## Queing theory:

"The Flow of customer from finite or infinite population towards service facilities is called as queue (Waiting line)".

**Def. customer:-**

"The arriving unit that requires some service to be perform is called a Customer".

**What are the Basic characteristics of Queing system:-**

1. The input (Arrival pattern)
2. The service mechanism (Service pattern)
3. The queue discipline.
4. Customer behaviour

**Define the following Reneging, Jockeying:- Balking?**

### Balking:

A condition in which a customer may leave the queue because the queue is too long and he has no time to wait (or) There is insufficient waiting space.

### Reneging:-

This occurs when waiting Customer leaves the queue due to impatience.

## Jockeying :-

Customer may Jockey from one waiting line to another

## Explain Kendall's notation :-

The kendall's notation is used for representing queing models. Generally queing model may be completely specified in the following symbol form. $(a/b/c):(d/e)$

where,

- a - arrival pattern
- b - Service pattern
- c - No. of channels
- d - capacity of the system
- e - queue discipline.

## Little's Formula :-

$$L_s = \lambda W_s$$

$$L_q = \lambda W_q$$

$$L_q = L_s - \lambda/\mu$$

## M/M/1 : FIFO :-

here first `M` stand for poisson's arrival

the second `M` stands for ~~Depatch~~ Departure

`1` - In the 3rd place stands for a single channel.

FIFO - means in queue Discipline (First In First out)

(i) Equation of study state of the system

(ii) The probability distribution of queue length.

(iii) Avg queue length of the system

(iv) Avg waiting time in the queue are in the system

The Following symbols are used in this models:-

'm' - No. of customers in the waiting line.

'n' - No. of customers in the system.

'w' - Waiting time of a customer in the queue.

'v' - Waiting time of a customers in the system.

'$\lambda$' - Mean arrivale rate that avg. No. of customers arriving per unit time.

'$\mu$' - Mean Service Rate the Avg. No. of Customers receiving service per unit time.

'$\rho$' - capacity utilization (or) traffic intensity.

($P(n>k)$ :-)

probability that there are more than 'k' Customers in the system using the mathematical equation describing the various

Characteristics of queuing problem are arrived at.

E(m) – The Avg No. of Customers in the Waiting Line.

$$\boxed{E(m) = \frac{\lambda^2}{\mu(\mu-\lambda)}}$$

E(n) – The Avg No. of Customers in the system

$$\boxed{E(n) = \frac{\lambda}{\cancel{\mu}(\mu-\lambda)}}$$

E(w) – The Avg waiting time of a customer in the queue.

$$\boxed{E(w) = \frac{\lambda}{\mu(\mu-\lambda)}}$$

E(v) – The Avg waiting time of a customer in the system

$$\boxed{E(v) = \frac{1}{(\mu-\lambda)}}$$

$\rho$ – Capacity utilization,

$$\boxed{\rho = \frac{\lambda}{\mu}}$$

$$P(n \geq k) = \left(\frac{\lambda}{\mu}\right)^{k+1}$$

$$P(n \geq 0) = \left(\frac{\lambda}{\mu}\right)$$

also the probability that there is no customer in the system

$$= 1 - P(n \geq 0)$$

$$= 1 - \frac{\lambda}{\mu}$$

01. A TV repair Name finds that time spend and his Job as an exponential distribution with means 30 min if he repairs sets in the order in which they coming and ~~with~~ if the arrival sets is approximately poission distribution ~~width~~ an Average rate 10 per 8 hours per day.

(i) how many Jobs are Ahead of the set in just brough in.

(ii) What is the repair man expected idle time each day.

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Solution :-

$\lambda = \frac{10}{8}$ for hour.

$\mu = \frac{1}{30} \times 60 = 2$ per hour.

(i) how many job are ahead of set of brought

(i.e) Avg No. of sets in the system.

$$E(n) = \frac{\lambda}{\mu-\lambda} = \frac{10/8}{2-10/8} = \frac{5}{3}.$$

(ii) Time taken to repair one set = 30 min = $\frac{1}{2}$ hr

$\left.\begin{array}{l}\text{No. of sets}\\ \text{arriving per day}\end{array}\right\} = 10.$

$\left.\begin{array}{l}\text{time taken to}\\ \text{repair 10 sets}\end{array}\right\} = 5$ hrs.

No. of working hrs = 8 hr.

Idle time = 8 − 5 = 3 hrs.

02 A one man barber shop customer arrive according to Poisson Distribution with mean arrival rate of 5 per hour and his hair cutting time is exponentially distributed with an average hair cut taking 10 mints. It is assumed that because of his excellence reputation customers are willing to wait. Calculate the following,

i) Avg No. of customer in shop and the Avg no. of customer waiting for a haircut.

(ii) The percentage of customer to have to wait prior to getting into the barbars chair.

Solution :-

$\lambda$ - Mean arrival rate - 5 per hr.

$\mu$ - Mean service rate - $\frac{1}{10} \times 60 = 6$ per hr.

(∵ 10 min)

Avg no. of customer in shop $\Big\}$ $E(n) = \frac{\lambda}{\mu - \lambda} = \frac{5}{6-5} = 5$

Avg no. of customers waiting for haircut $\Big\}$ $= E(m) = \frac{\lambda^2}{\mu(\mu - \lambda)}$

$= \frac{25}{6(6-5)} = \frac{25}{6}$

probability that a customer has to wait

= Probability there is at least one customer in the system

$P(n > 0) = \frac{\lambda}{\mu} = \frac{5}{6}$

percentage of customers to have to wait prior to get in to the barbor chair

$$= 100 \times \frac{5}{6} = 83\frac{1}{3}\%.$$

## Markov chain:-

**Definition:-**

Let $\{x(t)\}$ be a markov process which process markov property and which takes only discrete values. Whether '$t$' is discrete or continuous. then $\{x(t)\}$ called as "markov chain".

$$\text{If } P\{x_n = a_n \mid x_{n-1} = a_{n-1}, x_{n-2} = a_{n-2} \cdots x_0 = a_0\}$$

$$\Rightarrow P\{x_n = a_n \mid x_{n-1} = a_{n-1}\} \text{ for all } n.$$

then the process $\{x_n\}, n = 0,1,2 \ldots$ is called markov Chain.

Prob Q. The initial process of the Markovian Transition probability Matrix is given by

$$P = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.2 & 0.7 \\ 0.6 & 0.3 & 0.1 \end{bmatrix}$$

The initial probabilities,

$$P_1^{(0)} = 0.4,\ P_2^{(0)} = 0.3,\ P_3^{(0)} = 0.3$$

Find,

(i) $P_1^{(1)}$

(ii) $P_2^{(1)}$

(iii) $P_3^{(1)}$

Solu:-

$$P_1^{(1)} = p^{(1)} = p^{(0)} \cdot P$$

transcent probablity matrix (tpm) of a markov chain is,

$$P = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.2 & 0.7 \\ 0.6 & 0.3 & 0.1 \end{bmatrix}$$

$$p^{(0)} = [0.4, 0.3, 0.3]$$

$$P^{(1)} = (0.4, 0.3, 0.3) \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.2 & 0.7 \\ 0.6 & 0.3 & 0.1 \end{bmatrix}$$

$$\boxed{p^{(1)} = 0.29} \qquad \Rightarrow (0.29, 0.27, 0.44)$$

$$\boxed{p^{(2)} = 0.27}$$

$$\boxed{p^{(3)} = 0.44}$$

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