01. Solve the game 2x3 by graphically player A,

	B_1	B_2	B_3
A_1	1	3	11
A_2	8	5	2

Solution :-

$$\begin{array}{c|c|c c} & \begin{matrix} B_1 & B_2 & B_3 \end{matrix} & \text{min} & \\ \begin{matrix} A_1 \\ A_2 \end{matrix} & \begin{bmatrix} 1 & 3 & 11 \\ 8 & 5 & 2 \end{bmatrix} & \begin{matrix} 1 \\ 2 \end{matrix} & \left.\right\} \begin{matrix} \text{max} \\ 2 \end{matrix} \\ \text{max} & \underbrace{\begin{matrix} 8 & 5 & 11 \end{matrix}}_{\text{min } 5} & & \end{array}$$

Here check saddle point,

Maximini ≠ Minimax

$2 \neq 5$

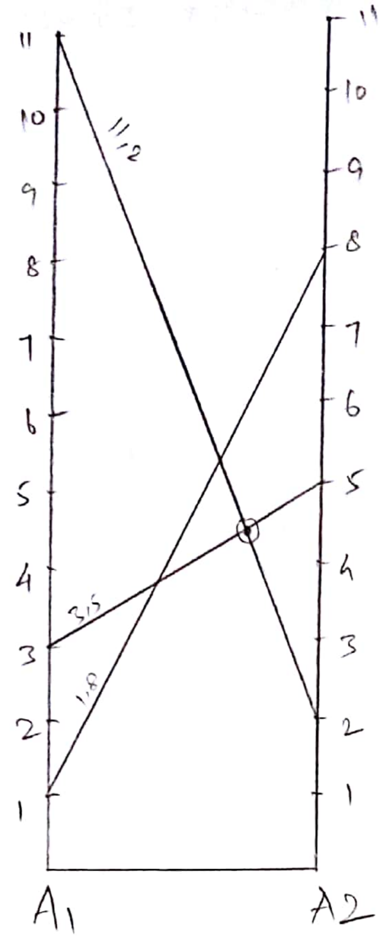
Graphical Method :-

Row < column

$2 < 3$

So we take A_1 & A_2 for graph axis.

Pick the point which is highest 1st intersection from downward side.



(3,5) (11,2) intersected

∴ Now the reduced Matrix

$$\begin{bmatrix} 3 & 11 \\ 5 & 2 \end{bmatrix}$$

Then check saddle point,

$$\begin{array}{c|c|c|c} & B_2 \quad B_3 & \text{min} & \text{max} \\ \begin{array}{c} A_1 \\ A_2 \end{array} & \begin{bmatrix} 3 & 11 \\ 5 & 2 \end{bmatrix} & \begin{array}{c} 3 \\ 2 \end{array} & 3 \end{array}$$

$$\text{max} \quad \underbrace{5 \quad 11}$$

$$\text{min} \quad 5$$

maximin $\neq$ minimax

$3 \neq 5$

(2x2) Without saddle point :-

$$S_A = \begin{bmatrix} A_1 & A_2 \\ p_1 & p_2 \end{bmatrix}$$

$$S_B = \begin{bmatrix} B_1 & B_2 & B_3 \\ 0 & q_1 & q_2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 11 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$p_1 = \frac{a_{22} - a_{21}}{\lambda}$$

$$\lambda = (a_{11} + a_{22}) - (a_{12} + a_{21})$$

$$= (3+2) - (11+5)$$

$$= 5 - 16$$

$$\boxed{\lambda = -11}$$

$$p_1 = \frac{2-5}{-11} = \frac{\cancel{-}3}{\cancel{-}11} \qquad \boxed{p_1 = \frac{3}{11}}$$

$$p_2 = 1 - p_1 = 1 - \frac{3}{11}$$

$$= \frac{11-3}{11}$$

$$\boxed{p_2 = \frac{8}{11}}$$

$$q_2 = 1 - q_1 \Rightarrow 1 - \frac{9}{11}$$

$$\boxed{q_2 = \frac{2}{11}}$$

$$q_1 = \frac{a_{22} - a_{12}}{\lambda} = \frac{2-11}{-11} \Rightarrow \frac{\cancel{-}9}{\cancel{-}11} \qquad \boxed{q_1 = \frac{9}{11}}$$

$$S_A = \left[\frac{3}{11}, \frac{8}{11}\right]$$

$$S_B = \left[\frac{9}{11}, \frac{2}{11}\right]$$

$$\gamma = \frac{a_{11}a_{22} - a_{12}a_{22}}{\lambda}$$

$$\gamma = \frac{(3\times 2) - (11\times 5)}{-11}$$

$$= \frac{6-55}{-11} = \frac{+49}{+11}$$

$$\boxed{\gamma = \frac{49}{11}}$$

Q2. Solve the game by Linear programming Method (L

(or) Simplex method.

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 1 & -1 & 3 \\ 3 & 5 & -3 \\ 6 & 2 & -2 \end{bmatrix} \end{array}$$

Soln :

$$\begin{array}{ccc} & \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} & \text{min} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} & \begin{bmatrix} 1 & -1 & 3 \\ 3 & 5 & -3 \\ 6 & 2 & -2 \end{bmatrix} & \left.\begin{array}{c} -1 \\ -3 \\ -2 \end{array}\right\} \begin{array}{c} \text{max} \\ -1 \end{array} \\ \text{max} & \underbrace{\begin{array}{ccc} 6 & 5 & 3 \end{array}} & \text{min } -3 \end{array}$$

Scanned with CamScanner

Maximini ≠ minimax

$-1 \neq 3$

(∴ No saddle point)

By using LPP:-

(Without Negative we may choose same matrix.)

here, '−3' is most Negative (have to change into '1').

∴ Add 4 for Each Element:-

$$\begin{bmatrix} 5 & 3 & 7 \\ 7 & 9 & 1 \\ 10 & 6 & 2 \end{bmatrix}$$

We assume (probability)

$y_1 + y_2 + y_3 = 1$, $x_1 + x_2 + x_3 = 1$

$$\max z = \frac{1}{v} = \frac{y_1 + y_2 + y_3}{v}$$

$$= \frac{y_1}{v} + \frac{y_2}{v} + \frac{y_3}{v}$$

take it as,

$$\max z = \frac{1}{v} = Y_1 + Y_2 + Y_3$$

Where $Y_1 = \frac{y_1}{v}$, $Y_2 = \frac{y_2}{v}$, $Y_3 = \frac{y_3}{v}$

lly $X_1 = \frac{x_1}{v}$, $X_2 = \frac{x_2}{v}$, $X_3 = \frac{x_3}{v}$

$\therefore$ Max $z = \frac{1}{v} = y_1 + y_2 + y_3$

S.to,

$5y_1 + 3y_2 + 7y_3 \leq 1$

$7y_1 + 9y_2 + y_3 \leq 1$

$10y_1 + 6y_2 + 2y_3 \leq 1$

$y_1, y_2, y_3 \geq 0.$

using simplex method,

$$\text{Max } z = \frac{1}{v} = y_1 + y_2 + y_3 + 0s_1 + 0s_2 + 0s_3.$$

$5y_1 + 3y_2 + 7y_3 + s_1 + 0s_2 + 0s_3 = 1$

$7y_1 + 9y_2 + y_3 + 0s_1 + s_2 + 0s_3 = 1$

$10y_1 + 6y_2 + 2y_3 + 0s_1 + 0s_2 + s_3 = 1$

$y_1, y_2, y_3, s_1, s_2, s_3 \geq 0.$

Initial iteration:

		0	1	1	1	0	0	0
C_B	Y_B	X_B	y_1	y_2	y_3	s_1	s_2	s_3
0	s_1	1	5	3	7	1	0	0
0	s_2	1	7	9	1	0	1	0
0	s_3	1	10	6	2	0	0	1
$z_j - c_j$		0	-1	-1	-1	0	0	0

First iteration :-

C_B	Y_B	X_B	y_1	y_2	y_3	s_1	s_2	s_3	θ min
0	s_1	$\frac{1}{2}$	0	0	6	1	0	$-1/2$	0.083
0	s_2	$\frac{3}{10}$	0	$24/5$	$-2/5$	0	1	$-7/10$	–
1	y_1	$\frac{1}{10}$	1	$3/5$	$1/5$	0	0	$1/10$	0.5
z_j-c_j		$\frac{1}{10}$	0	$-2/5$	$-4/5$	0	0	$1/10$	

↑

Second iteration:-

C_B	Y_B	X_B	y_1	y_2	y_3	s_1	s_2	s_3	θ min
1	y_3	$\frac{1}{12}$	0	0	1	$\frac{1}{6}$	0	$-\frac{1}{12}$	–
0	s_2	$1/3$	0	$24/5$	0	$1/15$	1	$-11/5$	0.069
1	y_1	$1/12$	1	$3/5$	0	$-1/30$	0	$7/60$	0.13
z_j-c_j		$\frac{1}{6}$	0	$-\frac{2}{5}$	0	$\frac{2}{15}$	0	$1/30$	

↑

Third iteration:-

C_B	Y_B	X_B	y_1	y_2	y_3	s_1	s_2	s_3	θ min
1	y_3	$1/12$	0	0	1	$1/6$	0	$-1/12$	–
1	y_2	$5/72$	0	1	0	$1/72$	$5/24$	$-11/72$	–
1	y_1	$1/24$	1	0	0	$-1/24$	$-1/8$	$5/24$	
z_j-c_j		$7/36$	0	0	0	$5/36$	$1/12$	$-1/36$	

↑

Fourth iteration :-

C_B	Y_B	X_B	Y_1	Y_2	Y_3	S_1	S_2	S_3
1	Y_3	$\frac{1}{10}$	$\frac{2}{5}$	0	0	$\frac{3}{20}$	$\frac{-1}{20}$	0
1	Y_2	$\frac{1}{10}$	$\frac{11}{15}$	1	0	$-1/60$	$\frac{7}{60}$	0
0	S_3	$\frac{1}{5}$	$\frac{24}{5}$	0	0	$-1/5$	$-3/5$	1
$Z_j - C_j$		$1/5$	$2/15$	0	0	$2/15$	$1/15$	0

$$Y_2 = \frac{1}{10}, \quad Y_3 = \frac{1}{10}, \quad Y_1 = 0, \quad \text{Max } Z = \frac{1}{5}$$

Verification,

$$\max Z = 0 + \frac{1}{10} + \frac{1}{10}$$

$$\boxed{\max Z = \frac{1}{5}}$$

$$\text{Max } Z = \frac{1}{v} = \frac{Y_1 + Y_2 + Y_3}{v}$$

$$\frac{1}{v} = \frac{1}{5}$$

$$Y_1 = \frac{y_1}{v}$$

$$Y_1 v = y_1$$

$$0 \times 5 = \boxed{y_1 = 0}$$

$$Y_2 = \frac{y_2}{v}$$

$$Y_2 v = y_2$$

$$\frac{1}{10} \times 5 = y_2$$

$$\boxed{\frac{1}{2} = y_2}$$

$$Y_3 = \frac{y_3}{v}$$

$$Y_3 v = y_3$$

$$\frac{1}{10} \times 5 = y_3$$

$$\boxed{\frac{1}{2} = y_3}$$

$$X_1 = S_1 = \frac{2}{15} \qquad \frac{x_1 + x_2 + x_3}{\vartheta} = \frac{1}{5}$$

$$X_1 = \frac{2}{15}, \; X_2 = \frac{1}{15}, \; X_3 = 0.$$

$$X_1 = \frac{x_1}{\vartheta} \qquad X_2 = \frac{x_2}{\vartheta} \qquad X_3 = \frac{x_3}{\vartheta}$$

$$X_1 \vartheta = x_1 \qquad X_2 \vartheta = x_2 \qquad X_3 \vartheta = x_3$$

$$\frac{2}{\cancel{15}\,3} \times \cancel{5} = x_1 \qquad \frac{1}{15} \times 5 = x_2 \qquad 0 \times 5 = x_3$$

$$\boxed{x_1 = \frac{2}{3}} \qquad \boxed{\frac{1}{3} = x_2} \qquad \boxed{x_3 = 0}$$

The strategy for A,

$$S_A = \begin{bmatrix} A_1 & A_2 & A_3 \\ x_1 & x_2 & x_3 \end{bmatrix}$$

Strategy for B,

$$S_B = \begin{bmatrix} B_1 & B_2 & B_3 \\ y_1 & y_2 & y_3 \end{bmatrix}$$

$$S_A = \begin{array}{c} \begin{array}{ccc} A_1 & A_2 & A_3 \end{array} \\ \left[\frac{2}{3}, \frac{1}{3}, 0 \right] \end{array}$$

$$S_B = \begin{array}{c} \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \left[0, \frac{1}{2}, \frac{1}{2} \right] \end{array}$$

the Value of the game is,

$$\vartheta = 5$$

$\therefore$ The original value of the game is $\vartheta = 5 - 4 = 1$

Q3. using the principle of Dominance property to solve following game.

$$\text{Player A} \begin{bmatrix} 3 & -2 & 4 \\ -1 & 4 & 2 \\ 2 & 2 & 6 \end{bmatrix}$$

Solution :-

$$\begin{array}{c|c|c|c} & & \text{min} & \text{max} \\ \text{Player A:} & \begin{bmatrix} 3 & -2 & 4 \\ -1 & 4 & 2 \\ 2 & 2 & 6 \end{bmatrix} & \begin{matrix} -2 \\ -1 \\ 2 \end{matrix} & 2 \\ \text{max} & 3 \quad 4 \quad 6 & & \end{array}$$

min 3

maximini ≠ minimax

$2 \neq 3$

Using Dominance Property :-

Column (1) & (3)

$3 < 4$
$-1 < 2$
$2 < 6$

(Optimal)

which one is Dominate another one is Eliminate.

So the column (3) is Deleted.

Below the reduced Matrix.

Scanned with CamScanner

Reduced matrix :-

$$\begin{array}{c|c|c}
 & \begin{bmatrix} 3 & -2 \\ -1 & 4 \\ 2 & 2 \end{bmatrix} & \begin{array}{c} \text{min} \\ -2 \\ -1 \\ 2 \end{array} \Big\} \text{ max } 2 \\
\text{max} & 3 \quad 4 & \text{min } 3
\end{array}$$

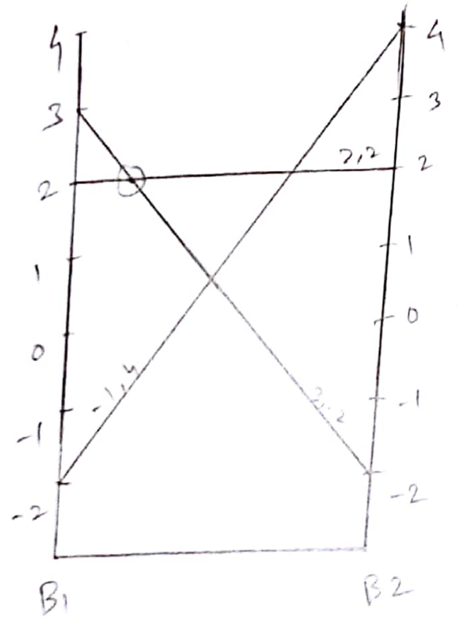
maximini ≠ minimax

$2 \neq 3$.

So use graphical method to solve.

row > Column. (So From upward First Lowest intersect line).

$3 > 2$



intersected.

$$\begin{pmatrix} 3 & -2 \\ 2 & 2 \end{pmatrix} \quad \begin{array}{c} \text{min} \\ -2 \\ 2 \end{array} \Big\} \text{ max } 2$$

max $\underbrace{3 \quad 2}$

min 2

maximini = minimax

$2 = 2$

So We found the saddle point 2.

04. Solve the problem by Arithmetic Method.

	B_1	B_2
A_1	3	1
A_2	2	4

Solution:-

Check saddle point;

$$\begin{array}{c|c} & \text{min} \\ \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \end{array} \end{array} \Big\} \begin{array}{c} \text{max} \\ 2 \end{array}$$

max $\underbrace{3 \quad 4}$

min 3

maximini $\neq$ minimax

$2 \neq 3$

By using Arithmetic Method:-

$$C = \begin{bmatrix} 2 \\ -2 \end{bmatrix} \quad R = \begin{bmatrix} 1 & -3 \end{bmatrix}$$

$|C_1| = |-2| = 2$

$|C_2| = |2| = 2$

$|R_1| = |-3| = 3$

$|R_2| = |1| = 1$

The Angumented Matrix,

3	1	2
2	4	2
3	1	4

The strategy player 'A' & 'B' :-

$$S_A = \left[\frac{2}{4}, \frac{2}{4}\right] = \left(\frac{1}{2}, \frac{1}{2}\right)$$

$$S_B = \left[\frac{3}{4}, \frac{1}{4}\right] = \left(\frac{3}{4}, \frac{1}{4}\right)$$

Value of the game :-

$$V = \left(\frac{1}{2} \times 3\right) + \left(\frac{1}{2} \times 2\right)$$

$$\boxed{V = \frac{5}{2}}$$

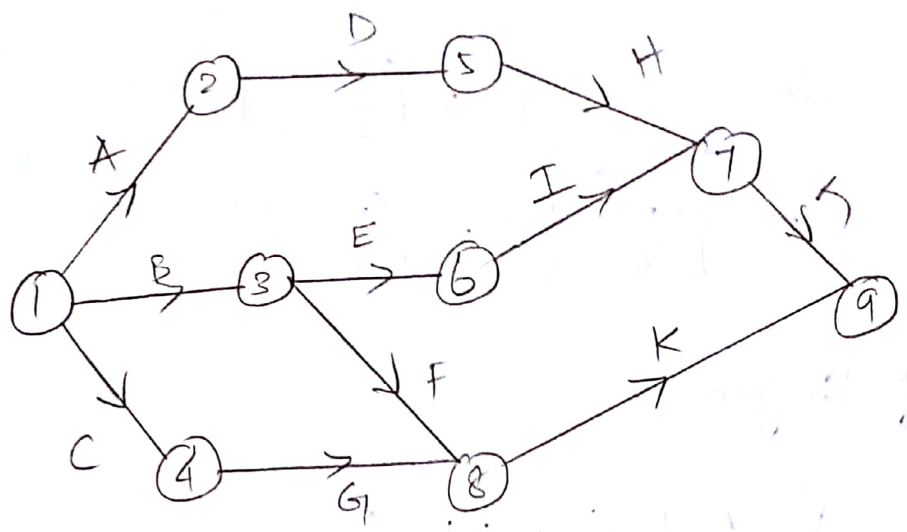
(or)

$$V = \left(\frac{3}{4} \times 3\right) + \left(\frac{1}{4} \times 1\right)$$

$$\boxed{V = \frac{5}{2}}$$

Q5. Draw the Network for the project use activities and the precedence relationships are given below,

Activities	A	B	C	D	E	F	G	H	I	J	K
Immediate Predessor	–	–	–	A	B	B	C	D	E	(H,I)	(F,G)



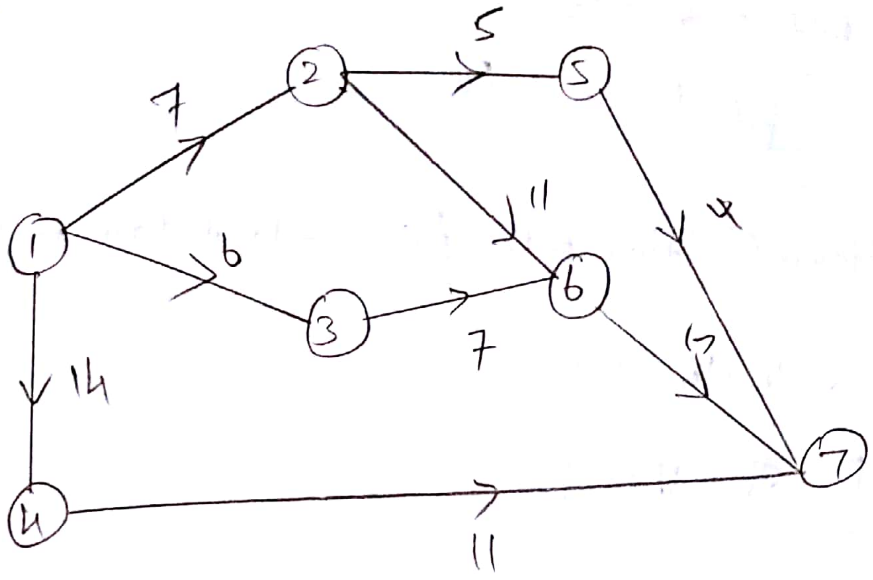
06. A Project consist of the following activities and Time estimate,

Activity	Least time (t_o)	Greatest Time (T_p)	mostly time (t_m)
1-2	3	15	6
1-3	2	14	5
1-4	6	30	12
2-5	2	8	5
2-6	5	17	11
3-6	3	15	6
4-7	3	27	9
5-7	1	7	4
6-7	2	8	5

(a) Draw the Network (b) What is the Probability that project will be completed In 27 days.

Scanned with CamScanner

Solution :-



Activity	load time (t_o)	(t_p)	(t_p)	$t_e = \frac{t_o + 4t_m + t_p}{6}$	$\sigma_c^2 = \left(\frac{t_p - t_o}{6}\right)^2$
1-2	3	15	6	7	4
1-3	2	14	5	6	4
1-4	6	30	12	14	16
2-5	2	8	5	5	1
2-6	5	17	11	11	4
3-6	3	15	6	7	4
4-7	3	27	9	14	16
5-7	1	7	4	4	1
6-7	2	8	5	5	1

To find critical path :-

1-2-5-7 = 7+5+4 = 16

1-2-6-7 = 7+11+5 = 23

$1-3-6-7 = 6+7+5 = 18$

$\boxed{1-4-7} = 14+11 = \boxed{25}$

Expected Variance (Critical path) :/ (Standard deviation)

$= 16+16 = 32.$

$$\boxed{S.D = \sigma_c = \sqrt{32}}$$

(b) To find probability that, the project will be completed in 27 days.

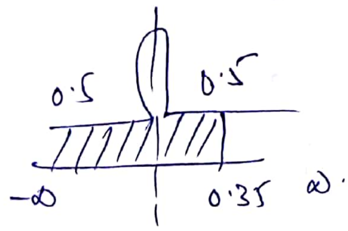
$$Z = \frac{T_S - T_E}{\sigma_c}$$

$(\because T_S = 27 \text{ days})$
$(T_E = 25 \text{days})$

$$= \frac{27-25}{\sqrt{32}}$$

$$= \frac{2}{\sqrt{32}} = 0.35$$

$P(T_S \leq 27) = P(Z \leq 0.35).$



$= 0.5 + 0.1368$

$= 0.6368$

$= 63.68\%$

$0.5 + \underline{0.1368}$

table value for

0.30 & 0.05.

marks :-

7. What are the different type of inventories:-

1. Transport inventories
2. Buffer inventories
3. Anticipation inventories
4. Decoupling inventories
5. Lot size inventories.

8. Define inventory cost :-

1. production / purchase cost
2. ordering / setup cost
3. Carrying / Holding cost
4. Shortage / stockout cost.