ZITTERBEWEGUNG MOTION:

In Elementary mechanics, We know that the angular momentum of a free particle Should be conserved. But in relativistic Dirac's particle, the angular momentum $L = r \times p$ is not conserved. This is ~~becomes~~ because, the operator L_x (ie component of momentum operator L) do not commute with P_x and P_y. This can only mean that L is not the complete angular momentum There must be another part (ie Spin - angular momentum S) Such that $L+S$ is conserved. The role of "Spin - orbit coupling" in the rectilinear motion of particle creates very rapid oscillations and this Jitterly motion (or) quivering motion (Or) Zitterbewegung. of the free electron was first discovered by E. Schrodinger in 1930.

The fast that the orbital motion of the electron. is coupled to spin through the term $c[\alpha . p]$ in the Dirac's Hamiltonian has peculiar consequences. The use of Heisenberg picture of time evolution facilitates the understanding of these.

According to Heisenberg equation of motion.

$$\left.\begin{aligned} A(t) &= e^{\frac{iHt}{\hbar}} A(0)\, e^{-\frac{iHt}{\hbar}} \\ \frac{dA}{dt} &= \frac{1}{i\hbar}[A, H] \end{aligned}\right\} \rightarrow (1)$$

For example.

$$\frac{dx}{dt} = \frac{1}{i\hbar}[x, H] = c\alpha_x \longrightarrow (2)$$

where $\pm c$ – eigen values of the operator (ie) velocity of the particle (at c)

As for the acceleration.

$$\frac{d^2x}{dt^2} = \frac{d}{dt}(c\alpha_x)$$

$$= \frac{1}{i\hbar}[c\alpha_x, H] \rightarrow (3)$$

$$\frac{d}{dt}(c\alpha_x) = -\frac{2H}{i\hbar} c\alpha_x + \frac{2}{i\hbar} c^2 p_x \rightarrow (4)$$

The above equation can be written as,

$$\frac{d}{dt}\left(e^{-\frac{2iHt}{\hbar}}(c\alpha_x)\right) = e^{-\frac{2iHt}{\hbar}} \frac{2c^2 p_x}{i\hbar} \rightarrow (5)$$

Integrating the above equation. one gets.

$$\int_0^t d\left(e^{-\frac{2iHt}{\hbar}} c\alpha_x\right) = \int_0^t e^{-\frac{2iHt}{\hbar}} \frac{2c^2 p_x}{i\hbar} p_x \, dt$$

$$\left[e^{-\frac{2iHt}{\hbar}} c\alpha_x\right]_0^t = \left(e^{-\frac{2iHt}{\hbar}} - 1\right)\frac{c^2 p_x}{H}$$

then. $e^{-\frac{2iHt}{\hbar}} c\alpha_x(t) - c\alpha_x(0)$

$$= \left(e^{-\frac{2iHt}{\hbar}} - 1\right)\frac{c^2 p_x}{H}$$

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Then. $c\alpha_x(0) = \frac{c^2 P_x}{H} + e^{\frac{-2iHt}{\hbar}} \left[c\alpha_x(t) - \frac{c^2 P_x}{H} \right]$ ⟶ (6)

The first term of equation denotes simply. The classical expression for the velocity of relativistic particle is $\frac{c^2 P}{E}$.

The actual operator $c\alpha_x$ differs from. This through the second term. which oscillates extremely rapidly, the frequency $\frac{2E}{\hbar}$ being > $\frac{2mc^2}{\hbar}$. This jittery motion of the particle due to spin-orbit coupling, is what makes the acceleration formally non-zero and leads to $\pm c$ an eigen values for velocity. But The zitterbewegung is much two fast to be observed, and for all practical purpose, the velocity operator is effectively $\frac{c^2 P_x}{H}$.