UNIT - 4

Measures of central tendency

Averages (or) Measure of central tendency

(or) Measures of location.

Averages are statistical constants which enable us to comprehend in a single effort the significance of the whole.

Uses :-

i) Arithmetric mean (or)

ii) median

iii) mode

iv) Geometrical mean

v) Harmonic mean

A.M of a set of observations is their sum divided by the No. of observation the Arithemetic mean $\bar{x}$ of number N observation. $x_1, x_2, x_3 \ldots x_n$ is given by

$\bar{X} = x_1 + x_2 + x_3 \ldots x_n / N = \frac{\Sigma x}{N}$

Arithmetic Mean :-

* Individual series

* discrete series

* Continuous series.

Individual series :-

problem :-

(1) Find the arithemetic mean for the following

150, 210, 140, 190, 170 .

Mean $\bar{x} = \frac{\Sigma x}{n}$

Solution :-

here $n = 5$

$\Sigma x = 150+210+140+190+170 = 860$

$= \frac{860}{5}$

$\therefore \bar{x} = \frac{860}{5} = 172$

$\boxed{\therefore \text{ mean} = 172}$

(2) Find the mean for the following

30, 41, 47, 54, 23, 34, 37, 51, 53, 47

Solution

$\bar{x} = \frac{\Sigma x}{n}$

where,

$\Sigma x = 30+41+47+54+23+34+$
$37+51+53+47$

$= \cancel{388}\ 417$

$n = 10$

$\therefore \bar{x} = \frac{\Sigma x}{n} = \frac{417}{10} = 41.7$

$\boxed{\therefore \text{ mean} = 41.7}$

Discrete series :-

$\bar{x} = \frac{\Sigma fx}{N}$

where,

f = frequency

X = The value of the data

$N = \Sigma f$ = The sum of frequency

problem :-

① Find mean from the following data.

X	2	4	6	8	10
Y	10	18	30	16	6

Solution :-

$$\bar{X} = \frac{\Sigma f x}{N}, \quad N = \Sigma f$$

x	f	fx
2	10	20
4	18	72
6	30	180
8	16	128
10	6	60

$\Sigma fx = 20 + 72 + 180 + 128 + 60$

$= 460$

$\Sigma f = 10 + 18 + 30 + 16 + 6$

$= 80$

$\bar{X} = \frac{\Sigma fx}{N} = \frac{460}{80} = 5.75$

2. Find mean from the following data

Marks obtained	4	6	9	12	16	20
No. of students	3	7	11	16	12	5

Solution:-

$\bar{X} = \frac{\Sigma fx}{N}$, $N = \Sigma f$

x	f	fx
4	3	12
6	7	42
9	11	99
12	16	192
16	12	192
20	5	100

$$\sum fx = 12+42+99+192+192+100$$

$$= 637$$

$$\sum f = 3+7+11+16+12+5$$

$$= 54$$

$$\therefore \bar{X} = \frac{\sum fx}{\sum f} = \frac{637}{54}$$

$$\boxed{\bar{X} = 11.7962}$$

$\therefore$ The mean is $\bar{x} = 11.7962$

Continuous series :-

problem :-

(1) From the following table find mean

C.I	20-30	30-40	40-50	50-60	60-70	70-80	80-90
f	(1) 3	(2) 61	(3) 132	(4) 153	(5) 140	(6) 51	(7) 2

Solution:-

$$\text{Mean } \bar{x} = \frac{\Sigma fx}{\Sigma f}$$

C.I	Mid values of x	f	fx
20 - 30	$\frac{20+30}{2}$ 25	3	75
30 - 40	$\frac{30+40}{2}$ 35	61	2135
40 - 50	45	132	5940
50 - 60	55	153	8415
60 - 70	65	140	9100
70 - 80	75	51	3825
80 - 90	85	2	170

$$\Sigma f = 542$$

$$\Sigma fx = 29,600$$

$$\bar{X} = \frac{29,600}{542}$$

$$\boxed{\bar{X} = 54.7232}$$

Unequal Intervals (Continuous series)

1. Find mean from the following data.

Class Intervals	0-9	10-19	20-29	30-39	40-49	50-59
Frequency	13	38	67	76	22	4

Given	changed	mid value	f	f(x)
0-9	-0.5-0.9	4.5	13	58.5
10-19	0.9-19.5	14.5	38	551
20-29	19.5-29.5	24.5	67	1641.5
30-39	29.5-39.5	34.5	76	2622
40-49	39.5-49.5	44.5	22	979
50-59	49.5-59.5	54.5	4	217.6
			220	6070

$$\text{Mean } \bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{6070}{220}$$

$$\bar{x} = 27.59$$

∴ The mean $\bar{x}$ is 27.59

2. Find mean from the following data

C.I	Below 50	Below 60	Below 70	Below 80	Below 90	Below 100
f	3	11	34	59	72	80

Solution:-

$$\text{Mean } \bar{x} = \frac{\Sigma fx}{\Sigma f}$$

Given	Mid values	f	fx
40-50	45	3	135
50-60	55	8	440
60-70	65	23	1495
70-80	75	25	1875
80-90	85	13	1105
90-100	95	8	760

$\Sigma f = 80$ $\Sigma fx = 5810$

$$\bar{X} = \frac{\Sigma fx}{\Sigma f} = \frac{5810}{80} = 72.625$$

$\therefore$ The mean $\bar{x}$ is 72.625

3. Find mean from the following data

C.I	above 40-50	above 50-60	above 60-70	above 70-80	above 80-90	above 90-100
frequency	80	77	69	46	21	8

Solution:-

$$\text{Mean } \bar{x} = \frac{\Sigma fx}{\Sigma f}$$

Given C.I	mid value	f	fx
40-50	45	80-77=3	135
50-60	55	77-69=8	440
60-70	65	69-46=23	1495
70-80	75	46-21=25	1875
80-90	85	21-8=13	1105
90-100	95	8	760
		$\Sigma f = 80$	$\Sigma fx = 5810$

$$\bar{x} = \frac{5810}{80}$$

$\therefore$ The mean $\bar{x}$ is 72.625

Median :-

i) Individual series

ii) Discrete series

iii) Continuous series

Individual series

i) Given data is odd

$$\left(\frac{N+1}{2}\right)^{th} \text{ value}$$

ii) Given data is even

$$\left(\frac{N}{2}\right)^{th} + \left(\frac{N}{2}+1\right)^{th} \text{ term}$$

(1) Find median from the following data

25, 20, 15, 35, 18,

solution :-

arrange the given data in ascending order

15, 18, 20, 25, 35

Total number of data $N = 5$

$$\text{Median} = \left(\frac{N+1}{2}\right)^{th} \text{term}$$

$$= \left(\frac{5+1}{2}\right) = \frac{6}{2} = 3^{rd} \text{ term}$$

$\therefore \quad M = 20$

2. Find Median from the following data

~~8 15 20 25 3~~

8 20 50 25 15 30

solution:-

Arrange the data in ascending order

8, 15, 20, 25, 30, 50

Total number of data $N = 6$

$$\text{Median} = \left(\frac{N^{th}}{2}\right) \& \left(\frac{N}{2}+1\right) \text{term}$$

$$= \left(\frac{6}{2}\right) = 3^{rd} \text{ term} \quad \therefore M = 20$$

$$\left(\frac{6}{2}+1\right) = 3+1 = 4^{rd} \text{ term} \quad \therefore M = 25$$

Mean of 3^{rd} & 4^{th} data

$$= \frac{20+25}{2}$$

$$\boxed{\text{Median} = 22.5}$$

DIS-crete series :-

① Find Median

x	1	2	3	4	5	6	7	8	9
f	8	10	11	16	20	25	15	9	6

Solution

$$\frac{N}{2} = \frac{\Sigma f}{2} = \frac{120}{2} = 60$$

x	f	cf (Cumulative frequency)
1	8	8
2	10	8+10 = 18
3	11	18+11 = 29
4	16	29+16 = 45
5	20	45+20 = 65
6	25	65+25 = 90
7	15	90+15 = 105
8	9	105+9 = 114
9	6	114+6 = 120
	$\Sigma f = 120$	

Cumulative frequency just greater than $\frac{N}{2}$ is 65 and the value of x corresponding to 65 is 5

$\boxed{\therefore \text{ Median } = 5}$

Continuous series

$$\text{median} = L + \left(\frac{\frac{N}{2} - cf}{f} \right) \times i$$

where,

L is lower limit of the median

f is the frequency of the median class

i is the length of class interval of median class

cf is the cumulative frequency of the class of median class

Here $N = \Sigma f$

7-12-19

① Find the median for the following data

Wages (in Rs)	20-30	30-40	40-50	50-60	60-70
No. of Laboures	3	5	20	10	5

Solution

C.I	f	Cf
20 - 30	3	3
30 - 40	5	8
40 - 50	20	28
50 - 60	10	38
60 - 70	5	43

$\Sigma f = 43$

Here $\Sigma f = 43$

Then $\frac{N}{2} = \frac{43}{2} = 21.5$

Cumulative frequency just greater than 21.5 and the corresponding class is 28 ∴ 40 - 50

$L = 40 \quad cf = 8 \quad f = 20 \quad i = 10$

$$M = L + \left(\frac{N/2 - c.f}{f} \right) \times i$$

$$= 40 + \left(\frac{21.5 - 8}{20} \right) \times 10$$

$$= 40 + (0.675) \times 0$$

$$= 46.75.$$

$\therefore$ The median wage is Rs 46.75

If

1. Calculate median from the following data

C.I	4-7	8-11	12-15	16-19	20-23	24-2	28-31	32-35
f	4	11	25	47	56	29	20	8

Solution:-

C.I	Converted C.I	f	c.f
4-7	3.5-7.5	4	4
8-11	7.5-11.5	11	15
12-15	11.5-15.5	25	40
16-19	15.5-19.5	47	87
20-23	19.5-23.5	56	143
24-27	23.5-27.5	29	172
28-31	27.5-31.5	20	192
32-35	31.5-35.5	8	200

$\Sigma f = 200$

$$\frac{N}{2} = \frac{\Sigma f}{2} = \frac{200}{2} = 100$$

Here $L = 19.5$, $c.f = 87$, $f = 56$,

$i = 4$

$$M = L + \left(\frac{\frac{N}{2} - c.f}{f}\right) \times i$$

$$= 19.5 + \left(\frac{100 - 87}{56}\right) \times 4$$

$$= 19.5 + \left(0.23214\right) \times 4$$

$$= 19.5 + 0.92856$$

$$= 20.428$$

$\therefore$ The median M is 20.428

12·12·19

Mode

Individual series

(1) Find mode from the following data

12, 14, 16, 18, 26, 16, 20, 16, 11, 12, 16, 15,

20, 24, 25

Solution :-

Arranging the data in Assending order

11, 12, 12, 14, 15, 16, 16, 16, 16, 18, 20, 20, 24,

25, 26

Here we get "16" 4 time, "12 & 20"

2 time each and other terms once only therefore

mode = (Z) = 16

2. Find mode 16, 18, 22, 4, 3, 1, 4, 5, 7, 9, 4

solution:-

Arrange the data in Assending order

1, 3, 4, 4, 4, 5, 7, 9, 16, 18, 22

Here we get 3 time '4' other terms once only therefore mode(z) = 16

Discrete series

(1) find mode for following data.

x	4	7	11	16	25
f	3	9	14	21	13

solution:-

In the above series highest frequency is 21 and variable x corresponding to this frequency is 16 $\therefore$ mode(z) = 16

2.

x	1	2	3	4	5	6	7	8
f	4	9	16	25	22	15	7	3

Solution:-

In the above series highest frequency is 25 and variable corresponding to x ~~is~~ ~~to~~ the frequency is 4

$\therefore$ mode = 4

Continuous series

formula

$$Z = L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times i$$

where L = The lower limit of the interval

f_1 = The frequency corresponding to the interval

f_0 = The frequency preceeding interval

f_2 = The frequency succeeding interval

i = The Length of interval

(1) Find mode

C.I	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
f	5	8	7	12	28	20	10	10

Solution:-

C.I	f
0-10	5
10-20	8
20-30	7
30-40	12 f_0
40-50	28 f_1
50-60	20 f_2
60-70	10
70-80	10

Here maximum frequency is 28

Thus the C.I 40 - 50

Here $L = 40$ $f_0 = 12$ $f_1 = 28$ $f_2 = 20$

$i = 10$

$$Z = 40 + \left[\frac{28 - 12}{2(28) - 12 - 20}\right] \times 10$$

$$= 40 + 6.6666$$

$$Z = 46.66.$$

1. The median and mode ...

$= 74.28541$

GEOMETRIC MEAN

Geometric mean of N value is the N^{th} root of the product of the N value

If $X_1, X_2, \ldots X_N$ are the values of the geometric mean is $\sqrt[N]{X_1, X_2 \ldots X_N}$

Formula :

Individual series (G.M) = Antilog $\left(\frac{\Sigma \log x}{n}\right)$

Discrete series (G.M) = Antilog $\left(\frac{\Sigma f \log x}{N}\right)$

Contineous series (G.M) = Antilog $\left(\frac{\Sigma f \log m}{N}\right)$

Weighted (G.M) = Antilog $\left(\frac{\Sigma w \log x}{\Sigma w}\right)$

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① Find G.M 3, 6, 24, 48

Solution:-

$$G.M = \text{Antilog}\left(\frac{\Sigma \log x}{n}\right)$$

n = no. of datas = 4

x	$\log x$
3	0.47712
6	0.77815
24	1.38021
48	1.68124

$$\sum \log x = 4.3167$$

$$G.M = \text{Antilog}\left(\frac{4.3167}{4}\right)$$

$$= \text{Antilog}\,(1.07918)$$

$$G.M = 12.0005$$

2. Find G.M

x	10	15	25	40	50
f	4	6	10	7	3

Formula:

$$G.M = \text{Antilog}\left(\frac{\sum f \log x}{N}\right)$$

x	f	$\log x$	$f \log x$
10	4	1.0000	4
15	6	1.1761	7.0566
25	10	1.3979	13.979
40	7	1.6021	11.2147
50	3	1.6990	5.097

$\sum f = 30$ $\qquad\qquad$ $\sum f \log x = 41.34473$

$$\therefore G.M = \text{Antilog}\left(\frac{41.34473}{30}\right)$$

$$= \text{Antilog}\ (1.3782)$$

$G.M = 23.89$

HORMONIC MEANS

* Hormonic mean is the reciprocal of the mean of the reciprocal of the values.

* If $x_1, x_2, \ldots x_N$ are the values their reciprocal of $\frac{1}{x_1}, \frac{1}{x_2}, \ldots \frac{1}{x_N}$

* The total number of reciprocal is $\Sigma(1/x)$

* The mean of the reciprocal is $\frac{\Sigma(1/x)}{N}$

$\therefore$ The reciprocal of the mean of the mean of the reciprocal is $\frac{N}{\Sigma\left(\frac{1}{x}\right)}$

That is Hormonic mean.

$$H \cdot M = \frac{N}{\left(\frac{1}{x_1}\right)+\left(\frac{1}{x_2}\right)+\cdots\left(\frac{1}{x_N}\right)}$$

INDIVIDUAL series of HM:-

$$H \cdot M = \frac{N}{\Sigma\left(\frac{1}{x}\right)}, \quad N = \text{No of data}$$

DISCRETE SERIES.

$$H \cdot M = \frac{N}{\Sigma\left(f/x\right)}, \quad N = \Sigma f$$

CONTINUOUS SERIES:

$$H \cdot M = \frac{N}{\Sigma\left(f/x\right)}, \quad N = \Sigma f$$

Weighted H·M

$$H \cdot M = \frac{\Sigma W}{\Sigma\left(\frac{W}{x}\right)}$$

Problem :-

1. Find the H·M for the following data
6, 15, 35, 40, 900, 520, 300, 400, 1800, 2000

Solution :-

X	$\overline{X}$
6	0.1667
15.	0.0667
35	0.0286
40	0.025
900	0.0011
520	0.0019
300	0.0033
400	0.0025
1800	0.0006
2000	0.0005

$$\Sigma\left(\frac{1}{x}\right) = 0.2969$$

$$H.M = \frac{N}{\Sigma(1/x)}$$

$$= \frac{10}{0.2969}$$

$$H.M = 33.68$$

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2. Find H.M

X	10	12	14	16	18	20
f	5	18	20	10	6	1

Solution:-

X	f	f/x
10	5	0.5
12	18	1.5
14	20	1.4286
16	10	0.625
18	6	0.3333
20	1	0.05

$\Sigma f = 60$ $\Sigma(f/x) = 4.4369$

$$H.M = \frac{60}{4.4569}$$

$$H.M = 13.52$$

i) H·M

C·I	f	Converted C·I	x	f/x
0-19	5	0.5-19.5	10	0.5
20-39	15	19.5-39.5	29.5	0.5084
40-59	35	39.5-59.5	49.5	0.7070
60-79	15	59.5-79.5	69.5	0.258
80-99	10	79.5-99.5	89.5	0.1117

$\Sigma f = 80$ $\qquad\qquad$ $\Sigma(f/x) = 2.0859$

$$H\cdot M = \left(\frac{N}{\Sigma(f/x)}\right), \quad N = \Sigma f$$

$$H\cdot M = \frac{80}{2.0859} = 38.3604$$

ii) To find G·M;

x	f	Converted x	M	log M	f log M
0-19	5	0.5-19.5	10	1	5
20-39	15	19.5-39.5	29.5	1.4698	22.04
40-59	35	39.5-59.5	49.5	1.6946	59.31
60-79	15	59.5-79.5	69.5	1.8419	27.62
80-99	10	79.5-99.5	89.5	1.9518	19.518

$\Sigma f = 80$ $\Sigma f \log m = 133.48$

$$G.M = \text{Antilog}\left(\frac{133.48}{80}\right)$$

$$= \text{Antilog}\,(1.6685)$$

$$G.M = 46.61224.$$

iii) To find A.M:-

C.I	mid values of x	f	fx
0.5-19.5	10	5	50
19.5-39.5	29.5	15	442.25
39.5-59.5	49.5	35	1732.5
59.5-79.5	69.5	15	1042.5
79.5-99.5	89.5	10	895

$\Sigma f = 80$ $\Sigma fx = 4162.5$

$$\text{Mean } \bar{x} = \frac{\Sigma fx}{\Sigma f}$$

$$\bar{x} = \frac{4162.5}{80}$$

$$\bar{x} = 52.03125$$

Mean, median, mode :

Their interrelation = mean - median

$= \frac{1}{3}$ (mean - mode)

$$\boxed{\text{mode} = 3\,\text{median} - 2\,\text{mean}}$$

problem

calculate median if mode $(z) = 32.1$

and mean$(\bar{x}) = 35.4$

Mode = 3 median - 2 mean

3 median = mode + 2 mean

$$= \frac{\text{mode} + 2\text{mean}}{3}$$

$$= \frac{32.1 + 2(35.4)}{3}$$

$= 34.3$

If $\bar{x} = 42.5$, $M = 41$. Find Z

mode = 3 median - 2 mean

$= \cancel{3(42.5)} - \cancel{2(41)}\ 3(41) - 2(42.5)$

$= \cancel{127.5} - \cancel{82}$

$= 38$.

Standard deviation.

Formula :-

$$S.D = \sigma = \sqrt{\frac{\Sigma (x-\bar{x})^2}{N}}$$

$$S.D = \sigma = \sqrt{\frac{\Sigma f (x-\bar{x})^2}{\Sigma f}}$$

(Individual series) N = No. of data, $\bar{x}$ = mean

σ = standard deviation

$$\text{Coefficient of } S.D = \frac{\sigma}{\bar{x}}$$

$$\left.\begin{array}{c}\text{Coefficient of variation (or)}\\ C.V\end{array}\right\} = \frac{\sigma}{\bar{x}} \times 100$$

1. 21, 22, 23, 24, 25.

Solution:-

Here $N = 5$

X	$X - \bar{X}$	$(X - \bar{X})^2$
21	-2	4
22	-1	1
23	0	0
24	1	1
25	2	4
$\Sigma X = 115$		10

$$\bar{X} = \frac{\Sigma X}{N} = \frac{115}{5} = 23$$

$$S.D = \sqrt{\frac{\Sigma(x-\bar{x})^2}{N}} = \frac{\sqrt{\cancel{(10)}\,2}}{\cancel{5}}$$

$$= 1.4142$$

Find S.D

①

C.I	below 10	below 20	below 30	below 40	below 50	below 60
f	15	32	51	78	97	109

Solution:-

C.I	f	mid values of x	fx	$x-\bar{x}$	$(x-\bar{x})^2$
0-10	15	5	75	-24.95	622.50
10-20	17	15	225	-14.95	223.50
20-30	19	25	475	-4.95	24.50
30-40	27	35	945	5.05	25.50
40-50	19	45	885	15.05	226.50
50-60	12	55	660	25.05	627.50
	$\Sigma f = 109$		$\Sigma fx = 3265$		

$f(x-\bar{x})^2$
9337.50
3799.50
465.50
688.50
4303.50
7530.08
26124.50

$$\bar{X} = \frac{\Sigma fx}{\Sigma f} = \frac{3265}{109} = 29.95$$

$$S.D = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}}$$

$$= \sqrt{\frac{26124.50}{109}} = \sqrt{239.67}$$

$$= 15.48$$

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positional measures

Quartiles deviation (Q.D)

percentiles (P)

Quartiles deviation

There are three quartiles Q_1, Q_2, and Q_3. $M = Q_2$ one quartile of the items are less than (or) equal to the lower quartile the remaining three quartiles of the item are more than (or) equal to Q_1. The second quartile Q_2 is nothing but the median two quartiles of the items are more than (or) equal to the median. The third quartile Q_3 is also called the upper quartile three quartiles of the items are less than (or) equal to Q_3 one quartile of the items are more than (or) equal to Q_3 The quartiles are used directly in measures of dispersion to find the quartile deviation.

Individual and Discrete series :-

$$Q_1 = \left(\frac{N+1}{4}\right)^{th} \text{ position}$$

$$Q_3 = \left(\frac{N+1}{2}\right)^{th} \text{ position}$$

$$Q_3 = 3\left(\frac{N+1}{4}\right)^{th} \text{ position}$$

Continuous series :-

$$Q_1 = L_1 + \left[\frac{i\,(N/4 - (f_1))}{f_1}\right]$$

$$Q_2 = M = L + \left[\frac{i\,(N/2 - (f))}{f}\right]$$

$$Q_3 = L_3 + \left[\frac{i_3 \left(\frac{3N}{4} - cf_3 \right)}{f_3} \right]$$

percentiles (P) :-

If $P_1, P_2, \ldots P_{99}$ are the are the 99 percentaile they divide the series in to hundred equal parts. one 100^{th} Item are less than (or) equal to P_1, one hundredth of the Item are more than (or) equal to P_{99} and one hundredth of the Item lies between any successive pairs of percentile. when all the Items are in Ascending order. percentiles are directly used in Kell's Coefficient of skeioness this is also called ~~a~~ a percentile.

Individual & Discrete series:

P_K is the value of the item at

$\left[\frac{K(N+1)}{100}\right]^{th}$ position.

Continuous series:

To find $\left(\frac{KN}{100}\right)^{th}$ position then

$$P_K = L_K + \left(\frac{i_K\left(\frac{KN}{100} - cf_K\right)}{f_K}\right)$$

Coefficient of Q.D

$$\frac{Q_3 - Q_1}{Q_3 + Q_1}$$

1. From The following data Q_1, Q_2, Q_3

1, 3, 4, 3, 4, 4, 5, 5, 2, 2 2, 2 9, 10

1, 1, 1, 1, 1

Solution:-

Arrange the data in Ascending order,

1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 3, 3, 4, 4, 4, 5, 5, 9, 10

total = 19

$N = 19$.

$Q_1 = \left(\frac{N+1}{4}\right)^{th}$ position $= \left(\frac{19+1}{4}\right)^{th} = 5^{th}$ position $= 1$

$Q_2 = \left(\frac{N+1}{2}\right)^{th}$ position $= \left(\frac{19+1}{2}\right)^{th} = 10^{th}$ position $= 2$

$Q_3 = 3\left(\frac{N+1}{4}\right)^{th}$ position $= 3\left(\frac{20}{4}\right)^{th} = 15^{th}$ position $= 4$

2. 25, 32, 15, 40, 40, 40, 50, 53, 55, 61, 72, 90,

95, 100.

Solution:-

Arrange the data in Ascending order.

15, 25, 32, 40, 40, 40, 50, 53, 55, 61, 72,

90, 95, 100.

$N = 14$

$Q_1 = \left(\frac{N+1}{4}\right)^{th}$ position $= \left(\frac{14+1}{4}\right)^{th}$ position $= 3.75^{th}$ position

$Q_1 = 3rd$ position $+ 0.75 \left(\text{Value of 4th position} - \text{Value at 3rd position}\right)$

$= 32 + 0.75(40 - 32)$

$= 32 + 0.75(8)$

$= 32 + 6 = 38$

$Q_2 = \left(\frac{N+1}{2}\right)^{th}$ position $= \left(\frac{14+1}{2}\right)^{th}$ position

$= \left(\frac{15}{2}\right)^{th}$ position $= 7.5^{th}$ position

$Q_2 = 7^{th}$ position $+ 0.5 \left(8^{th}\text{ position value} - 7^{th}\text{ position value}\right)$

$= 50 + 0.5\,(53 - 50)$

$= 50 + 0.5\,(3)$

$= 50 + 1.5 = 51.5$

$Q_3 = 3\left(\frac{N+1}{4}\right)^{th}$ position $= 3\left(\frac{14+1}{4}\right)^{th}$ position

$= 3\left(\frac{15}{4}\right)^{th}$ position

$= 11.25$ position

$Q_3 = 11^{th}$ position $+ 0.25$ (value of 12^{th} position $-$ value of 11^{th} position)

$= 72 + 0.25\,(90 - 72)$

$= 72 + 0.25\,(18)$

$= 72 + 4.5 = 76.5.$

Continuous series

1. Find median and Quartiles

X	6-8	8-10	10-12	12-14
f	85	65	59	50

Solution:-

X	f	c.f
6-8	85	85
8-10	65	150
10-12	59	209
12-14	50	259

$$Q_1 = L_1 + \left(\frac{i \, (N/4 - c.f)}{f_1} \right), N = 259$$

$$\frac{N}{4} = \frac{259}{4} = 64.78, \; L_1 = 6, \; f_1 = 85, \; i_1 = 2,$$

$c.f = 0$

$$Q_1 = 6 + \left(\frac{2 \, (64.75 - 0)}{85} \right)$$

$$= 6 + 1.52$$

$$Q_1 = 7.52$$

$$Q_2 = L_2 + \left(\frac{i_2 \left(N/2 - cf \right)}{f} \right) \qquad N = 259.$$

$$\frac{N}{4} = \frac{259}{4} = \cancel{64.78} ; \quad L_2 = 8 \quad f_1 = 65$$

$$i_2 = 2 \quad c.f = 0$$

$$= 8 + \left(\frac{2 \left(64.78 - 0 \right)}{65} \right)$$

$$= 8 + 1.99320$$

$$= \cancel{9.99320} \; 9.3692$$

$$Q_3 = L_3 + \left(\frac{i_3 \left(\frac{N}{2} - c.f \right)}{f} \right)$$

$$= 10 + \left(\frac{2 \left(64.78 - 0 \right)}{59} \right)$$

$$= 10 + 2.1959$$

$$= 12.1959$$

$$Q_4 = l_4 + \left(\frac{i_4 \left(\frac{N}{2} - c.f \right)}{f} \right)$$

$$= 12 + \left(\frac{2 \, (64.78 - 0)}{50} \right)$$

$$= 12 + 2.5912$$

$$= 14.5912.$$

Coefficient