BCA and B.Sc Computer Science

Numerical Analysis and Statistics

16SACMA2

Unit. I

1. The Four Numerical Methods are

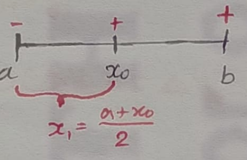
   1. Bisection Method
   2. Interation Method
   3. Regula falsi Method
   4. Newton Raphson Method.

Bisection Method

(i) $x_0 = \frac{a+b}{2}$ x_0 in the midpoint of a and b

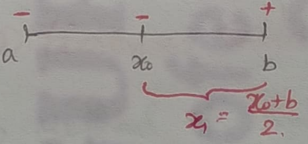
(ii) if $f(x_0) > 0 \Rightarrow f(x_0)$ is positive

There fore The root lies between a and x_0



(iii) if $f(x_0) < 0 \Rightarrow f(x_0)$ is Negative.

There fore The root lies between x_0 and b.



(iv) To find x_1, x_2, x_3 To find Same process.

(v) Continue the process until the root is desired accuracy.

Regula falsi Method.

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

Newton's Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2, 3 \ldots$$

Finite Differences

1. Forward difference operator Δ
2. Backward difference operator ∇
3. Center difference operator.

Forward Differences Table.

x	$y=f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y_0$
x_0	y_0					
		Δy_0				
x_1	y_1		$\Delta^2 y_0$			
x_2	y_2	Δy_1		$\Delta^3 y_0$		
x_3	y_3	Δy_2	$\Delta^2 y_1$		$\Delta^4 y_0$	
x_4	y_4	Δy_3	$\Delta^2 y_2$	$\Delta^3 y_1$		$\Delta^5 y_0$
x_5	y_5	Δy_4	$\Delta^2 y_3$	$\Delta^3 y_2$	$\Delta^4 y_1$	

$$\Delta y_0 = y_1 - y_0$$

$$\Delta^2 y_0 = \Delta(\Delta y_0) = \Delta y_1 - \Delta y_0 = (y_2 - y_1) - (y_1 - y_0)$$
$$= y_2 - 2y_1 - y_0$$

$$\Delta^3 y_0 = y_3 - 3y_2 + 3y_1 - y_0$$

Backward Differences Table

Similiar Method of Forward Table.

$$\nabla y_n = y_n - y_{n-1}$$

Newton's Interpolation Formulae.

$$y_n(x) = y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 +$$
$$+ \cdots + \frac{n(n-1)\cdots(n-(p-1))}{n!}\Delta^n y_0$$

p is last term of n.

where $x = x_0 + nh$ ie; $n = \frac{x - x_0}{h}$

The above formulae is Newton's forward interpolation formulae.

Newton's backward interpolation formulae

$$y_n(x) = y_n = y_n + n\nabla y_n + \frac{n(n+1)}{2!}\nabla^2 y_n + \frac{n(n+1)(n+2)}{3!}\nabla^3 y_n$$

$$+ \cdots + \frac{n(n+1)\cdots(n+(p-1))}{n!}\nabla^n y_n$$

where $x = x_n + nh$

Lagrange's Interpolation formula (unequal Interval).

$$y = f(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} \times y_0$$

$$+ \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)} \times y_1$$

$$+ \cdots$$

$$+ \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})} \times y_n$$

Unit-2

Derivatives using Newton's forward difference formula.

$$y(x) = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \cdots$$

where $n = \frac{x-x_0}{h}$

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \cdots\right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \cdots\right]$$

Newton's Backward difference formula.

$$y(x) = y_n + n\nabla y_n + \frac{n(n+1)}{2!}\nabla^2 y_n + \frac{n(n+1)(n+2)}{3!}\nabla^3 y_n + \cdots$$

where $n = \frac{x-x_n}{h}$

$$\frac{dy}{dx} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \cdots\right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \cdots\right].$$

Stirling's formula

$$y_n = y_0 + n\left[\frac{\Delta y_0 + \Delta y_{-1}}{2}\right] + \frac{n^2}{2!}\Delta^2 y_{-1} + \frac{n(n^2-1^2)}{2!}\left[\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right]$$

$$+ \frac{n^2(n^2-1)}{4!}\Delta^4 y_{-2} + \frac{n(n^2-1)(n^2-2^2)}{5!}\left[\frac{\Delta^5 y_{-2} + \Delta^5 y_{-3}}{2}\right] + \cdots$$

Numerical Integration.

(i) Trapezoidal Rule.

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + \cdots + y_{n-1})\right]$$

(or)

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{h}{2}\left\{\text{(sum of the first and last ordinates)} + 2\text{(sum of the remaining ordinates)}\right\}$$

(ii) Simpson's One third Rule (or) 1/3 Rule.

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{h}{3}\left[(y_0 + y_n) + 2(y_2 + y_4 + \cdots) + 4(y_1 + y_3 + \cdots)\right]$$

(iii) Simpson's Three eight rule (or) 3/8 Rule.

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{3h}{8}\left[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + y_7 + \cdots) + 2(y_3 + y_6 + y_9 + \cdots)\right]$$

Solutions To Linear Systems.

1. Gauss - Elimination Method
2. Gauss - Jordan Method
3. Gauss - Jacobi Method
4. Gauss - Seidal Method.

Gausss - Elimination Method.

Given linear equation

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

Set equation of the form $AX = B$

$$\Rightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\Rightarrow (A,B) = \begin{bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{bmatrix}$$

Using elementary row transformation to lower diagonal elements are Zero.

$$\Rightarrow (A,B) = \begin{bmatrix} C_{11} & C_{12} & C_{13} & d_1 \\ 0 & C_{22} & C_{23} & d_2 \\ 0 & 0 & C_{33} & d_3 \end{bmatrix}$$

$$\Rightarrow C_{11}x_1 + C_{12}x_2 + C_{13}x_3 = d_1$$
$$C_{22}x_2 + C_{23}x_3 = d_2$$
$$C_{33}x_3 = d_3$$

Solve the above equation to find x_1, x_2 and x_3.

Gauss - Jordon Method.

$$(A,B) = \begin{pmatrix} C_{11} & C_{12} & C_{13} & d_1 \\ 0 & C_{22} & C_{23} & d_2 \\ 0 & 0 & C_{33} & d_3 \end{pmatrix}$$

Using elementary row transformation to upper diagonal elements are Zero.

$$(A,B) = \begin{pmatrix} c_1 & 0 & 0 & d_1 \\ 0 & c_2 & 0 & d_2 \\ 0 & 0 & c_3 & d_3 \end{pmatrix}$$

Variable co-efficient only constant other element are zero, Equate the constant to variables to find x_1, x_2 and x_3.

Diagonally dominant.

An $n \times n$ matrix A is said to be diagonally dominant if the absolute value of each leading diagonal element is greater then or equal to the sum of the absolute values of the remaing elements in the row.

Thus the system of equations

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = d_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = d_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = d_3$$

is a diagonal system if

$$|a_{11}| \geq |a_{12}| + |a_{13}|$$
$$|a_{22}| \geq |a_{21}| + |a_{23}|$$
$$|a_{33}| \geq |a_{31}| + |a_{32}|$$

Gauss - Jacobi Method.

Consider the system of equations

$$a_1 x + b_1 y + c_1 z = d_1$$
$$a_2 x + b_2 y + c_2 z = d_2$$
$$a_3 x + b_3 y + c_3 z = d_3$$

Assume $|a_1| > |b_1| + |c_1|$

$|b_2| > |a_2| + |c_2|$

$|c_3| > |a_3| + |b_3|$ To check diagonally dominant.

The given equation can be written as

$$\left.\begin{aligned} x &= \frac{1}{a_1}(d_1 - b_1 y - c_1 z) \\ y &= \frac{1}{b_1}(d_2 - a_2 x - c_2 z) \\ z &= \frac{1}{c_3}(d_3 - a_3 x - b_3 y). \end{aligned}\right\} - (A).$$

if $x^{(0)}, y^{(0)}, z^{(0)}$ are the initial values of x, y, z respectively. then

$$x^{(1)} = \frac{1}{a_1}\left[d_1 - b_1 y^{(0)} - c_1 z^{(0)}\right]$$

$$y^{(1)} = \frac{1}{b_2}\left[d_2 - a_2 x^{(0)} - c_2 z^{(0)}\right]$$

$$z^{(1)} = \frac{1}{c_3}\left[d_3 - a_3 x^{(0)} - b_3 y^{(0)}\right].$$

Again using these values $x^{(1)}, y^{(1)}, z^{(1)}$ in (A)

$$x^{(2)} = \frac{1}{a_1}\left[d_1 - b_1 y^{(1)} - c_1 z^{(1)}\right]$$

$$y^{(2)} = \frac{1}{b_2}\left[d_2 - a_2 x^{(1)} - c_2 z^{(1)}\right]$$

$$z^{(2)} = \frac{1}{c_3}\left[d_3 - a_3 x^{(1)} - b_3 y^{(1)}\right]$$

Proceeding in the same way is continued till the convergence is assured.

Gauss - Seidel Method.

using equation (A)

Start the initial value $y^{(0)}, z^{(0)}$ for y and z and get $x^{(1)}$ from the first equation of (A).

$$\Rightarrow \quad x^{(1)} = \frac{1}{a_1}(d_1 - b_1 y^{(0)} - c_1 z^{(0)}).$$

Second equation $z^{(0)}$ for z and $x^{(1)}$ for x instead of $x^{(0)}$ as in the Jacobi method.

$$y^{(1)} = \frac{1}{b_2}(d_2 - a_2 x^{(1)} - c_2 z^{(0)})$$

$x^{(1)}$ and $y^{(1)}$, use $x^{(1)}$ for x and $y^{(1)}$ for y in the Third equation.

$$z^{(1)} = \frac{1}{c_3}\left(d_3 - a_3 x^{(1)} - b_3 y^{(1)}\right).$$

This process of iteration is continued untill the convergence is assured.

Unit - 3

Solution by Taylor's Series.

$$y(x_1) = y_1 = y_0 + \frac{h}{1!} y_0' + \frac{h^2}{2!} y_0'' + \cdots$$

Euler's formula.

$$y_{n+1} = y_n + h f(x_n, y_n), \quad n = 0, 1, 2, 3 \ldots$$

Runge - Kutta Method.

Runge - Kutta 2^{nd} order

$$y_1 = y_0 + \frac{1}{2}(k_1 + k_2)$$

where $k_1 = h f(x_0, y_0)$ where $\Delta y = \frac{1}{2}(k_1 + k_2)$

$k_2 = h f(x_0 + h, y_0 + k_1)$.

Runge - Kutta 4^{th} order

$$y_1 = y_0 + \Delta y$$

$$\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = h f(x, y)$$

$$k_2 = h f\left(x + \frac{1}{2}h, y + \frac{1}{2}k_1\right)$$

$$k_3 = h f\left(x + \frac{1}{2}h, y + \frac{1}{2}k_2\right)$$

$$k_4 = h f(x + h, y + k_3).$$

Predictor - Corrector Method.

1. Adams - Bash forth Method.

$$y_{n+1,p} = y_n + \frac{h}{24}\left[55y_n' - 59y_{n-1}' + 37y_{n-2}' - 9y_{n-3}'\right]$$

The above formula is Adams-Bashforth Predictor formula.

$$y_{n+1,c} = y_n + \frac{h}{24}\left[9y_{n+1}' + 19y_n' - 5y_{n-1}' + y_{n-2}'\right]$$

The above formula is Adams-Bashforth Corrector formula.

2. Milne's Method.

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}\left[2y_{n-2}' - y_{n-1}' + 2y_n'\right]$$

This is called Milne's Predicor formula.

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}\left[y_{n-1}' + 4y_n' + y_{n+1,p}'\right]$$

This is called Milne's Corrector formula.

Unit-4.

Mean.

Individual Series $\bar{x} = \frac{\Sigma x}{N}$

Discrete Series $\bar{x} = \left[\frac{\Sigma fx}{N}\right]$

where $N = \Sigma f$

Continuous Series $\bar{x} = \frac{\Sigma fx}{N}$

where $N = \Sigma f$

Median

Individual Series

Arrange ascending or decending order

$\left(\frac{N+1}{2}\right)^{th}$ observation is Median. if Totall is odd

$\left[\frac{\left(\frac{N}{2}\right)^{th} + \left(\frac{N}{2}+1\right)^{th}}{2}\right]$ is Median if Totall is even.

Discrete Series

(i) $N = \Sigma f$

(ii) $N/2$.

(iii) To find Cumulative frequency. (c.f)

Continuous Series.

$$\text{Median} = L + \left[\frac{\frac{N}{2} - c.f}{f}\right] \times i, \quad \text{Here } N = \Sigma f$$

The Relation Between Mean, Median and Mode.

Mode = 3 Median − 2 Mean.

Standard Deviation.

Individual Series.

$$\sigma = \sqrt{\frac{\Sigma (x - \bar{x})^2}{N}}$$

$\bar{x}$ is Mean value, N is number of data.

$\Sigma (x - \bar{x})$ is the squared deviation from actual Mean

Discrete Series.

$$\sigma = \sqrt{\frac{\Sigma f (x - \bar{x})^2}{N}}$$

Here, $N = \Sigma f$

Continuous Series.

$$\sigma = \sqrt{\frac{\Sigma f (x - \bar{x})^2}{N}}$$

Here, $N = \Sigma f$

Expected value and Variance

$$\begin{aligned} Var(x) &= E(x^2) - [E(x)]^2 \\ &= P(A) - [P(A)]^2 \\ &= P(A)[1 - P(A)] \end{aligned}$$

Covariance

$Cov(X, Y) = E(XY) - E(X)E(Y)$.

Unit-5

Karl pearson Coefficient of Correlation.

(i) $r(X,Y) = \dfrac{Cov(X,Y)}{\sigma_x \sigma_y}$

(ii) $r = \dfrac{N\Sigma XY - \Sigma X \Sigma Y}{\sqrt{N\Sigma X^2 - (\Sigma X)^2}\sqrt{N\Sigma Y^2 - (\Sigma Y)^2}}$

Properties of Correlation Coefficient.

(i) Correlation Coefficient lies between 1 and -1
ie, $-1 \leq r \leq 1$

(ii) It is independent of change of scale.

(iii) It is independent of Origin.

(iv) If b_{xy} and b_{yx} are two regression coefficient
Then c.c is $\sqrt{b_{xy} \times b_{yx}}$

Regression Line

Lines of x on y is

$$X - \bar{X} = r\frac{\sigma_x}{\sigma_y}(Y - \bar{Y})$$

or $X - \bar{X} = b_{xy}(Y - \bar{Y})$

Here $b_{xy} = r\dfrac{\sigma_x}{\sigma_y}$, where $b_{xy} = \dfrac{N\Sigma XY - \Sigma X \Sigma Y}{N\Sigma Y^2 - (\Sigma Y)^2}$

Lines of y on x is

$$Y - \bar{Y} = r\frac{\sigma_y}{\sigma_x}(X - \bar{X})$$

or $Y - \bar{Y} = b_{yx}(X - \bar{X})$

Here $b_{yx} = r\dfrac{\sigma_y}{\sigma_x}$ where $b_{yx} = \dfrac{N\Sigma XY - \Sigma X \Sigma Y}{N\Sigma X^2 - (\Sigma X)^2}$