$$S+1 = 1+1\left(\tfrac{1}{7}\right) - \frac{1(1+3)}{1\cdot 2}\left(\tfrac{1}{7}\right)^2 + \frac{1(1+3)(1+3\times 2)}{1\cdot 2\cdot 3}\left(\tfrac{1}{7}\right)^3 + \cdots \infty$$

$$(1+x)^{-p/q} = 1 + \frac{p}{1!}(x/q) - \frac{p(p+q)}{2!}(x/q)^2 + \frac{p(p+q)(p+2q)}{3!}(x/q)^3$$

(7) Sum the series $\frac{15}{16} + \frac{15\cdot 21}{16\cdot 24} + \frac{15\cdot 21\cdot 27}{16\cdot 24\cdot 32} + \cdots \infty$

let $S = \frac{15}{16} + \frac{15\cdot 21}{16\cdot 24} + \frac{15\cdot 21\cdot 27}{16\cdot 24\cdot 32} + \cdots \infty$

The numerator of the series has a common difference $2 = 6$ and at of multiplier of 8

Therefore the series is binomial

The factor 8 is missing in the dinominator therefore multiply LHS and RHS is

$$\frac{1}{8}S = \frac{15}{8\cdot 16} + \frac{15\cdot 21}{8\cdot 16\cdot 24}$$

Now the number of factor numerator and dinominator are not equal multiply the numerator is in LHS and RHS by 9

$$\frac{9}{8}S = \frac{9\cdot 15}{8\cdot 16} + \frac{9\cdot 15\cdot 21}{8\cdot 16\cdot 24} + \frac{9\cdot 15\cdot 21\cdot 27}{8\cdot 16\cdot 24\cdot 32} + \cdots \infty$$

$1 + 9/8$ both sides

$$1 + 9/8 + 9/8\,S = 1 + 9/8 + \frac{9\cdot 15}{8\cdot 16} + \cdots \infty$$

<u>compare with series</u>

$$(1-x)^{-p/q} = 1 + \frac{p}{1!}(x/q) + \frac{p(p+q)}{2!}(x/q)$$

$p = 9 \qquad q = 6$

$(x/q) = 1/8$

$x = 6/8$

$x = 3/4$

$9/8\,S = (1-x)^{-p/q} - 1 - 9/8$

$9/8\,S = (1-3/4)^{-9/6} - 1 - 9/8$

$9/8\,S = (1/4)^{-3/2} - 17/8$

$9/8 = 8 - 17/8 \Rightarrow 9/8\,S = 47/8$

$S = 47/9$

(12) Find the coeff of x^n the expansion of $\frac{(3x+2)}{e^{-2x}}$

$(3x+2)e^{2x}$

$$(3+2)\left(1+\frac{2x}{1!}+\frac{4x^2}{2!}+\frac{8x^3}{3!}+\frac{2^{n-1}x^{n-1}}{(n-1)!}+\frac{3^n x^n}{n!}\right)$$

$$=\frac{3(2^{n-1})}{(n-1)!}+\frac{2(2)^n}{n!}$$

$$=\frac{2^{n-1}[3^n+4]}{n!}$$

(18) Sum the series $\frac{1}{1\cdot 2} + \frac{1}{3\cdot 4} + \frac{1}{5\cdot 6} + \cdots$

n^{th} term of the series is $\frac{1}{(2n-1)(2n)}$ ...

$$t_n = \frac{1}{(2n-1)(2n)} = \frac{A}{(2n-1)} + \frac{B}{2n}$$

$$1 = A\,(2n) + B\,(2n-1)$$

Put $n = 0$, $1 = 0 + B(-1)$ $\quad B = -1$

Put $n = 1/2$, $1 = A(1) + B(0)$ $\quad A = 1$

$$t_n = \frac{1}{2n-1} - \frac{1}{2n}$$

$$S = \sum_{1}^{\infty} \frac{1}{2n-1} - \sum_{1}^{\infty} \frac{1}{2n}$$

$$S = (1/1 + 1/3 + 1/5 + 1/7) - (1/2 + 1/4 + 1/6 + 1/8)$$

$$S = (1 - 1/2 + 1/3 + 1/4 + 1/5)$$

$$S = \log 2$$

⑦ Sum the series $1 + \frac{2}{3}\cdot\frac{1}{2} + \frac{2\cdot5}{3\cdot6}\cdot\frac{1}{2^2} + \frac{2\cdot5\cdot8}{3\cdot6\cdot9}\cdot\frac{1}{2^3} + \dots\infty$

$$S = 1 + \frac{2}{1}\cdot\frac{1}{6} + \frac{2(2+3)}{1\cdot2}\cdot\frac{1}{6^2} + \frac{2(2+3)(2+2\times3)}{1\cdot2\cdot3}\,\frac{1}{6^3} + \dots\infty$$

$$= 1 + \frac{2}{1}(1/6) + \frac{2(2+3)}{2!}(1/6)^2 + \frac{2(2+3)(2+2\times3)}{3!}\left(\frac{1}{6}\right)^3 + \dots$$

Compare with expansion

$$(1-x)^{-p/q} = 1 + \frac{p}{1!}(x/q) + \frac{p(p+q)}{2!}(x/q)^2 + \frac{p(p+q)(p+2q)}{3!}\left(\frac{x}{q}\right)^3 + \dots$$

$p=2 \quad q=3 \qquad (x/q)(1/6)x = (3/6) = x = 1/2$

$$(1-x)^{-p/q} = (1-1/2)^{-2/3}$$
$$= (1/2)^{-2/3}$$
$$= (2)^{2/3}$$
$$= \sqrt[3]{4}$$

---

⑧ Prove $1 + \frac{1}{10^2} + \frac{1\cdot3}{1\cdot2}\cdot\frac{1}{10^4} + \frac{1\cdot3\cdot5}{1\cdot2\cdot3}\cdot\frac{1}{10^6} + 2\,\frac{5\sqrt{2}}{7}$

$$S = 1 + \frac{1}{1}\cdot\frac{1}{100} + \frac{1(1+2)}{1\cdot2}\cdot\frac{1}{100^2} + \frac{1(1+2)(1+2\times2)}{1\cdot2\cdot3}\,\frac{1}{100^3} + \dots$$

$$= 1 + \frac{1}{1}(1/100) + \frac{1(1+2)}{2!}\left(\frac{1}{100}\right)^2 + \dots$$

Compare with expansion:

$$(1-x)^{-p/q} = 1 + \frac{p}{1!}(x/q) + \frac{p(p+q)}{2!}(x/q)^2 + \frac{p(p+q)(p+2q)}{3!}(x/q)^3 + \dots$$

$p=1 \quad , q=2 \qquad (x/q) : (1/100)x = (2/100)x = 1/50$

$$S = (1-x)^{-p/q} = \left(1-\frac{1}{50}\right)^{-1/2}$$
$$= \left(\frac{49}{50}\right)^{-1/2}$$
$$S = \sqrt{\frac{50}{49}} = \frac{5\sqrt{2}}{7}$$
$$= \frac{\sqrt{50}}{7}$$
$$= \frac{\sqrt{5^2\times2}}{7}$$
$$= \frac{5\sqrt{2}}{7}$$

---

⑨ Find the coefficient of x^2 in the expansion $(1+x)$

The coefficient of x^2 in the expansion of $(1+x)^{-n}$ ..

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$$(2-x)\left[x^2-5x+4\right]-2\left[2-x\right]+1(x-1)=0$$

$$2x^2-10x+8-x^3+5x^2-4x-4+2x+x-1=0$$

$$-x^3+7x^2-11x+5=0$$

$$x^3-7x^2+11x-5=0$$

which is the characteristic equ of matrix of

$$A^3-7A^2+11A-5I=0$$

$$A^3=A\times A^2$$

$$A^2=\begin{bmatrix}2&2&1\\1&3&1\\1&2&2\end{bmatrix}\begin{bmatrix}2&2&1\\1&3&1\\1&2&2\end{bmatrix}$$

$$A^2=\begin{bmatrix}7&12&6\\6&13&6\\6&12&7\end{bmatrix}$$

$$A^3=A^2\times A$$

$$A^3=\begin{bmatrix}7&12&6\\6&13&6\\6&12&7\end{bmatrix}\begin{bmatrix}2&2&1\\1&3&1\\1&2&2\end{bmatrix}=\begin{bmatrix}32&62&31\\31&63&31\\31&62&32\end{bmatrix}$$

$$A^3-7A^2+11A-5I=0$$

$$=\begin{bmatrix}32&62&31\\31&63&31\\31&62&32\end{bmatrix}-7\begin{bmatrix}7&12&6\\6&13&6\\6&12&7\end{bmatrix}+11\begin{bmatrix}2&2&1\\1&3&1\\1&2&2\end{bmatrix}-5\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=0$$

$$=\begin{bmatrix}32-49+22-5&62-84+22-0&31-42+11-0\\31-42+11-0&63-91+33-5&31-42+11-0\\31-42+11-0&62-84+22-0&32-49+22-5\end{bmatrix}=0$$

$$=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}=0$$

Which is characteristic matrix .A to find `A'

$$A^3 - 5A^2 + 9A - 11 = 0$$

$$A^3A^{-1} - 5A^2A^{-1} + 9A^2A^{-1} - 11A^{-1} = 0$$

$$A^2 - 5A + 9I - 11A^{-1} = 0$$

$$-11A^{-1} = A^2 - 5A + 9I$$

$$A^{-1} = \frac{1}{11}[A^2 - 5A + 9I]$$

$$A^2 = \begin{bmatrix} 1 & -3 & 3 \\ 1 & 2 & -1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -3 & 3 \\ 1 & 2 & -1 \\ 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -6 & 12 \\ 2 & 0 & -1 \\ 4 & 1 & 6 \end{bmatrix}$$

$$5A = \begin{bmatrix} 5 & -15 & 15 \\ 5 & 10 & -15 \\ 5 & 5 & 10 \end{bmatrix}$$

$$9I = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$A^{-1} = \frac{1}{11}\left[\begin{bmatrix} 1 & -6 & 12 \\ 2 & 0 & -1 \\ 4 & 1 & 6 \end{bmatrix} - \begin{bmatrix} 5 & -15 & 15 \\ 5 & 10 & -15 \\ 5 & 5 & 10 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}\right]$$

$$= \frac{1}{11}\begin{bmatrix} 5 & 9 & -3 \\ -3 & -1 & 4 \\ -1 & -4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 5/11 & 9/11 & -3/11 \\ -3/11 & -1/11 & 4/11 \\ -1/11 & -4/11 & 5/11 \end{bmatrix}$$

$$\left|\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix} - \begin{bmatrix}x & 0\\0 & x\end{bmatrix}\right| = 0$$

$$\left|\begin{bmatrix}1-x & 2\\3 & 4-x\end{bmatrix}\right| = 0$$

$$(1-x)(4-x) - 6 = 0$$

$$4 - x - 4x + x^2 - 6 = 0$$

$$x^2 - 5x - 2 = 0$$

which is characteristic eau of matrix A

$$A^2 - 5A - 2I = 0$$

$$A^2 = A \times A = \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix} = \begin{bmatrix}7 & 10\\15 & 22\end{bmatrix}$$

$$A^2 - 5A - 2I = 0$$

$$\begin{bmatrix}7 & 10\\15 & 22\end{bmatrix} - 5\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix} - 2\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = 0$$

$$\begin{bmatrix}7-5-2 & 10-10-0\\15-15-0 & 22-20-2\end{bmatrix} = 0$$

$$\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix} = 0$$

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3 using cayley Hamiltan theorem A^{-1} for A $A = \begin{bmatrix}1 & -3 & 3\\1 & 2 & -1\\1 & 1 & 2\end{bmatrix}$

$$|A - xI| = 0$$

$$\left|\begin{bmatrix}1 & -3 & 3\\1 & 2 & -1\\1 & 1 & 2\end{bmatrix} - x\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\right| = 0$$

$$\begin{vmatrix}1-x & -3 & 3\\1 & 2-x & -1\\1 & 1 & 2-x\end{vmatrix} = 0$$

$$(1-x)[(2-x)(2-x)+1] + 3((2-x)+1) + 3(1-(2-x)) = 0$$

$$(1-x)((2-x)^2+1) + 3(2-x-1) + 3(1-2+x) = 0$$

$$(1-x)(2^2+x^2-2x+1) + 3(2-x-1) + 3(1-2+x) = 0$$

$$5 - 4x + x^2 - 5x + 4x^2 - x^3 + 9 - 3x - 3 + 3x = 0$$

$$-x^3 - 5x^2 - 9x + 11 = 0$$

$$(-)x \Rightarrow x^3 - 5x^2 + 9x - 10 = 0$$

⑤ using cayley Hambitian theorem A^{-1} for A $A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix}$

$|A - xI| = 0$

$$\left| \begin{bmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix} - \begin{bmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 1-x & 0 & -2 \\ 2 & 2-x & 4 \\ 0 & 0 & 2-x \end{bmatrix} \right| = 0$$

$$(1-x)\left[(2-x)(2-x) - 0\right] - 0(2(2-x) - 0) - 2(0-0) = 0$$

$$(1-x)(2-x)^2 - 0 - 0 = 0$$

$$(1-x)(4 + x^2 - 2x) = 0$$

$$4 + x^2 - 2x - 4x - x^3 + 2x^2 = 0$$

$$-x^3 + 3x^2 - 6x + 4 = 0$$

$$(-)x \Rightarrow x^3 - 3x^2 + 6x - 4 = 0$$

① using Cayley hamiltian theorem A^{-1} for A

characteristic matrix $A^3 - 3A^2 + 6A - 4 = 0$

$$A^3 - 3A^2 + 6A - 4 = 0$$

$$A^3A^{-1} - 3A^2A^{-1} + 6AA^{-1} - 4A^{-1} = 0$$

$$A^2 - 3A + 6I - 4A^{-1} = 0$$

$$-4A^{-1} = A^2 - 3A + 6I$$

$$A^{-1} = 1/4\,[A^2 - 3A + 6I]$$

$$A^2 = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0-4 & 0+0-0 & -2+0-4 \\ 2+4+0 & 0+4+0 & -4+8+8 \\ 0+0+0 & 0+0+0 & 0+0+4 \end{bmatrix} = \begin{bmatrix} -3 & 0 & -6 \\ 6 & 4 & 12 \\ 0 & 0 & 4 \end{bmatrix}$$

$$3A = \begin{bmatrix} 3 & 0 & -6 \\ 6 & 6 & 12 \\ 0 & 0 & 6 \end{bmatrix}$$

$$6I = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= 1/4 \begin{bmatrix} -3 & 0 & -6 \\ 6 & 4 & 12 \\ 0 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 0 & -6 \\ 6 & 6 & 12 \\ 0 & 0 & 6 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= 1/4 \begin{bmatrix} -3-3+6 & 0-0+0 & -6+6+0 \\ 6-6+0 & 4-6+6 & 12-12+0 \\ 0-0+0 & 0-0+0 & 4-6+6 \end{bmatrix}$$

$$= 1/4 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & \dots \end{bmatrix}$$

Case (iii)

$\lambda = 6$

$$\left[\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix} - 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right]\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\left[\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}\right]\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} -5 & 1 & 3 \\ 1 & -1 & 1 \\ 3 & 1 & -5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$-5x_1 + x_2 + 3x_3 = 0 \longrightarrow (1)$

$1x_1 - x_2 + x_3 = 0 \longrightarrow (2)$

$3x_1 + x_2 - 5x_3 = 0 \longrightarrow (3)$

$$\begin{array}{cccc} 1 & 3 & -5 & 1 \\ -1 & 1 & 1 & -1 \end{array}$$

$$= \frac{1+3}{x_1} = \frac{3+5}{x_2} = \frac{5-1}{x_3}$$

$$= \frac{4}{x_1} = \frac{8}{x_2} = \frac{4}{x_3}$$

$x_1 = 4 \quad , \quad x_2 = 8 \quad , \quad x_3 = 4$

$x = 1 \quad , \quad x_2 = 2 \quad , \quad x_3 = 1$

dividing throughout by a we get

$$x^2+y^2+z^2+\frac{2ux}{a}+\frac{2vy}{a}+\frac{2wz}{a}+d/a=0$$

The centre of this sphere $\sqrt{-u/a, -v/a, -w/a}$

and it is radius $=\sqrt{u^2/a^2+v^2/a^2+w^2/a^2-d/a}$

NOTE - 6 :

The eqn $x^2+y^2+z^2+2ux+2vy+2wz+d=0$

Here the coefficients x^2, y^2, z^2 all are equal and each equal to 1

This Eqn contains 4 independent orbitary constant u, v, w, d. Hence A Sphere can be found to satisfy 4 independent Geometrical conditions for instance a sphere can be found to passes through 4 Given points not laying in one plane.

NOTE - 7

Eqn of the Sphere described and the line Joining the Points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ as diameter

let $P(x, y, z)$ be any point on the Sphere describe on A, B as diameter. The directions ratios of AP are $x-x_1, y-y_1, z-z_1$ and those of BP are

$v - w = -1 \rightarrow (3)$

$2u - 4v = 5 \rightarrow (4)$

$u + 2v + 3w = -7 \rightarrow (5)$

$$\cancel{2u} - 4v = 5$$
$$\cancel{2u} + 4v + 6w = -14$$
$$-8v - \cancel{6w} = 19 \rightarrow (6)$$
$$6v - \cancel{6w} = -6$$
$$-14v = 25$$
$$v = \frac{-25}{14} \rightarrow (7)$$

$$v - w = -1$$
$$\frac{-25}{14} - w = -1$$
$$-w = -1 + 25/14$$
$$-w = -11/14$$
$$w = 11/14 \rightarrow (8)$$

$$(4) \Rightarrow 2u - 4v = 5$$
$$2u - 4(25/14) = 5$$
$$u = -15/14$$

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$$
$$x^2 + y^2 + z^2 - 15/7\, x - 25/7\, y - 11/7\, z + 0 = 0$$
$$7(x^2 + y^2 + z^2) - 15x - 25y - 11z = 0$$

(10) Find the eqn of the sphere whose centre (6, -1, 2) and touching the $2x - y + 2z = 2$

let

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0 \rightarrow (1)$$

Given

$-u = 6$ $-v = -1$ $-w = 2$

$u = -6$ $v = 1$ $w = -2$

$$(1) \Rightarrow x^2 + y^2 + z^2 + 2(-6)x + 2(1)y + 2(-2)z + d = 0$$
$$x^2 + y^2 + z^2 - 12x + 2y - 4z + d = 0 \rightarrow (2)$$

To find

$2x - y + 2z = 2$ touch the sphere

(8) Find the equation of the sphere $(1,-1,-1)$ and $(-3,4,5)$ as diameter

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)+(z-z_1)(z-z_2)=0 \rightarrow (1)$$

$x_1 = 1$ $\quad$ $x_2 = -3$

$y_1 = -1$ $\quad$ $y_2 = 4$

$z_1 = -1$ $\quad$ $z_2 = 5$

$$(x-1)(x+3)+(y+1)(y-4)+(z+1)(z-5)$$

$$x^2+3x-x-3+y^2-4y+y-4+z^2-5z+z-5=0$$

$$x^2+2x-3+y^2-3y-4+z^2-4z-5=0$$

$$x^2+y^2+z^2+2x-3y-4z-12=0$$

which is the required eqn of the sphere

(9) Find the eqn of the sphere passes through the point $(0,0,0)$ $(0,1,-1)$ $(-1,2,0)$ and $(1,2,3)$

let the eqn of the sphere

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0 \rightarrow (1)$$

(1) Passes through $(0,0,0)$ we get $d=0$

$$x^2+y^2+z^2+2ux+2vy+2wz=0 \rightarrow (2)$$

(2) passes through

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0$$

$$0+1+(-1)+2(0)+2(1)+2(-1)w=0$$

$$2+2v-2w=0$$

$$v-2w=-1 \rightarrow (3)$$

$(-1,2,0)$

$$1+4+0-2u+4v+0=0$$

$$5-2u+4v=0$$

$$2u-4v=5 \rightarrow (4)$$

$(1,2,3)$

$$1+4+9+2u+4v+6w=0$$

$$14+2u+4v+6w=0$$

$$2u+4v+6w=14$$

$$u+2v+3w=-7 \rightarrow (5)$$

Find the centre and radius of the sphere

$x^2+y^2+z^2+2x-4y-6z+5=0$

sphere $\rightarrow x^2+y^2+z^2+2x-4y-6z+5=0$

$2u=2$ $\quad 2v=-4$ $\quad 2w=-6$

$u=1$ $\quad v=-2$ $\quad w=-3$

centre $=$ ($-\frac{1}{2}$ coeff of x, $-\frac{1}{2}$ coeff of y, $-\frac{1}{2}$ coeff of z)

$= (-u, -v, -w)$

centre is $(-1, 2, 3)$

Radius $= \sqrt{u^2+v^2+w^2-d} = \sqrt{1^2+2^2+3^2-5} = \sqrt{1+4+9-5} = 3$

Hence centre is $(-1, 2, 3)$ radius is 3

NOTE 3:

If $u^2+v^2+w^2 > d$ then $r = \sqrt{u^2+v^2+w^2-d}$ is real

Sphere if $u^2+v^2+w^2 < d$ then $r = \sqrt{u^2+v^2+w^2-d}$ is imaginary and so we get an imaginary sphere

If $u^2+v^2+w^2 = d$ then $r = \sqrt{u^2+v^2+w^2-d} = 0$ the spheres reduces to a point and it is centre (Points Sphere)

NOTE - 4:

The equation of the sphere has a following three characteristics

it is of the second degree in (x, y, z).

The coefficients of x^2, y^2, z^2 are all equal.

The products terms xy, yz, zx are absent

NOTE - 5

The most General equation of Second degree in x, y and z is $ax^2+by^2+cz^2+2fyz+2gzx+2hxy+2ux+2vy+2wz+d=0$

This will represent a Sphere only if $a=b=c$ and $f=g=h=0$

In that case the eqn reduces to $a(x^2+y^2+z^2)+2ux+2vy+2wz+d=0$

UNIT - 3 SPHERE:

Sphere

A sphere is a locus of a point which moves. Such that it is distance from a fixed point is analyse or always equal to a constant. The fixed Point is called the centre of the Sphere and the constant distance is known as the radius of the sphere

NOTE - 1

equation of the sphere who centre and radius are given let (a, b, c) be the coordinate of the centre C and r radius of the sphere Put $P(x, y, z)$ the any point on the sphere

$$\boxed{(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2}$$

this is the equation of the required sphere.

The equation of the sphere the centre of origin and radius r is

$x^2 + y^2 + z^2 = a^2$

Note - 2

Therefore the point (x, y, z) lies on its sphere whose centre is $(-u, -v, -w)$ and the radius is $\sqrt{u^2 + v^2 + w^2 - d}$

$$\begin{matrix} 1 & 3 & -2 & 1 \\ 2 & 1 & 1 & 2 \end{matrix}$$

$$\Rightarrow \frac{1-6}{x_1} = \frac{3+2}{x_2} = \frac{-4-1}{x_3}$$

$$\Rightarrow -5/x_1 = 5/x_2 = -5/x_3$$

$$\Rightarrow x_1 = -5 \;,\; x_2 = 5 \;,\; x_3 = -5$$

equ divided by 5

$$x_1 = -5/5 \;,\; x_2 = 5/5 \;,\; x_3 = -5/5$$

$$x_1 = -1 \;,\; x_2 = 1 \;,\; x_3 = -1$$

Case -(ii)

$$\lambda = -2$$

$$(A - \lambda I)x = 0$$

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 2 & 1 & 3 \\ 1 & 7 & 1 \\ 3 & 1 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$2x_1 + 1x_2 + 3x_3 = 0 \rightarrow (1)$$

$$x_1 + 7x_2 + x_3 = 0 \rightarrow (2)$$

$$3x_1 + x_2 + 3x_3 = 0 \rightarrow (3)$$

$$\begin{matrix} 1 & 3 & 2 & 1 \\ 1 & 3 & 3 & 1 \end{matrix}$$

$$= \frac{3-3}{x_1} = \frac{9-6}{x_2} = \frac{2-3}{x_3}$$

$$= 0/x_1 = 4/x_2 = -1/x_3$$

$$x_1 = 0 \;,\; x_2 = 4 \;,\; x_3 = -1$$

(2) Find the eigen values and eigen vectors of the following

$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

Given $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

To eigen values $|A - \lambda I| = 0$

$$\left| \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \right| = 0$$

$$\begin{vmatrix} 1-\lambda & 1 & 3 \\ 1 & 5-\lambda & 1 \\ 3 & 1 & 1-\lambda \end{vmatrix} = 0$$

$(1-\lambda)[(5-\lambda)(1-\lambda)-1] - 1[(1-\lambda)-3] + 3[1-3(5-\lambda)] = 0$

$(1-\lambda)[5-5\lambda-\lambda-1\lambda^2-1] - 1[1-\lambda-3] + 3[1-15+3\lambda] = 0$

$(1-\lambda)(\lambda^2-6\lambda+4) - 1[-\lambda-2] + 3[3\lambda-14] = 0$

$\lambda^2 - 6\lambda + 4 - \lambda^3 + 6\lambda^2 - 4\lambda + \lambda + 2 + 9\lambda - 42 = 0$

$-\lambda^3 + 7\lambda^2 + 0 - 36 = 0$

$(-)x \Rightarrow \lambda^3 - 7\lambda^3 + 36 = 0$

3	1	-7	0	36
	0	3	-12	-36
	1	-4	-12	0

-12

-6 +2

-4

$\lambda = 3$ | $\lambda + 2$

$\lambda = 3$ | $\lambda = -2$

$\lambda - 6 = 0$

$\lambda = 6$

Case - i

$\lambda = 3$

$(A - \lambda I)x = 0$

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} -2 & 1 & 3 \\ 1 & 2 & 1 \\ 3 & 1 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$-2x_1 + x_2 + 3x_3 = 0 \longrightarrow (1)$

$x_1 + 2x_2 + x_3 = 0 \longrightarrow (2)$

$3x_1 + x_2 - 2x_3 = 0 \longrightarrow (3)$

⑤ find the eqn of the sphere whose centre is $(2,-3,4)$ and $r=3$

$$(x-a)^2+(y-b)^2+(z-c)^2=r^2$$

$a=2 \quad b=-3 \quad c=4 \quad r=3$

$$(x-2)^2+(y+3)^2+(z-4)^2=3^2$$

$$x^2+4x+4+y^2+6y+9+z^2-8z+16=9$$

$$x^2+y^2+z^2-4x+6y-8z+29=9$$

$$x^2+y^2+z^2-4x+6y-8z+29-9=0$$

$$x^2+y^2+z^2-4x+6y-8z+20=0$$

which is the required eqn

⑥ $(2,-3,4)$ $r=3$

$-u=2 \quad -v=-3 \quad -w=4 \quad r=3$

$u=-2 \quad v=3 \quad w=-4$

$$r^2=\sqrt{u^2+v^2+w^2-d}$$

$$(3)^2=\sqrt{4+9+16-d}$$

$$9=\sqrt{29-d}$$

$$d=29-9$$

$$d=20$$

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0$$

$$x^2+y^2+z^2-4x+6y-8z+20=0$$

⑦ find the $(1,1,1)$ $(-2,0,3)$

$-u=1 \quad -v=1 \quad -w=1$

$u=-1 \quad v=-1 \quad w=-1$

Shere of equation is

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0$$

$$x^2+y^2+z^2-2x-2y-2z+d=0$$

① ⇒ Passes through $(2,0,3)$

$$4+0+9-4-0-6+d=0$$

$$3+d=0$$

$$d=-3$$

hence the required equation of sphere

③ find the centre and radius of the sphere $2x^2+2y^2+2z^2-2x+4y+2z+3=0$

Given

$$2x^2+2y^2+2z^2-2x+4y+2z+3=0$$

$\div 2$

$$x^2+y^2+z^2-x+2y+z+3/2=0$$

($-1/2$ coeff of x , $-1/2$ coeff of y , $-1/2$ coeff of z)

$(-1/2(-1), -1/2(2), -1/2(1))$

$(1/2, -1, -1/2)$

$r = \sqrt{u^2+v^2+w^2-d}$

$= \sqrt{(1/2)^2+(-1)^2+(-1/2)^2-3/2}$

$= \sqrt{1/4+1+1/4-3/2}$

$= \sqrt{1/2+1-3/2}$

$= \sqrt{1/2-1/2}$

$= \sqrt{1-1}$

$= 0$

④ find the equation of the sphere whose centre is $(3,-4,5)$ radius $= 7$

Given

the equation of the sphere

$$(x-a)^2+(y-b)^2+(z-c)^2=r^2$$

$a=3$, $b=-4$, $c=5$, $r=7$

$(x-3)^2+(y+4)^2+(z-5)^2=7^2$

$x^2-6x+9+y^2+8y+16+z^2-10z+25=49$

$x^2+y^2+z^2-6x+8y-10z+50=49$

$x^2+y^2+z^2-6x+8y-10z+50-49=0$

$x^2+y^2+z^2-6x+8y-10z+1=0$

which is the required equation

$(1-x)^{-n}$ is $\frac{n(n+1)}{2!}$

$$= \frac{(-3)(-3+1)}{1\times 2}$$

$$= \frac{-3\times -2}{2}$$

$$= 6/2$$

$$= 3$$

(10) Find the coefficient of x^n the expansion of $\frac{1}{1-x^2}$

$$\frac{1}{1-x^2} = \frac{A}{1+x} + \frac{B}{1-x}$$

$$= \frac{A(1-x) + B(1+x)}{(1+x)(1-x)}$$

equating the numerator $1 = A(1-x) + B(1+x)$

Put $x = 1$

$$1 = A(1-1) + B(1+1)$$

$$1 = A(0) + B(2)$$

$$B = 1/2$$

The coefficient of x^n is the, expansion

$$= 1/2(-1)^n + 1/2$$

$$= 1/2\left[(-1)^n + 1\right]$$

(11) Coefficient of x^3 in the expansion of $\left(1+\frac{2x}{3}\right)^{3/2}$

The x^3 term are the expansion is

$\left(1+\frac{2x}{3}\right)^{3/2}$

$$\frac{P(P-q)(P-2q)}{3!}(x/q)^3$$

$P = 3$ $\quad q = 2$ $\quad x = \frac{2x}{3}$

$$= \frac{3\cdot(3-2)(3-2(2))}{3!}\left(\frac{2x/3}{2}\right)^3$$

$$= \frac{3(3-2)(3-4)}{3!}\left(\frac{2x/3}{2}\right)^3$$

$$= \frac{3(1)(-1)}{6!}\left(\frac{1}{27}\right)x^3$$

There the coefficient of x^3 is expansion of $\left(1+\frac{2x}{3}\right)^{3/2}$

(4) Obtain the characteristic Polynomial of the following

$A = \begin{bmatrix} -b & -c \\ 1 & 0 \end{bmatrix}$

$$|A - xI| = 0$$

$$\left| \begin{bmatrix} -b & -c \\ 1 & 0 \end{bmatrix} - x \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} -b & -c \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} -b-x & -c \\ 1 & -x \end{vmatrix} = 0$$

$$(-b-x)(-x) + c = 0$$

$$bx + x^2 + c = 0$$

$$x^2 + bx + c = 0$$

Cayley Hamilitan theorem:

Any Square matrix A Satisfied Its own characteristic equation (or) if $a_0 + a_1 x + a_2 x^2 + \dots a_n x^n$ where $a_0 I + a_1 x + a_2 A^2 + \dots + a_n A^n$ the characteristic polynomial of degree A.

$$a_0 I + a_1 A + a_2 A^2 + \dots + a_n A^n = 0$$

---

(1) Verify Cayley Hamilitan theorem $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

The characteristic equ $|A - xI| = 0$

$$\left| \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} - \begin{bmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 2-x & 2 & 1 \\ 1 & 3-x & 1 \\ 1 & 2 & 2-x \end{bmatrix} \right| = 0$$

$$(2-x)[(3-x)(2-x)-2] - 2[(2-x)-1] + 1[2-(3-x)] = 0$$

$$(2-x)[6-3x-2x+x^2-2] - 2[1-x] + 1[x-1] = 0$$

(2) Obtain characteristic polynomial equation the following matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Given $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$|A - xI| = 0$$

$$\left| \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - x \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 1-x & 2 \\ 3 & 4-x \end{vmatrix} = 0$$

$$(1-x)(4-x) - 6 = 0$$

$$4 - x - 4x + x^2 - 6 = 0$$

$$x^2 - 5x - 2 = 0$$

$$A = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$AA^T = A^TA = I$$

$$A^T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$AA^T = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & -\cos\theta \cdot \sin\theta + \sin\theta\cos\theta \\ -\sin\theta\cos\theta + \cos\theta\sin\theta & -\sin^2\theta + \cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= I$$

$$A^TA = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta \cdot & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= I$$

Equalent matrices:

Two matrices A and B of the Same order are to be equivalent one of them can be obtained from the otherby elementary the transformation is written by

$A \sim B$

$$A = \begin{bmatrix} 3 & 6 \\ 3 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 9 \\ 1 & 9 \end{bmatrix}$$

Transpose matrices:

In rows and columns are interchanged in a matrices A we obtaine second matrices that is called transpose of the original matrices and is denoted by A^T

$$Ex : A = \begin{bmatrix} 2 & 3 \\ 8 & 10 \end{bmatrix} \quad A^T = \begin{bmatrix} 2 & 8 \\ 3 & 10 \end{bmatrix}$$

Singular matrices:

A determinant of A is 0 , $|A| = 0$ is called a singular matrix

$$A = \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} , \ |A| = \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = (2-2) = 0$$

Non- Singular matrices

It is determinant of A is not equal to zero $|A| \neq 0$ is called a non- Singular matrices.

$$A = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = |A| = 2 \neq 0 \quad |A| \neq 0$$

Triangular matrix:

A singular matrix which is upper and lower triangular matrix, it is called a triangular matrix.

$$t(x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

③ If $3\begin{bmatrix}2 & a\\ b & c\end{bmatrix} = \begin{bmatrix}2 & b\\ 1 & -2c\end{bmatrix} + \begin{bmatrix}4 & a+2\\ b+c & 3\end{bmatrix}$ find a, b, c

$$3\begin{bmatrix}2 & a\\ b & c\end{bmatrix} = \begin{bmatrix}6 & a+8\\ 1 & -2c\end{bmatrix} + \begin{bmatrix}4 & a+2\\ b+c & 3\end{bmatrix}$$

$$\begin{bmatrix}6 & 3a\\ 3b & 3c\end{bmatrix} = \begin{bmatrix}6 & a+8\\ -1+b+c & 2c+3\end{bmatrix}$$

$3a = a+8$

$3a - a = 8$

$2a = 8$

$a = 8/2$

$a = 4$

$3b = -1+b+c$

$3b - b = -1+c$

$2b = -1+3$

$2b = 2$

$b = 2/2$

$b = 1$

$3c = 2c+3$

$3c - 2c = 3$

$c = 3$

---

④ Find the value of x, y, z and w that statisfy the matrix $\begin{bmatrix}x+3 & 2y+5\\ z+4 & 4x+5\\ w-2 & 3w+1\end{bmatrix} = \begin{bmatrix}1 & -5\\ -4 & 2x+1\\ 2w+5 & -20\end{bmatrix}$

$x + 3 = 1$

$x = 3 - 1$

$x = -2$

$2y + 5 = -5$

$2y = -5 - 5$

$2y = 10$

$y = 5$

$z + 4 = -4$

$z = -8$

$3w + 1 = -20$

$3w = -21$

$w = -7$

Addition and Subtraction

① $A = \begin{bmatrix} 2 & 3 \\ 10 & 15 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 7 \\ 8 & 24 \end{bmatrix}$ find (i) A+B (ii) A-B (iii) 2A+2B (iv) 2A + 3B

(i) A+B $= \begin{bmatrix} 2 & 3 \\ 10 & 15 \end{bmatrix} + \begin{bmatrix} 4 & 7 \\ 8 & 24 \end{bmatrix} = \begin{bmatrix} 6 & 10 \\ 18 & 39 \end{bmatrix}$

(ii) A - B $= \begin{bmatrix} 2 & 3 \\ 10 & 15 \end{bmatrix} - \begin{bmatrix} 4 & 7 \\ 8 & 24 \end{bmatrix} = \begin{bmatrix} -2 & -4 \\ 2 & -9 \end{bmatrix}$

(iii) 2A + 2B $= 2\begin{bmatrix} 2 & 3 \\ 10 & 15 \end{bmatrix} + 2\begin{bmatrix} 4 & 7 \\ 8 & 24 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 & 6 \\ 20 & 30 \end{bmatrix} + \begin{bmatrix} 8 & 14 \\ 16 & 48 \end{bmatrix}$

$= \begin{bmatrix} 12 & 20 \\ 36 & 78 \end{bmatrix}$

(iv) 2A + 3B $= 2\begin{bmatrix} 2 & 3 \\ 10 & 15 \end{bmatrix} + 3\begin{bmatrix} 4 & 7 \\ 8 & 24 \end{bmatrix}$

$= \begin{bmatrix} 4 & 6 \\ 20 & 30 \end{bmatrix} + \begin{bmatrix} 12 & 21 \\ 24 & 72 \end{bmatrix} \Rightarrow \begin{bmatrix} 16 & 27 \\ 44 & 102 \end{bmatrix}$

Square matrices:

A square matrices is one within the number of columns number of rows and the are equal

ex $A = \begin{bmatrix} 2 & 5 \\ 3 & 6 \end{bmatrix}_{2\times 2}$, $\begin{bmatrix} 2 & 6 & 5 \\ 2 & 7 & 6 \\ 2 & 8 & 5 \end{bmatrix}_{3\times 3}$

Diagonal matrices:

A diagonal matrices is a square matrices of any order with zero elements everywhere except on main diagonal

$A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

Scalar matrices:

A scalar matrices is a diagonal matrices in which all the elements along the main diagonal are equal

$A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

unit matrices:

unit matrices is a scalar matrices in which all the elements along the main diagonal are its unity

Ex $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Null or zero matrices

if all the elements in the matrices are zero is called a null or zero matrices

$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

equal matrices:

Two matrices A and B are equal . If and only lives they have save order That is $A = [a_{ij}]_{m\times n}$, $B = [a_{ij}]_{m\times n}$ than $A = B$ the element at corresponding places are equal that is $a_{ij} = b_{ij}$

Ex $= \begin{bmatrix} 3 & 6 & 5 \\ 1 & 2 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 6 & 5 \\ 1 & 2 & 3 \end{bmatrix}$

Transverse of matrix

$$B^T A^T = (AB)^T$$

If $A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}$ $B^T = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 1 \end{bmatrix}$

$B^T A^T = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 4 \\ 4 & 4 \end{bmatrix} \rightarrow (1)$

$(AB)^T = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 4 \\ 4 & 4 \end{bmatrix} \rightarrow (2)$

from (1) and (2) $B^T A^T = (AB)^T$

Find the value of determinant $A = \begin{bmatrix} 3 & -2 & 1 \\ 2 & 3 & -1 \\ 1 & 1 & 1 \end{bmatrix}$

Given $A = \begin{bmatrix} 3 & -2 & 1 \\ 2 & 3 & -1 \\ 1 & 1 & 1 \end{bmatrix}$

$|A| = 3(3+1) + 2(2+1) + 1(2-3)$

$= 3(4) + 2(3) + 1(-1)$

$= 12 + 6 - 1$

$|A| = 17$

Adjoint of matrix:

The adjoint matrix of Square matrix A is denoted by adjoint A is the transpose of the co-factor of example = adj A = (cofactor matrix)

If $A = \begin{bmatrix} 5 & 6 & 7 \\ 0 & 1 & -3 \\ -2 & 4 & 9 \end{bmatrix}$ find adj A

adj A = cofactor matrix

$\text{adj } A = \begin{bmatrix} (9+12) & -(0-6) & (0+2) \\ (54-28) & (-45-14) & -(20+12) \\ (-18-7) & -(-15) & (5) \end{bmatrix} = \begin{bmatrix} 21 & 6 & 2 \\ -26 & 59 & -32 \\ -25 & 15 & 5 \end{bmatrix}$

$\text{adj } (A)^T = \begin{bmatrix} 21 & -26 & -25 \\ 6 & 59 & 15 \\ 2 & -32 & 5 \end{bmatrix}$

UNIT - 2 MATRICES

Definition of matrices

A matrices is defined as a number of arranged into row and columns it is written as follows $\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ a_{m1} & a_{m2} & & a_{mn} \end{bmatrix}$ the above array is called am m by n matrices [written a $m \times n$] since it has m row and n column. The individual numbers in the array called the elements

Types of the matrices

* row matrices
* Column matrices
* Square matrices
* diagonal matrices
* Scalar matrices
* unit matrices
* null (or) zero matrices
* equal matrices
* equallen matrices
* transpose matrices
* Singular matrices
* Non-Singular matrices
* triangular matrices
* upward triangular matrices
* lower triangular matrices

* Row matrices:

A row matrices is a matrix with a only one Row

$$Ex = A = [3 \quad 6 \quad 5]_{1\times 3}$$

* Column matrices:

A column matrices is a matrices it only one column $Ex = A = \begin{bmatrix} 2 \\ 4 \\ 8 \end{bmatrix}_{3\times 1}$

(20) sum the series $\frac{x}{1\cdot 2} - \frac{x^2}{2\cdot 3} + \frac{x^3}{3\cdot 4} - \cdots$

Then n^{th} term of the series is

$$(-1)^{n-1}\frac{x^n}{(n)(n+1)}$$

we know that $\frac{1}{(n)(n+1)} = \frac{1}{n} - \frac{1}{n+1}$

$$t_n = (-1)^{n-1}(x^n/n) - (-1)^n(x^n/n+1)$$

$$S = \sum_1^{\infty}(-1)^{n-1}(x^n/n) - \sum_1^{\infty}(-1)^n(x^n/n+1)$$

$$S = (x/1) - x^2/2 + x^3/3 \cdots) - (x/2 - x^2/3 + x^3/4 - \cdots)$$

$$S = (x/1) - x^2/2 + x^3/3 - \cdots) - 1/x\,(x^2/2 - x^3/3 + x^4/4 \cdots)$$

$$S = (x/1 - x^2/2 + x^3/3 - \cdots) + 1/x\,[-x^2/2 + x^3/3 - x^4/4 \cdots)$$

$$S = \log(1+x) + 1/x\,[\log(1+x) - x]$$

$$(1 + 1/x)\log(1+x) - 1$$

⑤ sum the series $\frac{3}{18} + \frac{3 \cdot 7}{18 \cdot 24} + \frac{3 \cdot 7 \cdot 11}{18 \cdot 24 \cdot 30} + \dots \infty$

let $S = \frac{3}{18} + \frac{3 \cdot 7}{18 \cdot 24} + \frac{3 \cdot 7 \cdot 11}{18 \cdot 24 \cdot 30} + \dots \infty$

$q = 4$

Therefore the multiply LHS and RHS by $\frac{1}{6 \cdot 12}$

$$\frac{1}{6 \cdot 12} S = \frac{3}{6 \cdot 12 \cdot 18} + \frac{3 \cdot 7}{6 \cdot 12 \cdot 18 \cdot 24} + \frac{3 \cdot 7 \cdot 11}{6 \cdot 12 \cdot 18 \cdot 24 \cdot 30} + \dots \infty$$

Now, the number of factors and denominator are not equal

multiple numerator in LHS and RHS by $(-5)(-1)$

$$\frac{(-5)(-1)}{6 \cdot 12} S = \frac{(-5)(-1)(3)}{6 \cdot 12 \cdot 18} + \frac{(-5)(-1)(3)}{6 \cdot 12 \cdot 18 \cdot 24} + \frac{(-5)(-1)(3)(-1)(11)}{6 \cdot 12 \cdot 18 \cdot 24 \cdot 30} \dots \infty$$

add in $1 + \frac{(-5)}{6} + \frac{(-5)(-1)}{6 \cdot 12}$ both sides

$$1 + \frac{(6)}{6} + \frac{(-5)(-1)}{6 \cdot 12} + \frac{(-5)(-1)}{6 \cdot 12} S = 1 + \frac{(-5)}{6} + \frac{(-5)(-1)}{6 \cdot 12} + \frac{(-5)(-1)(3)}{6 \cdot 12 \cdot 18} + \dots \infty$$

Compare with series :

$$(1-x)^{-p/q} = 1 + \frac{p}{1!}(x/q) + \frac{p(p+q)}{2!}(x/q)^2 + \dots \infty$$

$p = -5 \quad q = 4 \quad (x/q) = (1/6)x = (4/6)x = (2/3)$

$$\frac{(-5)(-1)}{6 \cdot 12} S = (1-x)^{-p/q} - 1 - \frac{(-5)(-1)}{6 \cdot 12}$$

$$\frac{5}{72} S = (1 - 2/3)^{-(-5/4)} - 1 - \frac{(-5)}{6} - \frac{(-5)(-1)}{6 \cdot 12}$$

$$\frac{5}{72} S = -1 + \frac{5}{6} - \frac{(5)(1)}{6 \cdot 12}$$

$$S = \frac{72}{5}\left[(1/3)^{5/4} - \frac{17}{72}\right]$$

$$= \frac{72}{5}\left[\frac{1}{3(3)^{1/4}} - \frac{17}{72}\right]$$

$$S = 1/5\left[\frac{1}{8(3^{3/4})} - 17\right]$$

(15) Show that $\left(\frac{a-x}{a}\right) + \frac{1}{2}\left(\frac{a-x}{a}\right)^2 + \frac{1}{3}\left(\frac{a-x}{a}\right)^3 + \frac{1}{4}\left(\frac{a-x}{a}\right)^4$

Put $\left(\frac{a-x}{a}\right) = x$, $S = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots$

$$= -\left[-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \dots\right].$$

$$S = -\log(1-x) = -\log\left[1 - \left(\frac{a-x}{a}\right)\right]$$

$$S = -\log\left(\frac{a-(a-x)}{a}\right)$$

$$= -\log(x/a)$$

$$= \log(a/x)$$

---

(16) Show that $\log x = \frac{x-1}{x+1} + \frac{1}{2}\frac{(x^2-1)}{(x+1)^2} + \frac{1}{3}\frac{x^3-1}{(x+1)^3} + \frac{1}{4}\frac{(x^4-1)}{(x+1)^4} + \dots$

$$S = \frac{x-1}{x+1} + \frac{1}{2}\cdot\frac{(x^2-1)}{(x+1)^2} + \frac{1}{3}\cdot\frac{x^3-1}{(x+1)^3} + \frac{1}{4}\cdot\frac{x^4-1}{(x+1)^4}$$

$$S = \frac{x}{x+1} - \frac{1}{x+1} + \frac{1}{2}\frac{x^2}{(x+1)^2} - \frac{1}{2}\frac{1}{(x+1)^2} + \frac{1}{3}\frac{1}{(x+1)^3} - \frac{1}{3}\frac{1}{(x+1)^3} + \frac{1}{4}\frac{x^4}{(x+1)^4} - \frac{1}{4}\frac{1}{(x+1)^4} + \dots$$

Put $x = \frac{x}{x+1}$ $y = \frac{1}{x+1}$

$$S = x - y + \frac{1}{2}x^2 - \frac{1}{2}y^2 + \frac{1}{3}x^3 - \frac{1}{3}y^3 + \frac{1}{4}x^4 - \frac{1}{4}y^4 + \dots$$

$$S = \left(x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 \dots\right) - \left(y - \frac{1}{2}y^2 - \frac{1}{3}y^3 - \frac{1}{4}y^4 \dots\right)$$

$$S = -\left(x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4\right) + \left(-y - \frac{1}{2}y^2 - \frac{1}{3}y^3 - \frac{1}{4}y^4 \dots x\right)$$

$$S = \log(1-x) + \log(1-y)$$

$$= \log\left[\frac{1-y}{1-x}\right]$$

$$S = \log\left\{\frac{1 - 1/x+1}{1 - x/x+1}\right\}$$

$$= \log\left[\frac{x+1-1}{x+1} \Big/ \frac{x+1-x}{x+1}\right]$$

$$= \log\left[x/1\right]$$

$$= \log x$$

---

(17) Prove that $\log\left(\frac{n+1}{n-1}\right) = \frac{2n}{n^2+1} + \left(\frac{2n}{n^2+1}\right)^3 + \left(\frac{2n}{n^2+1}\right)^5 + \dots$

$$S = \frac{2n}{n^2+1} + \left(\frac{2n}{n^2+1}\right)^3 + \left(\frac{2n}{n^2+1}\right)^5 + \dots$$

Put $\frac{2n}{n^2+1} = x$ $S = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \dots$

Result

Replace x by $-x$

* $e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

* Put $x = 1$, $e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$

* Put $x = -1$, $e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots$

* Adding result (ii) and (iii)

$$\frac{e + e^{-1}}{2} = 1 + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!}$$

* Subtracting (iii) from (ii)

$$\frac{e - e^{-1}}{2} = \frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!}$$

Logarithm Series

$-1 < x < 1$ then $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$

Notes

(i) Replace x by $-x$

$$\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \cdots$$

(ii) $\log \frac{(1+x)}{(1-x)} = 2\left[x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots\right]$

(iii) $\log\left[\frac{1}{1-x^2}\right] = 2\left[x^2/2 + x^4/4 + x^6/6 + \cdots\right]$

Summation of Series using binomial Expansion

(i) The factors in the numerator are in arithmetic progression (AP)

(ii) The factor in denominators are in multiples of number

(iii) Each successive terms as one addition factor in the numerator and denominator

Sum the series $1 + 3/4 + \frac{3 \cdot 5}{4 \cdot 8} + \frac{3 \cdot 5 \cdot 7}{4 \cdot 8 \cdot 12} + \cdots$

Sol

Let $S = 1 + \frac{3}{4} + \frac{3.5}{4.8} + \frac{3.5.7}{4.8.12} + \cdots$

The numerator of the series in (AP) with a common two the denominator are

$$(P-x)^{-P/q} = 1 + \frac{P}{1!}(x/q) + \frac{P(P+q)}{2!}(x/q)^2 + \frac{P(P+q)(P+2q)}{3!}(x/q)^3$$

$P = 3$, $q = 2$ $\quad x/q = (1/4)$, $x = (2/4) = (1/2)$

$$S = (1-x)^{-P/q} = (1 - 1/2)^{-3/2}$$

$$= (1/2)^{-3/2}$$

$$= (2)^{3/2}$$

$$= 2^{1+1/2}$$

$$= 2^1 \times 2^{1/2}$$

$$= 2\sqrt{2}$$

(2) Sum the series $\frac{2}{5} + \frac{2\cdot4}{5\cdot10} + \frac{2\cdot4\cdot6}{5\cdot10\cdot15} + \dots \infty$

The numerator of the series is (A·P) common difference $q = 2$ and the dinominator of the multiply 5. The series is binomial

NOTE

Arrange the series similar to the binomial expansion for the rational index 1 and both sides as the first term one is missing the series

Sol

let $S = \frac{2}{5} + \frac{2\cdot4}{5\cdot10} + \frac{2\cdot4\cdot6}{5\cdot10\cdot15} + \dots \infty$

$q = 2$

$S+1 = 1 + \frac{2}{5} + \frac{2\cdot4}{5\cdot10} + \frac{2\cdot4\cdot6}{5\cdot10\cdot15} + \dots \infty$

$S+1 = 1 + \frac{2}{5} + \frac{2(2+2)}{1\cdot5\ \ 2\cdot5} + \frac{2(2+2)(2+4)}{1\cdot5\ \ 2\cdot5\ \ 3\cdot5} + \dots \infty$

$S+1 = 1 + \frac{2}{1}(1/5) + \frac{2(2+2)}{1\cdot2}(1/5)^2 + \dots$

Compare with expansion

$(1-x)^{-p/q} = 1 + \frac{p}{1!}(x/q) + \frac{p(p+q)}{2!}(x/q)^2 + \dots$

$p = 2, \quad q = 2 \qquad (x/q) = (1/5) \qquad q = 2/5$

$S+1 = (1-x)^{-p/q}$

$= (1 - 2/5)^{-2/2}$

$= (3/5)^{-1}$

$S+1 = 5/3$

$S = 5/3 - 1$

Syllabus

unit 1

Binomial and exponential and logarithmic series (formula only) - summation and approximation related problems only.

unit 2

Non-Singular, symmetric, skew symmetric, orthogonal hermitian, skew hermitation and unitary matrices - characteristics equation, values, eigen vectors - cayle theorem (proof not needed) simple applications only)

unit 3

finding the shortest distance between two skew lines and the equation of the plane containing them - contain condition for coplanarity equation of fear tangent plane - Plane section of sphere - finding the centre and radius of the circle of intersection - A sphere through a circle of intersection (only problems all the above.

unit 4

expansion of $\sin n\theta$, $\cos n\theta$, $\tan n\theta$ (n being a positive integer) expansion of $\cos^n\theta$, $\sin^n\theta$ in a series of sin and cos of multiples of θ (θ given in radians) Expansion of $\sin\theta$, $\cos\theta$, $\tan\theta$ in terms of Power of θ (only problems in all above)

unit - 5

Eulers formula for $e^{i\theta}$ - definition of hyperbolic functions - formula involving hyperbolic functions - Relations between hyperbolic & circular functions. expansion of $\sinh x$, $\tanh x$, $\cosh x$ in power of x - Expansion of inverse hyperbolic function $\sinh^{-1}x$, $\cosh^{-1}x$ and $\tanh^{-1}x$ - separation of real & imaginary part of $\sin(x+iy)$, $\cos(x+iy)$, $\tan(x+iy)$, $\sinh(x+iy)$, $\cosh(x+iy)$, $\tanh(x+iy)$

Binomial Expansion

$(x+a)^n = nC_0 x^n + nC_1 x^{n-1} a + nC_2 x^{n-2} a^2 + \dots + nC_r x^{n-r} a^r + nC_n$ *

Note

* The number of terms in the expansion are $(n+1)$

* The binomial co-efficient $\Rightarrow nC_1 = n/1$, $nC_2 = \frac{n(n-1)}{2!}$, $nC_3 = \frac{n(n-1)(n-2)}{3!}$, $nC_r = \frac{n(n-1)(n-2)n(r-1)}{r!}$

* $(1+x)^n = 1 + nC_1 x + nC_2 x^2 + \dots + nC_r x^r + \dots + nC_n x^n$

* Replace x by $-x$

$(1-x)^n = 1 - nC_1 + nC_2 x^2 + \dots + (-1)^r nC_r x^r + \dots + (-1)^n nC_n x^n$

If n is a rational numbers the number of terms in the expansions are infinite $(1+x)^n = 1 + \frac{n}{1!}x + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$

Provided $|x|<1$

Replace n by P/q

$$(1+x)^{P/q} = 1 + \frac{P}{1!}(x/q) + \frac{P(P-q)}{2!}(x/q)^2 + \frac{P(P-q)(P-2q)}{3!}(x/q)^3$$

Replace n by $-n$

$$(1+x)^{-n} = 1 - \frac{n}{1!}x + \frac{n(n+1)}{2!}x^2 - \frac{n(n+1)(n+2)}{3!}x^3 + \dots$$ Provided $|x|<1$

Particular cases

when $n=1$

$$(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 + \dots$$

$$(1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 + \dots$$

when $n=2$

$$(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + 5x^4 + \dots$$

$$(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + \dots$$

exponential series :

For all the series $-x$,

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

① Express $A = \begin{bmatrix} 0 & 5 & -3 \\ 1 & 1 & 1 \\ 4 & 5 & 9 \end{bmatrix}$

The sum of the symmetric and skew symmetric

Sol

Given $A = \begin{bmatrix} 0 & 5 & -3 \\ 1 & 1 & 1 \\ 4 & 5 & 9 \end{bmatrix}$, $A^T = \begin{bmatrix} 0 & 1 & 4 \\ 5 & 1 & 5 \\ -3 & 1 & 9 \end{bmatrix}$

where, symmetric matrix $S = \frac{1}{2}(A + A^T)$

Skew symmetric matrix $P = \frac{1}{2}(A - A^T)$

Transpose conjugate matrix

If A is any matrix the transp[ose] of conjugate of matrix A is called transpose conjugate of matrix, is denoted by

Herbitan matrix

If A is a square matrix and A [is] transpose of its conjugate such a matrix is called Herbitan matrix (ie), $A = (\bar{A})^T$

$$\text{If } A = \begin{bmatrix} a & b+ic & d-ic \\ b-ic & f & -g+ih \\ d+ie & -g-ih & k \end{bmatrix}$$

Skew habitan matrix

If A is a square matrix and if A is the negative transpose of its conjugate such a matrix is called a skew habitan matrix $A = (-\bar{A})^T$

$$\text{Ex : If } A = \begin{bmatrix} 1 & -1+2i & 5i \\ -1-2i & -2 & 6-7i \\ -5i & 6+7i & 3 \end{bmatrix} \text{ a habitan matrix}$$

$$A = (\bar{A}) = \begin{bmatrix} 1 & -1-2i & 5i \\ -1+2i & -2 & 6+7i \\ -5i & 6-7i & 3 \end{bmatrix}$$

$$= (\bar{A})^T \begin{bmatrix} 1 & -1+2i & -5i \\ -1-2i & -2 & 6-7i \\ 5i & 6+7i & 3 \end{bmatrix}$$

(e) To find the skew symmetric $A = \begin{bmatrix} 0 & 2 & 3 & -1 \\ -2 & 0 & 4 & -3 \\ -3 & 4 & 0 & 1 \\ 1 & 3 & -1 & 0 \end{bmatrix}$

$A = A^T$

$$A^T = \begin{bmatrix} 0 & -2 & -3 & 1 \\ 2 & 0 & 4 & 3 \\ 3 & 4 & 0 & -1 \\ -1 & -3 & 1 & 0 \end{bmatrix}$$

$$= - \begin{bmatrix} 0 & 2 & 3 & -1 \\ -2 & 0 & -4 & -3 \\ -3 & -4 & 0 & 1 \\ 1 & 3 & -1 & 0 \end{bmatrix}$$

$A = A^T$

The given matrix is skew matrix