\,(x_0, y_0)$

$y_1 = y_0 + hf\,(x_0, y_0)$

$\boxed{y_3 = 1.4675}$ //.

$x_4 = 0.4 \quad y_4 = y_3 + hf(x_3, y_3)$

$y(0.4) = 1.4675 + 0.1 \left[3(0.3) + (1.4675)^2\right]$

$= 1.4675 + 0.1 \left[0.09 + 2.1535\right]$

$= 1.4675 + 0.1 \, (\cancel{2.2435}\ 3.0535)$

$= 1.4675 + 0.\cancel{2243}\ 3053$

$\boxed{y_4 = \cancel{1.6918} \,/\!/.\ 1.7728 \,/\!/.}$

$x_5 = 0.5 \quad y_5 = y_4 + hf(x_4, y_4)$

$y(0.5) = \cancel{1.6918}\ 1.7728 + 0.1 \left[3(\cancel{1.6918}\ 0.4) + (1.\cancel{6918}\ 7728)^2\right]$

$= \cancel{1.6918}\ 1.7728 + 0.1 \left[1.2 + 3.1428\right]$

$= 1.7728 + 0.1 \left[4.3428\right]$ ③

$= 1.7728 + 0.4342$

$\boxed{y_5 = 2.2071 \,/\!/.}$

Given $dy/dx = \frac{y-x}{y+x}$ with $y = 1$ for $x = 0$, find y approximately for $x = 0.1$ by Euler's method in 5 steps: ②

$f(x, y) = \frac{y-x}{y+x}$

$y(0) = 0$

$x_0 = 0 \quad y_0 = 1$

Scanned with CamScanner

By Euler's method

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$x_i = x_0 + ih \qquad i = 1, 2, 3, 4, 5 \ldots$

$h = \frac{x_0 - x}{i} = 0 -$

$\boxed{h = 0.02}$

$y_1 = 1.02$
$y_2 = 1.0392$
$y_3 = 1.0577$
$y_4 = 1.0756$
$y_5 = 1.0928$

$x_1 = 0.1 \qquad y_1 = y_0 + hf(x_0, y_0)$

$y_{0.1} = 1 + 0.02\left(\frac{1-0}{1+0}\right)$

$= 1 + 0.02\left(\frac{1}{1}\right)$

$= 1 + 0.02$

$\boxed{y_{0.1} = 1.02}$

$x_2 = 0.02 \qquad y_2 = y_1 + hf(x_1, y_1)$

$= 1.02 + 0.02\left(\frac{1.02 - 0.02}{1.02 + 0.02}\right)$

$= 1.02 + 0.02\,(1/1.04)$

$= 1.02 + 0.02\,(0.9615)$

$= 1.02 + 0.01923$

$\boxed{y_2 = 1.0392}$

$x_3 = 0.04 \qquad y_3 = y_2 + hf(x_2, y_2)$

$= 1.0392 + (0.02)\left(\dfrac{1.0392 - 0.04}{1.0392 + 0.04}\right)$

$= 1.0392 + (0.02)\left(\dfrac{0.9992}{1.0792}\right)$

$= 1.0392 + 0.02\,(0.9258)$

$= 1.0392 + 0.0185$

$y_3 = 1.0577$ //.

$y_4 = \cancel{1.0577}\; y_3 + hf\,(x_3, y_3)$

$= 1.0577 + 0.02\left(\dfrac{1.0577 - 0.06}{1.0577 + 0.06}\right)$

$= 1.0577 + 0.02\left(\dfrac{0.9977}{1.1177}\right)$

$= 1.0577 + 0.02\,(0.8926)$

$= 1.0577 + 0.0178$

$\boxed{y_4 = 1.0756}$ //.

$y_5 = y_4 + hf\,(y_4, x_4)$

$= 1.0756 + 0.02\left(\dfrac{1.0756 - 0.08}{1.0756 + 0.08}\right)$

$= 1.0756 + 0.02\left(\dfrac{0.9956}{1.1556}\right)$

$= 1.0756 + 0.02\,(0.8615)$

$= 1.0756 + 0.0172$

$\boxed{y_5 = 1.0928}$ //.

Forth Order of Runge - Kutta Method :

Let $\frac{dy}{dx} = y' f(x, y)$

$$y(x_0) = y_0$$

The value $y_1 = y(x_1)$ where $x_1 = x_0 + h$

h - Step size

$K_1 = hf \; (x_0, y_0)$

$K_2 = hf \; (x_0 + h/2 , \; y_0 + K_1/2)$

$K_3 = hf \; (x_0 + h/2 , \; y_0 + K_2/2)$

$K_4 = hf \; (x_0 + h, \; y_0 + K_3)$

Finally The increment

$$\Delta y = \frac{1}{6} (K_1 + 2K_2 + 2K_3 + K_4)$$

Approximate y_1

$$y_1 = y_0 + \Delta y$$

$$= y_0 + \frac{1}{6} (K_1 + 2K_2 + 2K_3 + K_4)$$

Error = $O(h^5)$

In general

$$y_{m+1} = y_m + \frac{1}{6} (K_1 + 2K_2 + 2K_3 + K_4)$$

where

$K_1 = hf\,(x_m, y_m)$

$K_2 = hF\,(x_m + h/2,\ y_m + K_1/2)$

$K_3 = hf\,(x_m + h/2,\ y_m + K_2/2)$

$K_4 = hf\,(x_m + h,\ y_m + K_3)$, where $m = 0, 1, 2 \ldots$

Test Obtain the value of y at $x = 0.2$ if y satisfies $\frac{dy}{dx} = x^2 y + x$, $y(0) = 1$ taking $h = 0.1$ using

$f(x,y)$ $y' =$ Runge kutta method of fourth order:

$$\frac{dy}{dx} = x^2 y + x$$

x_0	y_0

$\boxed{h = 0.1}$

$x_0 = 0$; $y_0 = 1$

$y(0) = 1$

$x_1 = x_0 + h = 0 + 0.1 = 0.1$

$y(0.1) = y_1 = y_0 + 1/6\,(K_1 + 2K_2 + 2K_3 + K_4)$

$K_1 = hf\,(x_0, y_0)$

$K_2 = hf\,(x_0 + h/2,\ y_0 + K_1/2)$

$K_3 = hf\,(x_0 + h/2,\ y_0 + K_2/2)$

$K_4 = hf\,(x_0 + h,\ y_0 + K_3)$

$k_1 = hf(x_0, y_0)$

$= 0.1 f[(0)^2(1)+0]$

$\boxed{k_1 = 0}$

$k_2 = hf(x_0 + h/2, y_0 + k_1/2)$

$= 0.1 f\left(0 + \frac{0.1}{2}, 1 + \frac{0}{2}\right)$

$= 0.1 f(0.05, 1)$

$= 0.1[(0.05)^2(1) + 0.05]$

$= 0.1[0.0025 + 0.05]$

$= 0.1(0.0525)$

$\boxed{k_2 = 0.00525}$

$k_3 = hf(x_0 + h/2, y_0 + k_2/2)$

$= 0.1 f\left(0 + \frac{0.1}{2}, 1 + \frac{0.0052}{2}\right)$

$= 0.1 f(0.05, 1.0026)$

$= 0.1[(0.05)^2(1.0026) + 0.05]$

$= 0.1[(0.0025)(1.0026) + 0.05]$

$= 0.1[0.0025 + 0.05]$

$= 0.1[0.0525]$

$\boxed{k_3 = 0.00525}$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$= 0.1 f(0+0.1, 1 + 0.00525)$

$= 0.1 f(0.1, 1.00525)$

$= 0.1 [(0.1)^2 (1.00525) + 0.1]$

$= 0.1 [(0.01)(1.00525) + 0.1]$

$= 0.1 [0.010052 + 0.1] \Rightarrow 0.1 (0.11005)$

$\boxed{K_4 = 0.011005}$

$y(0.1) \quad y_1 = y_0 + 1/6 (k_1 + 2k_2 + 2k_3 + k_4)$

$= 1 + 1/6 [0 + 2(0.00525) + 2(0.00525) + 0.011005]$

$= 1 + 0.1666 [\cancel{0.01050}\ 0.01050 + 0.01050 + 0.011005]$

$= 1 + 0.1666 [0.03200) \Rightarrow 1 + 0.00533$

$\boxed{y_{(0.1)} = 1.0053}$

To find $y_2 = y(x_2)$

$x_2 = x_1 + h = 0.1 + 0.1$

$\boxed{x_2 = 0.2}$ $\boxed{h = 0.1}$

$y_{(0.2)} = y_2 = y_1 + 1/6 (k_1 + 2k_2 + 2k_3 + k_4)$

$k_1 = hf(x_1, y_1)$

$k_2 = hf(x_1 + h/2, y_1 + k_1/2)$

$k_3 = hf(x_1 + h/2, y_1 + k_2/2)$

$k_4 = hf(x_1 + h, y_1 + k_3)$

$k_1 = hf(x_1, y_1)$

$= 0.1 f(0.1, 1.0053)$

$= 0.1 [(0.1)^2(1.0053) + 0.1]$

$= 0.1 [(0.01)(1.0053) + 0.1]$

$= 0.1 [0.01005 + 0.1) \Rightarrow 0.1 (0.11005)$

$\boxed{k_1 = 0.0110}$

$k_2 = hf\left(x_1 + h/2, y_1 + \frac{k_1}{2}\right)$

$= 0.1 f\left(0.1 + \frac{0.1}{2}, 1.0053 + \frac{0.0110}{2}\right)$

$= 0.1 f(0.1 + 0.05, 1.0053 + 0.0055)$

$= 0.1 f [0.1500, 1.01080]$

$= 0.1 [(0.1500)^2(1.01080) + 0.1500]$

$= 0.1 [0.0225 + 0.1500] \Rightarrow 0.1 (0.17250)$

$\boxed{k_2 = 0.01727}$

$k_3 = hf\left[x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right]$

$= 0.1 f\left[0.1 + \frac{0.1}{2}, 1.0053 + \frac{0.01727}{2}\right]$

$= 0.1 f[0.1 + 0.05, 1.0053 + 0.0086]$

$= 0.1 f[0.15000, 1.01390]$

$= 0.1 [(0.15000)^2(1.01390) + \cancel{1.01390}\ 0.15000]$

$= 0.1 [(0.02250)(1.01390) + 0.15000]$

$= 0.1 [0.02281 + 0.15000] = 0.1 (0.1728)$

$\boxed{k_3 = 0.01728}$

$k_4 = hf\,(x_1+h,\ y_1+k_3)$

$= 0.1 f\,[0.1+0.1,\ 1.0053+0.01728]$

$= 0.1\,[0.2,\ 1.02258]$

$= 0.1\,[(0.2)^2(1.002258)+0.2]$

$= 0.1\,[0.04(1.002258)+0.2]$

$= 0.1\,[0.04009+0.2] \Rightarrow 0.1\,(0.24009)$

$\boxed{k_4 = 0.02409}$

$y_2 = y_1 + 1/6\,[k_1+2k_2+2k_3+k_4]$

$= 1.0053 + \frac{1}{6}\,[0.0110+2\,(0.01727)+2\,(0.01728)+0.02409]$

$= 1.0053 + 0.1666\,[0.0110+0.03454+0.034560+0.02409]$

$= 1.0053 + 0.1666\,[0.104190]$

$= 1.0053 + 0.01735$

$\boxed{y_2 = 1.0227}$ //.

$\boxed{y_{0.2} = 1.0227}$

R-K Method for Second Order differential Equation :

Second order differential Equation

$$\frac{d^2y}{dx^2} = f\left(x, y, \frac{dy}{dx}\right)$$

$$= f(x, y, y')$$

Initial Condition

$y(x_0) = y_0$

$y'(x_0) = y_0'$

put $z = \frac{dy}{dz}$

2^{nd} order differential Equation reduced to system of simultaneous 1^{st} order linear Equation.

$y' = \frac{dy}{dz} = z = g(x, y)$ with $y(x_0) = y_0$

$y'' = \frac{d^2y}{dx^2} = \frac{dz}{dx} = f(x, y, z)$ with $z(x_0) = y'(x_0) = y_0'$

(ie) $\frac{dy}{dx} = g(x, y, z)$ with $g(x, y, z) = z$

and $\frac{dz}{dx} = f(x, y, z)$ with $y(x_0) = y_0$

$z(x_0) = z_0$ where $z_0 = y_0'$

Initial values

x_0, y_0, z_0

Increments

$\Delta y, \Delta z$ in y and z

h - step size

$$k_1 = hg(x_0, y_0, z_0)$$

$$k_2 = hg\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right)$$

$$k_3 = hg\left(x_0 + \frac{h}{2}, y_0 + k_2/2, z_0 + \frac{l_2}{2}\right)$$

$$k_4 = hg(x_0 + h, y_0 + k_3, z_0 + l_3)$$

$$\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$l_1 = hf(x_0, y_0, z_0)$$

$$l_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right)$$

$$l_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 = \frac{l_2}{2}\right)$$

$$l_4 = hf(x_0 + h, y_0 + k_3, z_0 = l_3)$$

$$\Delta z = \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4)$$

$x_1 = x_0 + h$

$y_1 = y_0 + \Delta y = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$z_1 = z_0 + \Delta z = z_0 + \frac{1}{6}(l_1 + 2l_1 + 2l_2 + l_4)$

Similarly

x_2, y_2, z_2 are found repeatively

$x_2 = x_1 + h$

$y_2 = y_1 + \Delta y$

$z_2 = z_1 + \Delta z$

Test

Solve the Equation $\frac{dy}{dx} = xz + 1$, $\frac{dz}{dx} = -xy$

for $x_1 = 0.3$ and 0.6 Given that $y = 0$, $z = 1$

when $x = 0$.

$h = x_1 - x_0$

$= 0.3 - 0$

$\boxed{h = 0.3}$

$g(x, y, z) = \frac{dy}{dx} = xz + 1$

$f(x, y, z) = \frac{dz}{dx} = \frac{d^2y}{dx^2} = -xy$

$x_0 = 0$; $y_0 = 0$, $z_0 = 1 \Rightarrow$ ~~$y'(0)$~~ $y'_{(0)}$

To find $y_1 = y(0.3) = ?$ $y_2 = y(0.6) = ?$

$z_1 = z(0.3) = ?$ $z_2 = z(0.6) = ?$

To find $y(0.3)$ & $z(0.3)$

$k_1 = hg(x_0, y_0, z_0)$

$= 0.3\, g(0, 0, 1)$

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$= 0.3\ [(0)(1)+1]$

$= 0.3\ [1]$

$\boxed{k_1 = 0.3}$

$l_1 = hf(x_0\ y_0\ z_0)$

$= 0.3 f(0, 0, 1)$

$= 0.3 (-(0)(0))$

$\boxed{l_1 = 0}$

$k_2 = hg\left[x_0 + h/2,\ y_0 + \frac{k_1}{2},\ z_0 + \frac{l_1}{2}\right]$

$= 0.3 g\left[0 + \frac{0.3}{2},\ 0 + \frac{0.3}{2},\ 1 + \frac{0}{2}\right]$

$= 0.3 g[0.15,\ 0.15,\ 1]$

$= 0.3\ [(0.15)(1)+] \Rightarrow 0.3\ (1.15)$

$\boxed{k_2 = 0.3450}$

$l_2 = hf\left[x_0 + \frac{h}{2},\ y_0 + \frac{k_1}{2},\ z_0 + \frac{l_1}{2}\right]$

$= 0.3 f\left[0 + \frac{0.3}{2},\ 0 + \frac{0.3}{2},\ 1 + \frac{0}{2}\right]$

$= 0.3 f[0.15,\ 0.15,\ 1]$

$= 0.3\ [-(0.15)(0.15)] \Rightarrow 0.3\ (-0.0225)$

$\boxed{l_2 = -0.00675}$

$$K_3 = hg\left[x_0+\frac{h}{2},\ y_0+\frac{k_2}{2},\ z_0+\frac{l_2}{2}\right]$$

$$= 0.3\,g\left[0+\frac{0.3}{2},\ 0+\frac{0.3450}{2},\ 1-\frac{0.00776}{2}\right]$$

$$= 0.3\,g\left[0.15,\ 0.1725,\ 0.99612\right]$$

$$= 0.3\left[(0.15)(0.99612)+1\right]$$

$$= 0.3\left[0.14941+1\right] \Rightarrow 0.3\,(1.14941)$$

$$\boxed{K_3 = 0.3448}$$

$$l_3 = hf\left[x_0+\frac{h}{2},\ y_0+\frac{K_2}{2},\ z_0+\frac{l_2}{2}\right]$$

$$= 0.3\,f\left[0+\frac{0.3}{2},\ 0+\frac{0.3450}{2},\ 1-\frac{0.00776}{2}\right]$$

$$= 0.3\left[0.15,\ 0.1725,\ 0.99612\right]$$

$$= 0.3\left[-(0.15)(0.1725)\right] \Rightarrow 0.3\,(-0.02587)$$

$$\boxed{l_3 = -0.00776}$$

$$K_4 = hg\left[x_0+h,\ y_0+K_3,\ z_0+l_3\right]$$

$$= 0.3\,g\left[0+0.3\ \ 0+0.3448\ \ 1-0.00776\right]$$

$$= 0.3\,g\left[0.3,\ 0.3448,\ 0.99224\right]$$

$$= 0.3\left[(0.3)(0.99224)+1\right]$$

$$= 0.3\left[0.2976+1\right] \Rightarrow 0.3\,(1.2976)$$

$$\boxed{K_4 = 0.3893}$$

$l_4 = hf(x_0+h, y_0+k_3, z_0+l_3)$

$= 0.3 f(0+0.3, 0+0.3448, 1-0.00776)$

$= 0.3 f(0.3, 0.3448, 0.99224)$

$= 0.3[-(0.3)(0.3448)] \Rightarrow 0.3(-0.10344)$

$\boxed{l_4 = -0.03103}$

$y_{(0.3)} = y_1 = y_0 + (k_1+2k_2+2k_3+k_4)$

$\Rightarrow 0 + \frac{1}{6}(0.3+2(0.3450)+2(0.3448)+0.3893]$

$\Rightarrow 0 + 0.16666[0.3+0.69+0.6896+0.3893]$

$\Rightarrow 0.16666[2.0689]$

$\boxed{y_{(0.3)} = y_1 = 0.3448}$

$z_{(0.3)} = z_1 = z_0 + \frac{1}{6}[l_1+2l_2+2l_3+l_4]$

$\Rightarrow 1 + 0.16666[0+2(-0.00675)+2(-0.00776) - 0.03103]$

$\Rightarrow 1 - 0.16666[0+0.0135+0.01552+0.03103]$

$\Rightarrow 1 - 0.16666[0.06005] \Rightarrow 1 - 0.01000$

$\boxed{z_{(0.3)} = z_1 = 0.9899}$

To find $y(0.6)$ and $z(0.6)$

$\boxed{h = 0.3}$

$x_1 = x_0 + h$

$= 0 + 0.3$

$\boxed{x_1 = 0.3}$

$\boxed{y_1 = 0.3448}$ $\boxed{z_1 = 0.9899}$

$k_1 = hg(x_1, y_1, z_1)$

$= 0.3\, g(0.3, 0.3448, 0.9899)$

$= 0.3\left[(0.3)(0.9899) + 1\right]$

$= 0.3\left[0.29697 + 1\right] \Rightarrow 0.3(1.29697)$

$\boxed{k_1 = 0.3891}$

$l_1 = hf(x_1, y_1, z_1)$

$= 0.3(0.3, 0.3448, 0.9899)$

$= 0.3\left[-(0.3)(0.3448)\right] \Rightarrow 0.3\left[-0.10344\right]$

$\boxed{l_1 = -0.0310}$

$k_2 = hg\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}, z_1 + \frac{l_1}{2}\right)$

$= 0.3\, g\left[0.3 + \frac{0.3}{2}, 0.3448 + \frac{0.3891}{2}, 0.9899 - \frac{0.0310}{2}\right]$

$\Rightarrow 0.3g\left[0.3 + 0.15, 0.3448 + 0.19455, 0.9899 - 0.0155\right]$

$= 0.3g\left[0.45, 0.53935, 0.9744\right]$

$= 0.3\left[(0.45)(0.9744) + 1\right]$

$= 0.3 \ [0.43848 + 1] \Rightarrow 0.3 (1.43848)$

$\boxed{k_2 = 0.4315}$

$l_2 = hf \left[x_1 + \frac{h}{2}, \ y_1 + \frac{k_1}{2}, \ z_1 + \frac{l_1}{2} \right]$

$= 0.3 f \left[0.3 + \frac{0.3}{2}, \ 0.3448 + \frac{0.3891}{2}, \ 0.9899 - \frac{0.0310}{2} \right]$

$= 0.3 f \ [0.45, \ 0.53935, \ 0.9744]$

$= 0.3 \ [-(0.45)(0.53935)]$

$= 0.3 \ [-0.24270]$

$\boxed{l_2 = -0.0728}$

$k_3 = hg \left[x_1 + \frac{h}{2}, \ y_1 + \frac{k_2}{2}, \ z_1 + \frac{l_2}{2} \right]$

$= 0.3 g \left[0.3 + \frac{0.3}{2}, \ 0.3448 + \frac{0.4315}{2}, \ 0.9899 - \frac{0.0728}{2} \right]$

$= 0.3 g \ [0.3 + 0.15, \ 0.3448 + 0.21575, \ 0.9899 - 0.0364]$

$= 0.3 \ [0.45, \ 0.56055, \ 0.9535]$

$= 0.3 \ [(0.45)(0.9535) + 1]$

$= 0.3 \ [0.429075 + 1] \Rightarrow 0.3 (1.42907)$

$\boxed{k_3 = 0.4287}$

$$l_3 = hf\left[x_1+\frac{h}{2},\ y_1+\frac{k_2}{2},\ z_1+\frac{l_2}{2}\right]$$

$$= 0.3f\left[0.2+\frac{0.3}{2},\ 0.3448+\frac{0.4315}{2},\ 0.9899-\frac{0.0728}{2}\right]$$

$$= 0.3f[0.45,\ 0.56055,\ 0.9535]$$

$$= 0.3[-(0.45)(0.56055)]$$

$$= 0.3(-0.252247)$$

$$\boxed{l_3 = -0.0757}$$

$$k_4 = hg[x_1+h,\ y_1+k_3,\ z_1+l_3]$$

$$\Rightarrow 0.3g[0.3+0.3,\ 0.3448+0.4287,\ 0.9899-0.0757]$$

$$\Rightarrow 0.3g[0.6,\ 0.7735,\ 0.9142]$$

$$\Rightarrow 0.3[(0.6)(0.9142)+1]$$

$$\Rightarrow 0.3[0.54852+1] \Rightarrow 0.3(1.54852)$$

$$\boxed{k_4 = 0.4645}$$

$$l_4 = hf[x_1+h,\ y_1+k_3,\ z_1+l_3]$$

$$\Rightarrow 0.3f[0.3+0.3,\ 0.3448+0.4287,\ 0.9899-0.0757]$$

$$\Rightarrow 0.3f[0.6,\ 0.7735,\ 0.9142]$$

$$\Rightarrow 0.3[-(0.6)(0.7735)] \Rightarrow 0.3[-0.4641]$$

$$\boxed{l_4 = -0.1392}$$

$$y_{(0.6)} = y_2 = y_1 + \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$$

$$\Rightarrow 0.3448 + \frac{1}{6}\left[0.3891 + 2(0.4315) + 2(0.4287) + 0.4645\right]$$

$$\Rightarrow 0.3448 + 0.16666\left[0.3891 + 0.863 + 0.8575 + 0.4645\right]$$

$$\Rightarrow 0.3448 + 0.16666\left[2.5741\right]$$

$$\Rightarrow 0.3448 + 0.42899$$

$$\boxed{y_{(0.6)} = y_2 = 0.7738}$$

$$z(0.6) = z_2 = z_1 + \frac{1}{6}\left[l_1 + 2l_2 + 2l_3 + l_4\right]$$

$$\Rightarrow 0.9899 + \frac{1}{6}\left[(-0.0310) + 2(-0.0728) + 2(-0.0757) - 0.1392\right]$$

$$\Rightarrow 0.9899 - 0.16666\left[0.0310 + 0.1456 + 0.1514 + 0.1392\right]$$

$$\Rightarrow 0.9899 - 0.16666\left[0.4672\right]$$

$$\Rightarrow 0.9899 - 0.07786$$

$$\boxed{z(0.6) = z_2 = 0.9120}$$

Using R-K method of 4^{th} order solve

$y'' = x(y')^2 - y^2$ for $x = 0.2$ given that

$y = 1$ and $y' = 0$ when $x = 0$

$z' = y'' = x(y')^2 - y''$

$\boxed{z = y'} \Rightarrow z' = y'' = x(z)^2 - y^2$

Initial condition

$\boxed{x_0 = 0}$ $\boxed{y_0 = 1}$, $\boxed{z_0 = y_0 = 0}$

$y(0.2) = ?$

$\boxed{h = 0.2}$

$g(x, y, z) = z$

$f(x, y, z) = xz^2 - y^2$

$k_1 = h\,g(x_0, y_0, z_0)$

$= 0.2\,g(0, 1, 0)$

$= 0.2\,(0)$

$\boxed{k_1 = 0}$

$f(x, y, z) = xz^2 - y^2$

$l_1 = hf(x_0, y_0, z_0)$

$= 0.2\,f(0, 1, 0)$

$= 0.2\,[0.0^2 - 1^2] \Rightarrow 0.2\,(-1)$ $\boxed{l_1 = -0.2}$

$$K_2 = hg\left(x_0 + h/2, y_0 + K_1/2, z_0 + l_1/2\right)$$

$$= 0.2\, g\left(0 + \frac{0.2}{2}, 1 + 0/2, 0 + \frac{-0.2}{2}\right)$$

$$= 0.2\, g\,(0.1, 1, -0.1)$$

$$= 0.2\,(-0.1)$$

$$\boxed{K_2 = -0.02}$$

$$l_2 = hf\left(x_0 + h/2, y_0 + K_2/2, z_0 = l_1/2\right)$$

$$= 0.2 f\,(0.1, 1, -0.1)$$

$$= 0.2\left((0.1)(-0.1)^2 - (1)^2\right)$$

$$= 0.2\left[(0.001) - 1\right] \Rightarrow 0.2\,(-0.9999)$$

$$\boxed{l_2 = -0.1998}$$

$$K_3 = hg\left(x_0 + h/2, y_0 + K_2/2, z_0 + \frac{l_2}{2}\right)$$

$$= 0.2\, g\left(0 + \frac{0.2}{2}, 1 + \frac{-0.02}{2}, 0 + \frac{-0.1998}{2}\right)$$

$$= 0.2\, g\,(0.1, 0.99, -0.0999)$$

$$= 0.2\,(-0.0999)$$

$$\boxed{K_3 = -0.01998}$$

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$l_3 = hf\left(x_0 + h/2,\ y_0 + \frac{k_2}{2},\ z_0 + \frac{l_2}{2}\right)$

$= 0.2 f(0.1,\ 0.99,\ -0.9999)$

$= 0.2\left[(0.1)(-0.9999)^2 - (0.99)^2\right]$

$= 0.2\left[0.0999980 - 0.9801\right]$

$= 0.2\left[-0.880119\right]$

$\boxed{l_3 = -0.1958}$

$K_4 = hg(x_0+h,\ y_0+K_3,\ z_0+l_3)$

$= 0.2g(0+0.2,\ 1-0.01998,\ 0-0.1958)$

$= 0.2\,g(0.2,\ 0.98002,\ -0.1958)$

$= 0.2\,(-0.1958)$

$\boxed{K_4 = -0.03916}$

$l_4 = hf(x_0+h,\ y_0+k_3,\ z_0+l_3)$

$= 0.2f(0.2,\ 0.98002,\ -0.1958)$

$= 0.2\left[(0.2)(-0.1958)^2 - (0.98002)^2\right]$

$= 0.2\left[(0.007667 - 0.960439\right]$

$= 0.2\,(-0.952772)$

$\boxed{l_4 = -0.1905}$

Scanned with CamScanner

$$y_{(0.2)} = y_1 = y_0 + \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$$

$$\Rightarrow y_1 = 1 + \frac{1}{6}\left[0 + 2(-0.02) + 2(-0.01998) + (-0.03916)\right]$$

$$= 1 - \frac{1}{6}\left[(0.04) + 0.03996 + 0.03916\right]$$

$$= 1 - 0.16666\left[0.11912\right]$$

$$= 1 - 0.019852$$

$$\boxed{y_1 = 0.9801}$$

$$z_{(0.2)} = z_1 = z_0 + \frac{1}{6}\left[l_1 + 2l_2 + 2l_3 + l_4\right]$$

$$z_1 = 0 + \frac{1}{6}\left[(-0.2) + 2(-0.1998) + 2(-0.1958) + (-0.1905)\right]$$

$$\Rightarrow -0.16666\left[0.2 + 0.3996 + 0.3916 + 0.1905\right]$$

$$= -0.16666\left[1.1817\right]$$

$$\boxed{z_{(0.2)} = z_1 = -0.1969}$$

Find $y(0.2)$ using fourth order R-K method, given that $y'' = xy$, $y(0) = 1$ and $y'(0) = 1$

$z' = y'' = xy$

$z' = y''$; $z = y'$

$h = \frac{x - x_0}{h}$

$= \frac{0.2 - 0}{}$

$\frac{dy}{dx} = z$ and $\frac{dz}{dx} = xy$

Initial condition

$\boxed{x_0 = 0}$ $\boxed{y_0 = 1}$ $\boxed{z_0 = 1}$

$y(0.2) = ?$ $z(0.2) = ?$

$\boxed{h = 0.2}$

$g(x, y, z) = z$

$f(x, y, z) = xy$

$k_1 = hg(x_0, y_0, z_0)$

$= 0.2\, g(0, 1, 1)$

$= 0.2(1)$

$\boxed{k_1 = 0.2}$

$l_1 = hf(x_0, y_0, z_0)$

$= 0.2\, f(0, 1, 1) \Rightarrow 0.2[(0)(1)]$

$\boxed{l_1 = 0}$

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$k_2 = hg\left[x_0 + h/2,\ y_0 + \frac{k_1}{2},\ z_0 + \frac{l_1}{2}\right]$

$= 0.2g\left[0 + \frac{0.2}{2},\ 1 + \frac{0.2}{2},\ 1 + \frac{0}{2}\right]$

$= 0.2\ g[0.1,\ 1.1,\ 1]$

$k_2 = 0.2\ (1)$

$\boxed{k_2 = 0.2}$

$l_2 = \overset{hf}{\cancel{hg}}\left[x_0 + h/2,\ y_0 + \frac{k_1}{2},\ z_0 + \frac{l_1}{2}\right]$

$= 0.2\ f[0.1,\ 1.1,\ 1]$

$= 0.2\ [(0.1)(1.1)] \Rightarrow 0.2\ [0.11]$

$\boxed{l_2 = 0.022}$

$k_3 = hg\left[x_0 + \frac{h}{2},\ y_0 + \frac{k_2}{2},\ z_0 + \frac{l_2}{2}\right]$

$= 0.2g\left[0 + \frac{0.2}{2},\ 1 + \frac{0.2}{2},\ 1 + \frac{0.022}{2}\right]$

$= 0.2\ g[0.1,\ 1.1,\ 1.011]$

$= 0.2\ [1.011]$

$\boxed{k_3 = 0.2022}$

$l_3 = hf\left[x_0 + h/2,\ y_0 + \frac{k_2}{2},\ z_0 + \frac{l_2}{2}\right]$

$= 0.2\ f[0.1,\ 1.1,\ 1.011]$

Scanned with CamScanner

$= 0.2\,[(0.1)(1.1)] \Rightarrow 0.2\,(0.11)$

$\boxed{l_3 = 0.022}$

$k_4 = hg\,[x_0+h,\ y_0+k_3,\ z_0+l_3]$

$= 0.2\,g\,[0+0.2,\ 1+0.2022,\ 1+0.022]$

$= 0.2\,g\,[0.2,\ \overset{1.2022}{\cancel{1.02022}},\ 1.022]$

$= 0.2\,[1.022]$

$\boxed{k_4 = 0.2044}$

$l_4 = hf\,[x_0+h,\ y_0+k_3+\ z_0+l_3]$

$= 0.2\,f\,[0.2,\ 1.2022,\ 1.022]$

$= 0.2\,[(0.2)(1.2022)]$

$= 0.2\,[0.24044]$

$\boxed{l_4 = 0.0481}$

$y_{(0.2)} = y_1 = y_0 + \frac{1}{6}\,[k_1 + 2k_2 + 2k_3 + k_4]$

$y_1 = 1 + \frac{1}{6}\,[(0.2) + 2(0.2) + 2(0.2022) + 0.2044]$

$= 1 + 0.16666\,[0.2 + 0.4 + \overset{0.4044}{\cancel{2.022}} + 0.2044]$

$= 1 + 0.16666\,[\overset{1.2088}{\cancel{2.8264}}]$

$= 1 + 0.2015$

$$\boxed{y_{(0.2)} = y_1 = 1.2015}$$

$$z(0.2) = z_1 = z_0 + \frac{1}{6}\left[l_1 + 2l_2 + 2l_3 + l_4\right]$$

$$z_1 = 1 + \frac{1}{6}\left[0 + 2(0.022) + 2(0.022) + 0.0481\right]$$

$$= 1 + 0.16666\left[0.044 + 0.044 + 0.0481\right]$$

$$= 1 + 0.16666\left[0.1361\right]$$

$$= 1 + 0.0227$$

$$\boxed{z_{(0.2)} = z_1 = 1.0227}$$

2 marks:

1. State the order of convergence and converge condition for newton's Raphson method

Order of convergence is two

Converge condition is

$$|f(x)\, f''(x)| < |f'(x)|^2$$

2. Construct a linear interpolating polynomial given the point (x_0, y_0) and (x_1, y_1)

$$y(x) = \frac{(x - x_1)}{(x_0 - x_1)} y_0 + \frac{(x - x_0)}{(x_1 - x_0)} y_1$$

Similiary 3 points (x_0, y_0) (x_1, y_1) (x_2, y_2)

$$y(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2$$

3. Obtain the divided difference table following data.

x	-1	0	2	3
y	-8	3	1	12

x	$y(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$
-1	-8			
		$\frac{3-(-8)}{0-(-1)} = 11$		
0	3		$\frac{-1-(11)}{2-(-1)} = \frac{12}{3} = -4$	
		$\frac{1-3}{2-0} = -1$		$\frac{4-(-4)}{3-(-1)} = 2$
2	1		$\frac{11-(-1)}{3-0} = 4$	
		$\frac{12-1}{3-2} = 11$		
3	12			

4. Find ~~and~~ an iterative formula to find root $\sqrt{N}$, where N is positive number

Let $x = \sqrt{N}$

$$x^2 = N \Rightarrow x^2 - N = 0$$

$$f(x) = x^2 - N = 0$$

$$f'(x) = 2x$$

By Newton's Raphson method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n^2 - N}{2x_n} = \frac{2x_n x_n - x_n^2 + N}{2x_n}$$

$$x_{n+1} = \frac{x_n^2 + N}{2x_n}$$

Improved Euler's Method (or) Modified

Euler's Method:

In general

1. $y'_{n+1} = y_n + hf(x_n, y_n)$

2. $y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y'_{n+1})\right]$

where $n = 0, 1, 2, \ldots$

$x_{n+1} = x_n + h$

Error = order of (h^3)

1. Given $\frac{dy}{dx} = -xy^2$, $y(0) = 2$ using improved Euler's method find $y(0.2)$ in two steps of 0.1 each.

$y(0) = 2$ $\boxed{x_1 = x_0 + h}$

$\boxed{x_0 = 0}$; $\boxed{y_0 = 2}$ $\frac{dy}{dx} = f(x, y) = -xy^2$

$\boxed{h = 0.1}$

Improve Euler's method

(i). $y'_1 = y_0 + hf(x_0, y_0)$

(ii). $y_1 = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1)\right]$

Step 1 :- $x_1 = x_0 + h$

$x_1 = 0 + 0.1 = 0.1$

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(i) $y_1' = 2 + 0.1\,[-0(2)^2=0]$ $x_1 = x_0 + nh = 0.1$

$y_0 = 2$ $y_1 = ?$

$y_1' = 2$

(ii). $y_1 = y(0.1) = 2 + \frac{0.1}{2}\left[-0(2)^2 + (-0.1(2)^2)\right]$

$= 2 + 0.05\,[-0.4]$

$y_1(0.1) = 2 - 0.020$

$\boxed{y_1 = 1.98}$

Step - 2 :-

(i). $\cancel{y_2'} = y_1 + \cancel{h}f\,(\cancel{x_1}, \cancel{y_1})$ $x_1 = 0.2$

$= \cancel{1.98 + 0.1\,f\,[-0(2)^2]}$ $y_1 = 1.98$

$= \cancel{1.98 + 0.1}$

$y_1' = \cancel{2.08}$

$y'_{n+1} = y_n + hf\,(x_n, y_n)$

$y_{n+1} = y_n + \frac{h}{2}[f(x_n, y_n)$

~~(ii). $y_2' = y_{(0.2)} = y_1 + \frac{h}{2}$ [~~

(i). $y_2' = y_1 + hf\,[x_1, y_1]$

$= 1.98 + 0.1\,[-(0.1)(1.98)^2]$

$= \cancel{1.98 + 0.1\,[-0.7840]}$ =) 0.0198

$= 1.98 - \cancel{0.07840}$ 0.0198

$= \cancel{1.9016}$ /. $y_2' = 1.94079$ //.

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(ii). $y_2 = y(0.2) = y_1 + \frac{h}{2}[f(x_1, y_1) + f(x_2, y_2')]$

$= 1.98 + \frac{0.1}{2}[-(0.1)(1.98)^2 - (0.2)(1.94079)]$

$= 1.98 + 0.05[-0.1(3.9204) - 0.2(3.76666)]$

$= 1.98 + 0.55[-0.39204 - 0.753332]$

$= 1.98 + 0.55[-1.145372]$

$= 1.98 - 0.6299546$

1.3500454 //.

$y_2 = 1.35005$ //.

Given that $dy/dx = \log(x+y)$, with $y = 1$ when $x = 0$ find y for $x = 0.2$ and $x = 0.5$ using improved Euler's method correct to four decimal places.

$$f(x,y) = \frac{dy}{dx} = \log(x+y)$$

$x_0 = 0$ $\quad y_0 = 1$ $\quad \boxed{h = 0.2}$

$x_1 = 0.2$ $\quad y_1 = ?\ (y_1(0.2)) = ?$

$x_2 = 0.5$ $\quad y_2 = ?\ y(0.5) = ?$ $\quad \boxed{h = 0.3}$

General

$y'_{n+1} = y_n + hf(x_n, y_n)$

$y_{n+1} = y_n + \frac{h}{2}[f(x_n, y_n) + f(x_{n+1}, y'_{n+1})]$

Step - 1 :- $x_0 = 0$; $y_0 = 1$

$x_1 = 0.2$ $y_1 = ?$

(i). $y_1' = y_0 + hf\ (x_0, y_0)$

(ii). $y_1 = y_0 + h/2\ [f(x_0, y_0) + f(x_1, y_1')]$

(i). $y_1'(0.2) = 1 + 0.2\ [\log(x_0+y_0)]$

$= 1 + 0.2\ [\log(0+1)]$

$= 1 + 0.2\ (0) \Rightarrow 1+0$

$= 1$

(ii). $y_1(0.2) = y_0 + h/2\ [\log(x_0+y_0) + \log(x_1+y_1)]$

$= 1 + \frac{0.2}{2}\left[\log(0+1) + \log(0.2+1)\right]$

$= 1 + 0.1\ [\log(1) + \log(1.2)]$

$= 1 + 0.1\ [0 + 0.07918] \Rightarrow 1 + 0.1(0.017918)$

$= 1 + 0.00791812$

$y_1(0.2) = 1.0079$ //.

Step - 2 :-

$y_2' = y_1 + hf\ (x_1, y_1)$

$y_2 = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, y_2')\right]$

$y_2'(0.5) = 1.0079 + 0.3 [\log (0.2 + 1.0079)]$

$= 1.0079 + 0.3 [\log (1.2079)]$

$= 1.0079 + 0.3 (0.08203)$

$= 1.0079 + 0.02460$ 1.040

$y_2' = 1.03250$ //.

$y_2(0.5) = 1.0079 + \frac{0.3}{2} [\log (0.2 + 1.0079) + \log (0.5) + 1.0325)]$

$= 1.0079 + 0.15 [\log (1.2079) + \log (1.5325)]$

$= 1.0079 + 0.15 [0.08203 + 0.18540]$

$= 1.0079 + 0.115 [0.26743]$

$= 1.0079 + 0.0401$

$y_2 = 1.0480$ //.

$\therefore \quad y(0.2) = 1.0079$

$y(0.5) = 1.0480.$

2 mark :

By Gauss elimination method, Solve $x + y = 2$;

$2x + 3y = 5$

$A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 5 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix}$

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$$(A/B) = \left(\begin{array}{cc|c} 1 & 1 & 2 \\ 2 & 3 & 5 \end{array}\right) \quad R_2 \rightarrow R_2 - 2R_1$$

$$= \left(\begin{array}{cc|c} 1 & 1 & 2 \\ 2-2\times1 & 3-2\times1 & 5-2\times2 \end{array}\right)$$

$$= \left(\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 1 & 1 \end{array}\right)$$

$y = 1$ $\qquad x + y = 2$

$\boxed{y = 1}$ $\qquad x + 1 = 2 \Rightarrow x = 2 - 1$

$\boxed{x = 1}$

Solve the system of Equations $x - 2y = 0$; $2x + y = 5$ By Gauss Elimination method.

$$A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

$$(A/B) = \left(\begin{array}{cc|c} 1 & -2 & 0 \\ 2 & 1 & 5 \end{array}\right) \quad R_2 \rightarrow R_2 - 2R_1$$

$$= \left(\begin{array}{cc|c} 1 & -2 & 0 \\ 2-2\times1 & 1-(-2\times2) & 5-(0\times2) \end{array}\right)$$

$$= \left(\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 5 & 5 \end{array}\right)$$

$5y = 5$ $\qquad x - 2y = 0$

$y = 5/5$ $\qquad x - 2(1) = 0$

$\boxed{y = 1}$ $\qquad \boxed{x = 2}$

Obtain the divided difference table for the following data

x	2	3	5
y	0	14	102

x	y	Δy	$\Delta^2 y$
2	0		
		$\frac{14-0}{3-2} = 14$	
3	14		$\frac{44-14}{5-2} = \frac{30}{3} = 10$
		$\frac{102-14}{5-3} = 44$	
5	102		

Find the polynomial for the following data by Newton's backward difference formula:

x	0	1	2	3
y	-3	2	9	18

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	-3			
		5		
1	2		2	
		7		0
2	9		2	
		9		
3	18			

$$y = f(x) = y_n + P\Delta y_n + \frac{P(P+1)}{2!}\nabla^2 y_n + \cdots$$

$$P = \frac{x - x_n}{h} = \frac{x-3}{1}$$

$$\boxed{P = x-3}$$

$$y = 18 + (x-3)9 + \frac{(x-3)[(x-3)+1](2)}{2}$$

$$= 18 + 9x - 27 + \frac{(x-3)(x-2)(\cancel{2})}{\cancel{2}}$$

$$= 18 + 9x - 27 + x^2 - 2x - 3x + 6$$

$$y = x^2 + 4x - 3$$

write down the expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = x_0$ by

By newton's forward difference table formula.

$$\frac{dy}{dx} = \frac{1}{h}\left[\nabla y_0 - \frac{\nabla^2 y_0}{2} + \frac{\nabla^3 y_0}{3} - \frac{\nabla^4 y_0}{4} \cdots\right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\nabla^2 y_0 - \nabla^3 y_0 + \frac{11}{12}\nabla^4 y_0 \cdots\right]$$

Find $y(1.1)$ given $dy/dx = x+y$; $y(1) = 2$ by Euler's method.

$f(x,y) = x+y$ $\quad x_0 = 1$; $y_0 = 2$ $\quad h = \frac{x-x_0}{}$

$y(x_1) = y_0 + hf(x_0+y_0)$ $\quad h = 1.1-1$

$\boxed{h = 0.1}$

$y(1.1) = 2 + 0.1(1+2)$

$= 2 + 0.1(3) \Rightarrow 2 + 0.3$

$\boxed{y(1.1) = 2.3}$

12 a). (i). Find an iteration formula to find $\sqrt{N}$ where N is a positive integer using Newton's method and hence find $\sqrt{11}$

$x = \sqrt{N}$

$x^2 = N \Rightarrow x^2 - N = 0$

$f(x) = x^2 - N$

$f'(x) = 2x$

By newton's formula,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2, \ldots$$

$$x_{n+1} = \frac{2x_n^2 - (x_2^2 - N)}{2x_n} \Rightarrow \cancel{\frac{2x_n^2}{}} = \frac{1}{2}\left[x_n + \frac{N}{x_n}\right]$$

$N = 11$ and $x_0 = 3$, $x_1 = 3.3333$, $x_2 = 3.3167$

$x_3 = 3.3166$ and $x_4 = 3.3166$

$\therefore \sqrt{11} = 3.3166$

Find the inverse of a matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ By Gauss - Jordan method.

$$(A/I) = \left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 2 & 3 & 1 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 \to 2R_1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 1 & -3 & -2 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 2 & -1 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & -4 & -1 & -1 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 7/4 & -5/4 & 1/4 \\ 0 & 1 & 0 & -5/4 & 3/4 & 1/4 \\ 0 & 0 & 1 & 1/4 & 1/4 & -1/4 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 + R_3/4 \\ R_2 \to R_2 + R_3/4 \\ R_3 \to (-1/4\, R_3) \end{array}$$

$$A^{-1} = \begin{bmatrix} 7/4 & -5/4 & 1/4 \\ -5/4 & 3/4 & 1/4 \\ 1/4 & 1/4 & -1/4 \end{bmatrix}$$

~~Compare~~ Compute $\int_0^1 dx/_{1+x^2}$ by using Trapezoidal rule, taking $h = 0.5$ and $h = 0.25$ hence the find the value of the above integral by Romberg's method.

x	0	0.25	0.5	0.75
$fx = \frac{1}{1+x^2}$	1.0	0.9412	0.8	0.64

By Trapezoidal rule, taking $h = 0.5$,

$$I_1 = \int_0^1 \frac{dx}{1+x^2} = \frac{0.5}{2}\left[1.5+2(0.08)\right]$$

$$= 0.25\left[1.5+0.16\right] \Rightarrow 0.25(1.66)$$

$$= 0.775$$

By Trapezoidal rule taking $h = 0.25$,

$$I_2 = \int_0^1 dx/1+x^2 = \frac{0.5}{2}\left[1.5+2(0.9412+0.8+0.64)\right]$$

$$= 0.7828$$

By Romberg's method, the value of integral is $I\} = \dfrac{4I_2 - I_1}{3} = 0.7854$

Using Runge-kutta method of fourth order $\dfrac{dy}{dx} = \dfrac{y^2-x^2}{y^2+x^2}$

given $y(0) = 1$ at $x = 0.2$

$$y' = f(x,y) = \frac{y^2-x^2}{y^2+x^2}$$

$\boxed{x_0 = 0}$, $\boxed{h = 0.2}$ $\boxed{y_0 = 1}$

$$K_1 = hf(x_0, y_0) = 0.2$$

$$K_2 = hf(x_0 + h/2, y_0 + K_1/2) \Rightarrow 0.2\left(0+\frac{0.2}{2}, 1+\frac{0.2}{2}\right)$$

$$= 0.19672$$

$$k_3 = hf\left(x_0 + h/2, y_0 + \frac{k_2}{2}\right) \Rightarrow 0.2\left(0+\frac{0.2}{2}, 1+\frac{0.19672}{2}\right)$$

$$= 0.1967$$

$$K_4 = hf(x_0 + h, y_0 + K_4) \Rightarrow 0.2(0+0.2, 1+0.1967)$$

$$= 0.1891$$

$$y(0.2) = y_0 + 1/6\,(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 1.19598$$

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Euler's Method

$y(x_0) = y_0$

$\frac{dy}{dx} = y' = z = f(x, y)$

$\frac{dz}{dx} = f(x, y, z)$

$y'' = -y' + \sin xy$

$y(0) = 1$ $\boxed{x_0 = 0}$ $\boxed{y_0 = 1}$ $\boxed{z_0 = 2}$

$y''(0) = 2$ $x_1 = 1$ $y_1 = ?$

$y' = z$.

$h = x_1 - x_0$

$z' = -z + \sin xy$

$h = 1 - 0$

$y'(0) = z(0) = 2$

$\boxed{h = 1}$

$y'' = f(x, y, y')$

$y(x_0) = y_0 \Rightarrow y'(x_0) = y_0$

$x_1 = x_0 + h$

$y_1 = y_0 + h\Delta y$

$z_1 = z_0 + h\Delta z$

$y_1 = y_0 + hg(x_0, y_0, z_0)$

$= 1 + 1(0, 1, 2)$

$= 1 + 1[(-2) + \sin(0)(1)]$

$= 1 + 1 \, (2)$

$= 1 + 2$

$\boxed{y_1 = 3}$

$z_1 = z_0 + hf\,(x_0, y_0, z_0)$

$= 2 + 1\,[(-2) + \sin(0)(1)]$

$= 2 + 1\,[(-2) + 0(0.8415)]$

$= 2 + 1(-2) \Rightarrow 2 - 2$

$\boxed{z_1 = 0}$

1^{st} order differenciate Equation 2 - point formula :

$f'(x_0)$	$\simeq$	$\dfrac{f(x+h) - f(x)}{h}$	(forward)
$f'(x_0)$	$\simeq$	$\dfrac{f_0 - f_{-1}}{h}$	(backward)
$f'(x_0)$	$\simeq$	$\dfrac{f_1 - f_{-1}}{2h}$	(Centre difference)

Four point formula

$$f'(x_0) = \frac{-f_2 + 8f_1 - 8f_{-1} + f_{-2}}{12h}$$

$$\frac{d^2y}{dx^2} = \frac{dz}{dx} = z' = y'' \Rightarrow f(x, y, z)$$

$$dy/dx = z = y' = g(x, y, z)$$

Solve $2\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 5y = 11e^{-x}$ using Euler's method using $y(0) = 7$. $dy/dx\,(0) = 13$, $h = 0.25$

find y ? (0.5)

Let $dy/dx = z$

$y_0 = 7$

$x_0 = 0$

$$\frac{d^2y}{dx^2} = \frac{dz}{dx} = z'$$

$dy/dx\,(0) = z_0 = 13$

$\boxed{h = 0.25}$

$$2\frac{dz}{dx} + 3z + 5y = 11e^{-x}$$

$$\frac{dy}{dx} = z = g(x, y, z) \longrightarrow (1)$$

$$\frac{dz}{dx} = \frac{11e^{-x} - 3z - 5y}{2} = f(x, y, z) \longrightarrow (2)$$

$x_0 = 0$	$y_0 = 1$	$z_0 = 13$
$x_1 = 0.25$	$y_1 = ?$	$z_1 = ?$
$x_2 = 0.5$	$y_2 = ?$	$z_2 = ?$

From Euler's formula

$$y_{n+1} = y_n + hg(x_n, y_n, z_n)$$

$$z_{n+1} = z_n + hf(x_n, y_n, z_n)$$

$\boxed{n = 0}$

$$y_1 = y_0 + hg(x_0, y_0, z_0)$$

$= y_0 + hg\ (0, 1, 3)$

$= 7 + 0.25\ (13)$

$= 7 + 3.25$

$\boxed{y_1 = 10.25}$

$z_1 = z_0 + hf\,(x_0, y_0, z_0)$

$= z_0 + hf\left[\dfrac{11e^{-x_0} - 3z_0 - 5y_0}{2}\right]$

$= 13 + 0.25\left[\dfrac{11(1) - 3(13) - 5(7)}{2}\right]$

$= 13 + 0.25\left[\dfrac{11 - 39 - 35}{2}\right] \Rightarrow 13 + 0.25\left[\dfrac{-63}{2}\right]$

$= 13 + 0.25\,(-3.15) \Rightarrow 13 - 7.875$

$\boxed{z_1 = 5.125}$

$y(0.5) = y_2 = y_1 + hg\ (x_1, y_1, z_1)$

$= 10.25 + 0.25\ g\ (0.25, 10.25, 5.125)$

$= 10.25 + 0.25\ (5.125)$

$= 10.25 + 1.28125$

$\boxed{y_2 = 11.5312}$

$z(0.5) = z_2 = z_1 + hf\ (x_1, y_1, z_1)$

$= z_1 + hf\left[\dfrac{11e^{-x_1} - 3z_1 - 5y_1}{2}\right]$

$$= 5.125 + 0.25 \left[\frac{11e^{-0.25} - 3(5.125) - 5(10.25)}{2} \right]$$

$$= 5.125 + 0.25 \left[\frac{11(0.77880) - 3(5.125) - 5(10.25)}{2} \right]$$

$$= 5.125 + 0.25 \left[\frac{8.5668 - 15.375 - 5.125}{2} \right]$$

$$= 5.125 + 0.25 \left[\frac{-11.9332}{2} \right]$$

$$= 5.125 + 0.25 (-5.9666)$$

$$= 5.125 - 1.49165$$

$$z_2 = 3.63335$$ //

04.03.19 Monday

2nd Order Differential Equation

3 point formula

$$f''(x_0) = \frac{f_1 - 2f_0 + f_{-1}}{h^2}$$

$x_0 = 1$	$y_0 = 0$
$x_1 = 1.25$	$y_1 = ?$
$x_2 = 1.5$	$y_2 = ?$
$x_3 = 1.75$	$y_2 = ?$
$x_4 = 2$	$y_4 = 2$

5 point formula:

$$f''(x_0) = \frac{-f_2 + 16f_1 - 30f_0 + 16f_{-1} - f_{-2}}{12h^2}$$

$f_0 = f(x_0)$

$f_1 = f(x_0 + h)$

$f_2 = f(x_0 + 2h)$

$f_{-1} = f(x_0 - h)$

$f_{-2} = f(x_0 - 2h)$

1. Solve the boundary problem $x^2 y''(x) + xy'(x) + (x^2-3)y(x) = 0$ given $y(1)=0$, $y(2)=2$ and take $h = 0.25$

To find $= y_0, y_1, y_2$

Given $y(1) = 0$, $\boxed{x_0 = 1}$ $\boxed{y_0 = 0}$

$y(2) = 2$ $x_1 = 2$ $y_1 = 2$

The approximations

$$y''(x) = \frac{f_1 - 2f_0 + f_{-1}}{h^2} = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

$$y'(x) = \frac{f_1 - f_{-1}}{2h} = \frac{y_{i+1} - y_{i-1}}{2h}$$

The approximation $h = 0.25 = 1/4$

$$x_i^2\left[\frac{y_{i+1} - 2y_i + y_{i-1}}{(1/4)^2}\right] + x_i\left[\frac{y_{i+1} - y_{i-1}}{2(1/4)}\right] + (x_i^2-3)y_i = 0$$

for $i = 1, 2, 3$

$$16x_i^2[y_{i+1} - 2y_i + y_{i-1}] + 2x_i[y_{i+1} - y_{i-1}] + (x_i^2 - 3)y_i = 0$$

$$i=0 \Rightarrow 16 x_0^2 [y_{0+1} - 2y_0 + y_{-1}] + 2x_0[y_1 - y_{-1}] + (x_0^2 - 3)y_0 = 0$$

$i=1 \Rightarrow 16x_1^2\left[y_2-2y_1+y_0\right]+2x_1\left[y_2-y_0\right]+$

$$(x_1^2-3)\,y_1=0$$

$$16\left(\frac{5}{4}\right)^2\left[y_2-2y_1\right]+2\left(\frac{5}{4}\right)\left[y_2\right]+\left(5/4\right)^2-3y_1=0$$

$$\cancel{16}\left(\frac{25}{\cancel{16}}\right)\left[y_2-2y_1\right]+\frac{5}{2}y_2+\frac{25}{16}-3y_1=0$$

$$25y_2-50y_1+\frac{5}{2}y_2+\frac{25}{16}-3y_1=0$$

$$\times 16 \Rightarrow 400y_2-800y_1+40y_2+\cancel{16}\times\frac{25}{\cancel{16}}-3(16)-y_1=0$$

$$\Rightarrow 823\,y_1-440\,y_2=0$$

$i=2 \Rightarrow 16x_2^2\left[y_3-2y_2+y_1\right]+2x_2\left[y_3-y_1\right]+(x_2^2-3)y_2=0$

$$16\left(\frac{6}{4}\right)^2\left[y_3-2y_2+y_1\right]+2\left(6/4\right)\left[y_3-y_1\right]+\left(\frac{6}{4}\right)^2-3y_2=$$

$$\cancel{16}\left(\frac{36}{\cancel{16}}\right)\left[y_3-2y_2+y_1\right]+3\left[y_3-y_1\right]+\frac{36}{16}-3y_2=0$$

2 marks:-

1. Under the conditions that f(a) and f(b) have opposite signs and a<b, find the first approximation to the root of $f(x)=0$ by the method of flase position.

$$x = \frac{af(a) - bf(a)}{f(b) - f(a)}$$

2. write the itration formula for Newton - Rapshon method to find a root of $f(x)=0$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2, \ldots$$

3. State the Criterin for the Convergence in Newton_Rapshon method?

Newton Rapshon method convergens if $|f(x)f''(x)| < [f'(x)]^2$ in the interval considered.

4. Define order of convergence. What is order of convergence of Newton's method?

Let $\phi(x_n) = x_{n+1}$ be an iteration method for solving the Equation $x = \phi(x)$. If α is a root of the Equation, then

$x_n = \alpha + \epsilon_n$

5. In an approximation value of the root of the Equation $x^x = 1000$ is 4.5. find a better approximation of the root of by Newton's method

$$f(x) = x\log_e x - \log_e 1000$$

$$f'(x) = 1 + \log_e x$$

Approximation is $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 4 + \dfrac{0.0605}{1.6532} = 4.5366$

6. Write Newton's formula to find the cube root of N

$x^3 - N = 0 \qquad f(x) = x^3 - N \qquad f'(x) = 3x^2$

By Newton's method

$$x_{n+1} = x_n - \frac{x_n^3 - N}{3x_n^2} \Rightarrow \frac{2x_n^3 + N}{3x_n^2}$$

7. Establish an iteration formula to find the reciprocal of a positive number N by Newton's method

$$N = \frac{1}{x} \Rightarrow f(x) = \frac{1}{x} - N \qquad f'(x) = \frac{1}{x^2}$$

$n = 0, 1, 2,$

$$x_{n+1} = x_n - \frac{(1/x_n - N)}{-1/x_n^2} \Rightarrow x_{n+1} = x_n(2 - Nx_n),$$

8. Find the first approximation to the root lying between 0 and 1 of $x^3 + 3x - 1 = 0$ by Newton's method

$$f(x) = x^3 + 3x - 1 \qquad f'(x) = 3x^2 + 3$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \Rightarrow x_1 = 0 - \left(-\tfrac{1}{3}\right)$$

$= 0.3333$

9. Find an iteration formula for finding the square root of N by Newton's method.

$$f(x) = x^2 - N \Rightarrow f'(x) = 2x$$

$$x_{n+1} = x_n - \frac{(x_n^2 - N)}{2x_n} \Rightarrow x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{x_n}\right) \quad n = 0, 1, 2, \ldots$$

10. What is the order of convergence of fixed point iteration $x = g(x)$ method?

The order of convergence of $x = g(x)$ method is one

11. What is the sufficient condition for the convergence of $x = g(x)$ method?

The sufficient condition for the convergence is $|\phi'(x)| < 1$ for all x in the interval I containing the root $x = \alpha$ of the Equation $f(x) = 0$. Which can be written as $x = \phi(x)$

12. How do you express the Equation $x^3 + x^2 - 1 = 0$ in the interval $[0, 1]$ by iteration method?

$$x^2(x+1) = 1 \Rightarrow x = \frac{1}{\sqrt{x+1}} = \phi(x)$$

13. Distinguish Gauss Elimination and Gauss Jordan

Gauss Elimination	Gauss Jordan
(i). Co-efficient matrix A of the system reduces into upper triangular matrix	Co-efficient matrix A of the system reduces into diagonal of unit matrix
(ii). Back substitution process gives solution	solution obtained directly.

14. State the principal involved in Gauss-elimination method of solving a system of Equation.

Augmented matrix (A, B) reduces into (U, K) and solution is obtained from the Equivalent upper triangular system of Equation by back substitution.

15. By Gauss Elimination, solve $x + y = 2$; $2x + 3y = 5$

$$(A, B) = \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 2 & 3 & 5 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 1 & 1 \end{array}\right]$$

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$x+y=2 \Rightarrow x+1=2 \Rightarrow x=2-1 \Rightarrow \boxed{x=1}$

$\boxed{y=1}$

16. Solve $3x+2y=4$; $2x-3y=7$ by Gauss Elimination method.

$$(A,B)\sim\left[\begin{array}{cc|c}3 & 2 & 4\\ 2 & -3 & 7\end{array}\right] \Rightarrow \left[\begin{array}{cc|c}3 & 2 & 4\\ 0 & -13/3 & 13/3\end{array}\right]$$

$\boxed{y=-1}$ $\boxed{x=2}$

17. Gauss Jordan method - $3x+2y=4$, $2x-3y=7$

$$\left[\begin{array}{cc|c}3 & 2 & 4\\ 2 & -3 & 7\end{array}\right] \sim \left[\begin{array}{cc|c}1 & 0 & 2\\ 0 & 1 & -1\end{array}\right]$$

$\boxed{x=2}$ $\boxed{y=-1}$

18. State the basic principal involved for finding A^{-1} by Gauss -Jordan method

Reduce the agumented matrix (A/I) into (I, X) then $X=A^{-1}$

19. State Lagrange's formula to find $y(x)$ if three sets of values (x_0, y_0) (x_1, y_1) and (x_2, y_2) are given

$$y(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}y_2$$

20. Explain the use of Lagrange's interpolation formula for inverse interpolation

Lagrange interpolation formula is a relation between two variables x and y in which x or y is taken as independent variable Replacing x by y and y by x in Lagrange formula, we can use the resulting formula for finding x ~~from~~ for a given y.

21. If $f(3)=5$ and $f(5)=3$, what is the form of $f(x)$ by Lagrange's formula ?

$$y = \frac{x-5}{3-5}(5) + \frac{x-3}{5-3}(3) \Rightarrow y = -\frac{5}{2}(x-5) + \frac{3}{2}(x-3)$$

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$y = -x + 8 \qquad f(x) = 8 - x$

2. State any two properties of divided differences.

(i). The divided differences are symmetrical in all their arguments

(ii). The divided difference of sum or difference of two functions is equal to the sum or difference of the corresponding separate divided difference

3. Find the divided difference for the data

x	2	5	10
y	5	29	109

$f(2,5) = \frac{29-5}{3} = 8$

$f(5,10) = \frac{80}{5} = 16$

$f(2,5,10) = \frac{8}{8} = 1$

4. Newton's divided difference interpolation formula

$f(x) = f x_0 + x - x_0 \ f x_0, x_1 + x - x_0 \ \ x - x_1 \ f \ x_0, x_1, x_2 + \dots$

$+ x - x_0 \ \ x - x_1 \dots \ x - x_{n-1} \ f \ x_0, x_1, \dots x_n.$

25. Find the divided differences of $f(x) = x^2 + x + 2$ for the arguments 1, 3, 6, 11.

The corresponding $f(x)$ values are 4, 32, 224, 1344

The divided difference are $f(1,3) = \frac{32-4}{3-1} = 14$

$f(3,6) = \frac{224-32}{6-3} = 64$; $f(6,11) = \frac{1344-224}{11-6} = 224$;

$f(1,3,6) = \frac{64-14}{6-1} = 10$, $f(3,6,11) = \frac{224-64}{11-3} = 20$ and

$f(1,3,6,11) = \frac{20-10}{11-1} = 1$

26. Show that $f(x_0, x_1) = f(x_1, x_0) = \frac{f(x_0)}{x_0 - x_1} + \frac{f(x_1)}{x_1 - x_0}$

$f(x_0, x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{f(x_0) - f(x_1)}{x_0 - x_1} = f(x_1, x_0)$

Again $f(x_0, x_1) = \frac{f(x_0)}{x_0 - x_1} + \frac{f(x_1)}{x_1 - x_0}$

In the same way $f(x_1, x_0) = \frac{f(x_1)}{x_1 - x_0} + \frac{f(x_0)}{x_0 - x_1}$

27. Fit a polynomial of least degree to the following data

x	1	2	4
y	5	10	26

$\boxed{x_0 = 1}$ $\boxed{x_1 = 2}$ $\boxed{x_2 = 4}$

$\boxed{y_0 = 5}$ $\boxed{y_1 = 10}$ $\boxed{y_2 = 26}$

By divided difference table

$f(x_0) = 5$, $f(x_0, x_1) = 5$, $f(x_1, x_2) = 8$ $f(x_0, x_1, x_2) = 1$

$y = f(x) = f(x_0) + (x - x_0) f(x_0, x_1) + (x - x_0)(x - x_1) f(x_0, x_1, x_2)$

$= 5 + (x-1) 5 + (x-1)(x-2)$

$= x^2 - 3x + 2 + 5x - 5 + 5$

$= x^2 + 2x + 2$

28. What are the n^{th} divided difference of a polynomial of the n^{th} degree?

The n^{th} divided difference of an n^{th} degree polynomial are constant.

29. Divided difference table given

x	1	2	4	7
y	22	30	82	106

$f(1,2) = 8$ $f(2,4) = 26$ $f(4,7) = 8$

$f(1,2,4) = 6$, $f(2,4,7) = -3.6$ $f(1,2,4,7) = -1.6$

30. Newton's backward interpolation formula:

$f(x) = f(x_n + ph) = y_n + p\nabla y_n + \frac{p(p+1)}{2!} \nabla^2 y_n + \frac{p(p+1)(p+2)}{3!} \nabla^3 y_n + \cdots$

where $p = \frac{x - x_n}{h}$

31. When will use Newton's backward interpolation formula?

Newton's backward difference formula is used to interpolate the values of $y = f(x)$ nearer end of a set of tabular values and for extrapolation closer to the right of y_n.

31. Given $f(0) = -1$, $f(1) = 1$ and $f(2) = 4$. Find the newton's interpolating polynomial.

$y_0 = -1$, $\Delta y_0 = 1-(-1) = 2$ $\Delta y_1 = 4-1 = 3$ $\Delta^2 y_0 = 3-2 = 1$

$$p = \frac{x-0}{1} = x$$

$$f(x) = -1 + 2x + \frac{x(x-1)}{2}$$

$$= \tfrac{1}{2}(x^2+3x-2)$$

32. Find the inverse of $\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$ by Gauss-Jordan method.

$$(A/I) = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\sim \left[\begin{array}{cc|cc} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 \end{array}\right] = (I/A^{-1})$$

$$A^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$$

33. Find the inverse of $\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$ by Gauss-Jordan method.

$$(A/I) = \left[\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|cc} 1 & 0 & 0 & 1 \\ 2 & 1 & 1 & 0 \end{array}\right] \Rightarrow \sim \left[\begin{array}{cc|cc} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & -2 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}$$

34. State Newton's Backward interpolation formula to find dy/dx and d^2y/dx^2 at $x = x_n$.

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{\nabla^2 y_n}{2} + \frac{\nabla^3 y_n}{3} + \cdots\right]$$

$$\left(\frac{d^2x}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n \cdots\right]$$

35. State Trapezoidal rule with the error order.

For the given data (x_i, y_i) where $x_i = x_0 + ih$ $i = 0, 1, 2, \ldots n$

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{3}\left[(y_0+y_n)+2(y_1+y_3+\cdots+y_{n-1})\right]$$

Error is of order h^2.

36. State Simpson's 1/3 rule

If (x_i, y_i) $i = 0,1,2,\ldots n$ where $x_i = x_0 + ih$

Simpson's 1/3 rule : $$\int_{x_0}^{x_n} y\,dx = \frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+y_5+\cdots)+2(y_2+y_4+y_6+\cdots)\right]$$

37. Basic principle for deriving Simpson's 1/3 rule

The curve passing through three consecutive points is replaced by a parabola.

38. State the order of error in Simpson's 1/3 rule.

Error in Simpson's 1/3 rule is of order h^4.

39. Using Simpson's rule, find $\int_0^4 e^x\,dx$ $e^0 = 1$, $e^1 = 2.72$ $e^2 = 7.39$, $e^3 = 20.09$ and $e^4 = 54.6$

$$\int_0^4 e^x\,dx = \frac{1}{3}\left[(1+54.6)+4(2.727\ 20.09)+2(7.39)\right]$$

$$= \frac{1}{3}\left[55.6+4(22.81)+14.78\right]$$

$$= \frac{1}{3}\left[55.6+91.24+14.78\right] \Rightarrow \frac{1}{3}\left[161.62\right]$$

$$= 53.873$$

40. A curve passes through $(2,8)$, $(3,27)$, $(4,64)$ and $(5,125)$. Find the area of the curve between x-axis and lines $x=2$ and $x=5$, by Trapezoidal rule.

$$\int_2^5 y\,dx = \frac{1}{2}\left[(8+125)+2(27+64)\right] = \frac{1}{2}\left[133+182\right]$$

$$= \frac{315}{2} = 157.5 \text{ sq. units.}$$

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41. find $\int_{-2}^{2} x^4\,dx$ by simpson's rule taking $h=1$

x	-2	-1	0	1	2
y	16	1	0	1	16

$$\int_{-2}^{2} x^4\,dx = \frac{1}{3}\left[(16+16)+4(2)\right] \Rightarrow \frac{1}{3}\left[32+8\right] = \frac{1}{3}\left[40\right]$$

$$= 13.3 \text{ sq. units.}$$

42. Compute $\int_{1}^{2} dx/x$ using simpson's rule with $h=0.25$

$0.6931.$

43. Evaluate $\int_{0}^{1} dx/1+x^2$ by Trapezoidal rule with $h=0.5$

$$\int_{0}^{1} dx/_{1+x^2} = \frac{0.5}{2}\left[1.5+2(0.8)\right] = 0.25\,(1.5+1.6)$$

$$= 0.25\,(3.1) = 0.775\,//.$$

44. use simpson's $1/3$ rule with $h=0.5$ to evaluate $\int_{0}^{1} dx/_{1+x}$

$$I = \frac{1}{6}\left[1+\frac{4}{1.5}+\frac{1}{2}\right] = \frac{1}{6}\left[1+2.66+0.5\right]$$

$$= \frac{1}{6}\left[4.16\right] = 0.6944$$

45. Evaluate $\int_{-1}^{1} |x|\,dx$ with two sub intervals by simpson's $1/3$ rule by Trapezoidal rule.

By simpson's $1/3$ rule $I = 1/3\,[1+0+1] = 2/3$

By Trapezoidal rule $I = \frac{1}{2}\,[1+1] = \frac{2}{2} = 1$

46. Find $I = \int_{-1}^{1} dx/_{1+x^2}$ by two point formula

$$I = \frac{1}{1+1/3} + \frac{1}{1+1/3} = \frac{3}{2} = 1.5$$

47. Find $I = \int_0^1 dx/1+x$ by Gauss two point formula

$$I = \int_{-1}^{1} \frac{dt}{t+3} \text{ using } x = \frac{1+t}{2}$$

$$I = \frac{1}{3+\frac{1}{\sqrt{3}}} + \frac{1}{3-\frac{1}{\sqrt{3}}} = 0.6923$$

48. Evaluate $\int_{-1}^{1} \cos x \, dx$ using two point Gaussian formula

$$\int_{-1}^{1} \cos x \, dx = 2\cos\left(\frac{1}{\sqrt{3}}\right) = 1.6758$$

49. Find $y(0.1)$ from $y' = x+y$, $y(0) = 1$ by Euler's ~~formula~~ method

$$y_1 = 1 + 0.1[0+1] = 1.10$$

$$y(0.1) = 1.10$$

50. Given $y'+y=0$ and $y(0)=1$ find $y(0.01)$, $y(0.02)$ by Euler's method.

$$y_1 = 1 + (0.01)[-1] = 0.99$$

$$y(0.01) = 0.99$$

$$y_2 = 0.99 + (0.01)[-0.99] = 0.9801$$

$$y(0.02) = 0.9801$$

51. State algorithm for modified Euler method, to solve

$dy/dx = f(x,y)$, $y(x_0) = y_0$

$$y_{n+1}^{(1)} = y_n + h f(x_n, y_n)$$

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y_{n+1}^{(1)})\right]$$

Where $n = 0, 1, 2, \ldots$ and $x_{n+1} = x_n + h$

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52. State Runge-Kutta fourth order formula for solving $dy/dx = f(x,y)$, $y(x_0) = y_0$

$$y_1 = y(x_0+h) = y_0 + \frac{1}{6}[k_1+2k_2+2k_3+k_4]$$

$k_1 = hf(x_0, y_0)$; $k_2 = hf[x_0 + h/2, y_0 + k_1/2]$

$k_3 = hf(x_0+h/2, y_0 + k_2/2)$; $k_4 = hf[x_0+h, y_0+k_4]$

53. What are the distinguished properties of Runge-Kutta methods?

(i). These methods do not require the higher order derivatives and requires only the function values at different points.

(ii). To evaluate y_{n+1}, we need only y_n but not previous of y's.

(iii). The solution by these methods agree with Taylor series solution upto the terms of h^r where r is the order of the Runge-kutta method.

Local ~~transition~~ truncation error:

The amount of transition error that occurs in one step of a numerical approximation.

Global ~~transition~~ truncation error:

The amount of transition error that occurs in the use of numerical approximation to slove a problem.

Error :-

The term used to denote the amount by which an approximation fails to equal the exact solution

Error occurs in approximation for several reation

Truncation error

found of error

How can we reduces the truncation error?

Truncation error can be reduces by applying the same approximation to large number of smaller intervals or by switching to a better approximation.

Composite Trapezoidal rule :-

function $y = f(x)$

find the area over $[a, b]$

In the intervals $[x_K, x_{K+1}]$

Suppose that intervals $[a, b]$ is sub divider in M

width $= h = \frac{b-a}{M}$

using Equally spaced notes $x_K = a + kh$ $K = 0, 1, 2, 3, \ldots$

The composite Trapezoidal rule from M intervals expressed in 3 equivalent ways.

$$T(f, h) = \frac{h}{2} \sum_{k=1}^{M} \left(f(x_{K-1}) + f(x_K) \right)$$

Trapezoidal error analysis :

$$y = f(x)$$

Suppose that $[a, b]$ is sub divided in to M subintervals $[x_K, x_{K+1}]$

with width $h = \frac{b-a}{M}$

Composite Trapezoidal rule :

$$T(f, h) = \frac{h}{2} \left[f(a) + f(b) \right] + h \sum_{K=1}^{M-1} f(x_K) \simeq \int_a^h f(x)\, dx$$

is approximation to the integral.

$$\int_a^b f(x)dx = T(f,h) + E_T(f,h)$$

if $F \in c^2[a,b]$

There exits a value c with $a<c<b$

so error term $E_T(f,h)$

$$E_T(f,h) = -\frac{(b-a)}{12} f^{(2)}(c)h^2 \simeq O h^2 \ [\text{order of } h^2]$$

Error in simpson 1/3 rule :-