

Home Work

50	30	220
90	15	170
250	200	50

Using VAM's Method

$\sum a_i = 1+3+4 = 8$
 $\sum b_j = 1+2+2 = 5$

Step 1

50	30	220
90	15	170
250	200	50

$\{r_1, c\} = R_2$

$t =$

Step 2

50	30	200
90	15	2
250	2	250

$r_{32} = \{2, 2\}$

$= 2$

Step 3

50	1	(50)
3	3	(90) *

t
(90)

$r_{21} = \{1, 3\}$

$= 3$

Step 4

50	1	
----	---	--

Step 5

50	30	220
90	15	170
250	200	50

$m+n = 1+2 = 3$
 $3+3 = 6$
 $6-1 = 5$

$\Rightarrow (1 \times 50) + (3 \times 90) + (2 \times 200) + (2 \times 50) + (250 \times 2)$

$= 2 \rightarrow 0$

$Z = 820$

Using VAM's Method

9	12	9	6	9	10
7	3	7	7	5	5
6	5	9	11	3	11
6	8	11	12	2	10

t 2 2 6 2 2 2

Step 1

$\sum a_i = 5 + 6 + 2 + 7 + 9 = 22$
 $= 22$

$\sum b_j = 4 + 4 + 6 + 2 + 4 + 2 = 22$
 $= 22$

$Z_{ai} = Z_{bj}$

Step 1 + (0x1x) + (00x1x) + (0p1x) + (0x1x) <

9	12	9	6	9	10	5	(3)
7	3	7	7	5	$\frac{2}{5}$	$\frac{6}{5}$	(2)
6	5	9	11	3	11	2	(2)
6	8	11	12	2	10	9	(4)

h h 6^0 2^0 4^0 2^0 5^1 7^1
 (1) (2) (2) (1) (1) (5) 6 7

$$r_{26} = \min \{ 6, \frac{2}{5} \}$$

$$= 2$$

Step 2

9	12	9	6	9
7	3	7	7	5
6	5	9	11	3
6	8	11	12	2

$$r_{26} = r_{26}$$

③ Using VAN's Method

20	28	32	55	70	50
48	36	40	44	25	100
35	55	22	25	48	150
100	70	50	40	40	

Sol:-

$$\sum a_i = 150 + 100 + 50 = 300$$

$$\sum b_j = 100 + 70 + 50 + 40 + 40 = 300$$

$$\sum a_i = \sum b_j$$

Step 1

20	28	32	55	70	50 (8)
48	36	40	44	25	100 - 40 = 60 (11)
35	55	22	25	48	150 (13)

100	70	50	40	40
(15)	(8)	(10)	(1)	(23)

$$x_{25} = \min \{100, 40\}$$

$$= 40$$

Step 2

50 20	28	32	55	50 (8)
48	36	40	44	60 (4)
35	55	22	25	150 (13)

100 - 50 = 50	70	50	40
(15)	(8)	(10)	(1)

$$x_{11} = \{50, 100\}$$

$$= 50$$

Step 3

18	$\frac{50}{36}$	40	44	50 (14)
35	55	22	25	150 (13)

$$50 \quad 70-50 \quad 50 \quad 40$$

$$(12) \quad (9) \quad (8) \quad (1)$$

$$x_{22} = \{60, 70\}$$

$$= 60$$

Step 4

35	$\frac{10}{55}$	22	25	150-10(12)
50	10	50	40	=140

$$(35) \quad (55) \quad (22) \quad (25)$$

$$x_{32} = \min \{150, 110\}$$

$$= 10$$

Step 5

35	22	$\frac{40}{45}$	25	150-10(12)
50	50	50	40	=140

$$(35) \quad 50 \quad (22) \quad 50 \quad (45) \quad 40$$

$$x_{34} = \min \{20, 40\}$$

$$= 20$$

Step 6

$\frac{50}{35}$	22	100-50(13)
50	50	=50

$$(35) \quad (22)$$

$$x_{21} = \min \{100, 50\}$$

$$= 50$$

Step 7

$\frac{50}{22}$	50
50	

$$50$$

Step 8

$\frac{50}{20}$	28	22	55	70	50
18	$\frac{50}{36}$	40	44	$\frac{40}{25}$	100
$\frac{50}{35}$	$\frac{10}{55}$	$\frac{50}{22}$	$\frac{40}{45}$	28	150

$$100 \quad 70 \quad 50 \quad 20 \quad 20$$

$$m+n-1 = 0$$

$$3+5-1 = 0$$

$$8-1 = 7 //$$

$$\rightarrow (50 \times 20) + (60 \times 26) + (20 \times 25) + (80 \times 85) + (10 \times 55) + (50 \times 22) + (20 \times 25)$$

$$\boxed{Z_i = 9,360}$$

Unbalanced LP Method using VAM's Method

② P Q T S

X	95	80	70	60	70
Y	75	65	80	80	20
T	70	25	80	20	90
Z	60	20	20	30	20
	70	50	60	60	

Ans:-

$$Z_i = 70 + 20 + 90 + 20 = 220$$

$$Z_j = 20 + 80 + 60 + 60 = 220$$

$$\boxed{Z_i = Z_j}$$

Step 1

95	80	70	60	20	70 (60) *
75	65	80	80	0	20 (50)
70	25	80	20	0	90 (20)
60	20	20	30	0	20 (20)

70 50 60 60 20
(a) (5) (6) (6)

$z_{ai} = 230, z_{bj} = 230$

$\therefore z_{ai} = z_{bj}$

$x_{15} = \min\{20, 70\}$
 $= 20$

step 2 :-

95	80	70	50 60	50 (10) *
75	65	60	50	40 (10)
70	45	50	40	90 (5)
60	20	70	20	30 (10)
40	50	60	60-50 =10	
(10)	(5)	(6)	(10)	

$x_{12} = \min\{50, 60\}$
 $= 50$

step 3 :-

75	65	60	50	40 (10)
70	45	50	40	90 (10)
60 30	20	70	30	30 (10)
40-30 =10	50	60	10	
(10)	(5)	(6)	(10)	

$x_{21} = \min\{20, 40\}$
 $= 20$

step 4 :-

75	65 30	60	50	40 (10)
70	45	50	40	90-50 =40 (5)
10	50	60	10	
(5)	(20)	(10)	(10)	

$x_{22} = \min\{90, 50\}$
 $= 50$

step 5 :-

75	60	50 10	40 (10)
70	50	40	40-10 =30
10	60	10	
(3)	(10)	(10)	

$x_{24} = \min\{40, 10\}$
 $= 10$

step 6 :-

75	60	40 (5)
70	50 30	30 (20)
10	60-30 =30	
(5)	(10)	

$$x_{22} = \min\{30, 60\}$$

$$= 30$$

step 7:-

$\frac{10}{45}$	60	20	(15)
10	80		
(75)	(60)		

$$x_{21} = \min\{40, 10\}$$

$$= 10$$

step 8:-

$\frac{30}{60}$	20-10
30	

step 9:-

95	80	70	$\frac{10}{60}$	$\frac{10}{60}$
$\frac{10}{45}$	65	$\frac{20}{60}$	80	0 20
70	$\frac{10}{45}$	$\frac{30}{60}$	20	0 90
$\frac{30}{60}$	20	40	80	0 30
10	80	60	60	

$$m+n-1 =$$

$$4+5-1 =$$

$$9-1 = 8 //$$

$$= (80 \times 60) + (20 \times 0) + (10 \times 75) + (20 \times 60) + (50 \times 25) + (30 \times 80)$$

$$+ (10 \times 40) + (30 \times 60)$$

$$Z = 11,500 //$$

8/11/16

Modi MethodTransportation AlgorithmStep 1

Find the initial basic feasible solution by using of the three method discussed above

Step 2

check the number of occupied cells if they are less than $m+n-1$ then degenerate and use very small positive assignment if ≤ 0 so that the number of occupied cells is exactly equal to $m+n-1$.

Step 3

Each occupied cell in the current solution solve the system of equation $u_i + v_j = C_{ij}$ with some $u_i = 0$ (or) $v_j = 0$ the values of u_i and v_j in the T_p table.

Step 4

Compute the net evaluations.

$D_{ij} = U_j + V_j - U_{ij}$ for all unoccupied basic cells and the lower left of the corresponding cells.

Step 5

Each D_{ij} if all $D_{ij} \leq 0$ then the current basic feasible solution. If an obtain one atleast one $D_{ij} > 0$ selected the unoccupied cell having the largest positive net evaluations to enter the basic.

Step 6

The unoccupied cell (i, j) enter the basic allocated and unknown quantity. say identified a loop start and end at the cell (i, j) and end connected sum of the basic cells and end subtract, inter changing. The loop is choose away that the firm requirements satisfied.

Step 7

A minimum value to a increase away that the value of one basic variable becomes 0 and a non basic variable one negative whose allocated sell has been reduced to 0.

Step 8

Return to step 5 and repeat the process until an optimum basic feasible solution has been obtained

① Find the starting solution in the following transportation problem in matrix.

	D ₁	D ₂	D ₃	D ₄	
3	7	6	4	5	5
2	2	3	2	2	2
1	3	8	5	3	3
3	3	2	2		

Ans:-

$\sum a_i = 5 + 2 + 3 = 10$
 $\sum b_j = 3 + 3 + 2 + 2 = 10$

Step 1:-

3	7	6	4	5	(1)
2	2	3	2	2	(2)
1	3	8	5	3	(3)

$x_{23} = \min \{2, 2\} = 2$
 (1) (2) (3) (4)
 *

Step 2:-

3	7	4	5	(1)
4	3	5	3	(2)
3	3	2		

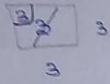
$x_{32} = \min \{3, 3\} = 3$
 *

Step 3:-

3	2	5	(1)
3	2		
(3)	(1)		

$x_{14} = \min \{2, 5\} = 2$
 *

Step 4 :-



3	7	6	4	5	
2	2	3	2	2	
2	3	2	5	5	
5	5	2	2		

$$u_1 = 4$$

$$u_2 = 2$$

$$u_3 = 5$$

$$v_1 = -1 \quad v_2 = 2 \quad v_3 = 1 \quad v_4 = 0$$

$$m+n-1 = 0$$

$$3+4-1 = 7-1$$

$$= 6$$

$$m+n-1 = 4$$

$$3+4-1 = 4$$

$$7-1 = 4$$

$$b = 4$$

Allocated cells :-

$$C_{11} = u_1 + v_3$$

$$C_{12} = u_1 + v_4$$

$$4 = 4 + 0$$

$$u_1 = 4$$

$$C_{23} = u_2 + v_3$$

$$3 = 2 + 1$$

$$u_2 = 2$$

Unallocated cells :-

$$A_{11} = u_1 + v_1 - C_{11}$$

$$A_{12} = u_2 + v_2 - C_{12}$$

$$= 2 - 2 - 7$$

$$A_{12} = -5$$

$$A_{13} = u_1 + v_3 - C_{13}$$

$$= 4 + 1 - 6$$

$$= 4 + 1 - 6$$

$$A_{13} = -1$$

$$C_{24} = u_2 + v_4$$

$$5 = 2 + 3$$

$$u_3 = 5$$

$$C_{25} = u_2 + v_5$$

$$3 = 2 + 1$$

$$v_3 = 2$$

$$C_{32} = u_3 + v_2$$

$$3 = 5 + 2$$

$$v_2 = -2$$

$$C_{14} = v_1 + u_1$$

$$3 = 2 + 1$$

$$v_1 = -1$$

$$A_{21} = u_2 + v_1 - C_{21}$$

$$= 2 - 1 - 2$$

$$A_{21} = -1$$

$$A_{31} = u_3 + v_1 - C_{31}$$

$$= 5 - 1 - 2$$

$$A_{31} = 0$$

$$A_{22} = u_2 + v_2 - C_{22}$$

$$= 2 - 2 - 7$$

$$A_{22} = -4$$

$$A_{33} = u_3 + v_3 - C_{33}$$

$$= 5 + 1 - 8$$

$$A_{33} = -2$$

$A_{ij} < 0$ the current basic feasible solution is a obtain, it is possible to present a more compact form for computing the unknown u_i and v_j than evaluate the each unoccupied cells in a more cumbersome way by working the transportation table.

1	2	3	4
4	3	2	0
0	2	2	1

The optimum solution is $x_{12} = 6$, $x_{22} = 2$,
 $x_{24} = 6$, $x_{31} = 4$, $x_{32} = 2$, $x_{33} = 2$

The transportation cost associated with the optimum is

$$\Rightarrow 3x_3 + 2x_4 + 2x_3 + 2x_4 + 2x_3 + 5x_2$$

$$\Rightarrow x_1 = 0$$

$$\Rightarrow 38 \times 2x_1 + 5x_2$$

$$\Rightarrow 38$$

② Obtain an initial basic feasible

solution to the following problem.

1	2	3	4	6
4	3	2	0	8
0	2	2	1	10
4	6	8	6	

Step 1:-

$$z_{ai} = 6 + 8 + 10 = 24$$

$$z_{bj} = 4 + 6 + 8 + 6 = 24$$

Step 1:-

1	2	3	4	6	(1)
4	3	2	0	8	(2) *
0	2	2	1	10	(1)
4	6	8	6		
(1)	(1)	(1)	(1)		

$$x_{24} = \min \{8, 6\} = 6$$

Step 2

1	2	3	6	(1)
4	3	2	2	(1)
0	2	2	10	(2) *
4	6	8		
(1)	(1)	(1)		

$$x_{31} = \min \{10, 4\} = 4$$

Step 3

2	3	6	(1)
3	2	2	(1) *
2	2	6	(1)
6	8		
(1)	(1)		

$$x_{23} = \min \{8, 2\} = 2$$

Step 1

2	3	6	(1)
$\frac{b}{2}$	$\frac{b}{2}$	6	(0)
6	6		
(0)	(1)		

$x_{33} = \min\{6, 6\}$
 $= 6$

Step 2

$\frac{b}{2}$	6
6	

Step 3

1	$\frac{b}{2}$	3	4
4	3	2	$\frac{b}{2}$
$\frac{b}{2}$	$\frac{b}{2}$	2	1

$u_1 = 0$
 $u_2 = 0$
 $u_3 = 0$

$v_1 = 0, v_2 = 2, v_3 = 2, v_4 = 0$

$m+n-1 =$

$3+4-1 =$

$7-1 = 6 //$

Allocated cells

$C_{ij} = u_i + v_j$

$C_{31} = u_3 + v_1$

$0 = 0 + v_1$

$v_1 = 0$

$C_{32} = u_3 + v_2$

$2 = 0 + v_2$

$v_2 = 2$

$C_{33} = u_3 + v_3$

$2 = 0 + 2$

$v_3 = 2$

$C_{23} = u_2 + v_3$

$2 = 0 + 2$

$v_4 = 0$

$C_{24} = u_2 + v_4$

$2 = 2 + 0$

$u_1 = 0$

$C_{12} = u_1 + v_2$

$2 = 2 + 0$

$u_1 = 0$

Unallocated cells

$D_{ij} = u_i + v_j - C_{ij}$

$D_{11} = u_1 + v_1 - C_{11}$

$= 0 + 0 - 1$

$D_{11} = -1$

$D_{13} = u_1 + v_3 - C_{13}$

$= 0 + 2 - 3$

$D_{13} = -1$

$D_{12} = u_1 + v_2 - C_{12}$

$= 0 + 2 - 4$

$D_{12} = -4$

$D_{21} = u_2 + v_1 - C_{21}$

$= 0 + 0 - 4$

$D_{21} = -4$

$D_{22} = u_2 + v_2 - C_{22}$

$= 0 + 2 - 3$

$D_{22} = -1$

$D_{34} = u_3 + v_4 - C_{34}$

$= 0 + 0 - 1$

$D_{34} = -1$

As ≤ 0 the current basic feasible solution is a optimum, it is possible to present a more compact form for comparison the unknown u_i and v_j than evaluate the each unallocated cells in a more convenient way by working the transportation table.

1	2	3	4
2	3	2	0
0	2	2	1

The obtain solution is $x_{12} = 6, x_{22} = 2$

$x_{24} = 6, x_{21} = 4, x_{32} = 2, x_{33} = 2$

The transportation cost associated with

the optimum is

$$= 6 \times 2 + 2 \times 2 + 6 \times 0 + 4 \times 0 + 2 \times 2 + 2 \times 6$$

$$Z = 0$$

$$= 12 + 4 + 0 + 0 + 4 + 12$$

$$= 28 //$$

① Assignment problem:-

18	26	17	11
13	28	14	26
28	19	18	18
19	26	24	10

How should the tasks be allocated one to a man so as to minimum the total hours.

Sol:-

Step 1

7	15	6	0
0	15	1	13
23	2	3	0
9	16	14	0

Step 2

7	11	5	0
0	11	0	13
23	0	2	0
9	12	13	0

Step 3

2	6	0	0
0	1	0	16
23	0	2	0
9	12	13	0

$$= A \rightarrow G + B \rightarrow E + C \rightarrow F + D \rightarrow H$$

$$= 14 + 18 + 19 + 10$$

$$= 59 //$$

(2)

8	7	6
5	7	8
6	8	7

Using Assignment Problem.

Step 1

2	1	0
0	0	8
0	0	1

Step 2

2	0	8
0	1	8
0	1	1

$$= A \rightarrow 2 + B \rightarrow 1 + C \rightarrow 3$$

$$= 7 + 5 + 7$$

$$= 19 //$$

Step 3

2	0	8
0	1	8
0	0	0

(3)

9	26	15	0
13	27	6	0
35	20	15	0
18	30	20	0

Ans:-

	1	2	3	4
1	0	17	6	0
2	7	21	0	0
3	20	5	0	0
4	0	12	2	0

Step 2

	1	2	3	4	
I	0	12	6	0	1
II	4	16	0	0	1
III	20	0	0	0	1
IV	0	4	2	0	1

$$= 9 + 6 + 20 + 0$$

$$= 35 //$$

20/7/16

①

-	5	2	0
4	4	5	6
5	8	4	3
3	6	6	2

using assignment problem.

Ans:

Step 1

3	5	2	0
0	3	1	2
2	5	1	0
1	4	4	0

Step 2

3	2	1	0
0	0	0	2
2	2	0	0
1	1	3	0

Step 3

3	2	1	0
0	0	0	2
2	2	0	0
0	0	2	0

$$= 1 \rightarrow D + 2 \rightarrow A + 3 \rightarrow C + 1 \rightarrow B$$

$$= 0 + 1 + 4 + 6$$

$$= 14$$

1	2	3	4
5	6	7	8
9	10	11	12

②

9	11	15	10	11
12	9	-	10	9
-	11	14	11	7
14	8	12	4	8
0	0	0	0	0

Using assigned
Problem.

Ans:

0	0	6	1	2
3	0	3	1	0
4	1	4	2	0
4	1	0	0	1
0	0	0	0	0

$$= 1 \rightarrow A + 2 \rightarrow B + 3 \rightarrow C + 4 \rightarrow D + 5 \rightarrow E + 6 \rightarrow F + 7 \rightarrow G + 8 \rightarrow H + 9 \rightarrow I + 10$$

$$= 9 + 9 + 11 + 11 + 0$$

$$= 32$$

Home Work.

Ans: 15

①

1	2	3	4
5	6	-	2
7	4	2	3
9	3	5	-
1	2	6	7

Unit - 1

Linear Programming Problem

A linear programming problem must have an objective which should be clearly identified and measurable in quantitative terms. It can be of profit maximisation, cost minimisation. The relationship among the variables representing objective must be linear.

* The Decision Variables :-

Refer to the activities that are one another for using the resource available all the decision variables are considered as continuous and non-negative operation Research (1940) methodology. County name United Kingdom.

* Definition :- (OR) \rightarrow Operations Research (1940, methodology)

This new approach to systematic and scientific study of the operations

of system was called the operations Research (OR)

Applications of OR (8 Mark)

OR is mainly concerned with the aspects of applying scientific knowledge, besides the development of science. It provides and understanding which gives the manager new insights and capabilities to determine better solutions with greater speed in recent years OR has successfully entered many different areas of research in different Government service organizations and industry. Some applications of OR in the functional areas.

N:1 Finance

N:2 Marketing

N:3 Physical distributions

N:4 explorations

N:5 Personal

N:6 Product

N:7 development

2/2/2010

Linear programming Problem :-

(LPP - Graphical Solutions)

Graphical Solution Method

Step 1 :-

Identified the problem the Decision the variables, the Objective and the restriction.

Step 2 :-

Set up the mathematical formula of the problem.

Step 3 :-

Set up the problem and identified the feasible region. The feasible region is the intersection of all the region. Represented by the constraints the problem and the feasible only.

Step 4 :-

The feasible region obtained in step 3 may be bounded (or) unbounded compute the

Coordinate of all the corner points of the feasible region.

Steps :-

Find out the values of objective functions. Each corner point determined in step 1.

Steps :-

Select the corner point that optimizes the value of the objective function. It gives the optimum feasible solution.

feasible solutions (5 mark) 2005-2006

Any solution to the general LPP which also satisfied the non-negative restrict of the problem is called the feasible solution to the general LPP.

2.6.19 of 2.4.1:-
Optimum Solutions (2m)

Any feasible solution which optimize (minimize) or (maximize) the objective function of a general LPP is called optimum solution to general LPP.

Slack Variables (2m)

Let the constraints of a general LPP $\sum_{j=1}^n a_{ij} x_j \leq b_i ; i=1,2,3,\dots,k$.

Then the non-negative variable x_{n+i} which satisfied $\sum_{j=1}^n a_{ij} x_j + x_{n+i} = b_i ; i=1,2,3,\dots,k$ called -slack variables.

Surplus Variables (2m) (negative slack)

Let the constraint of a general LPP $\sum_{j=1}^n a_{ij} x_j = b_j ; i=k+1, k+2, \dots, k$.

Or then the non-negative variables x_{n+1} which satisfied $\sum_{j=1}^n a_{ij} x_j - x_{n+1} = b_i$ $i=1, 2, \dots, m$ are

called the surplus variables.

The Canonical form (SM)

The general formulation of

LPP in the previous section can always be put in the following form maximize,

$$\text{Max } Z = C_1 x_1 + C_2 x_2 + \dots + C_n x_n$$

$$a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n \leq b_1$$

$$x_1, x_2, \dots, x_n \geq 0$$

where $i = 1, 2, \dots, m$

this form of LPP is called the Canonical form of the characteristics of this form are,

① The objective function of the maximization type, $\text{min } f(x) = -\text{max } \{-f(x)\}$ for example the linear objective function min.

$$z = C_1 x_1 + C_2 x_2 + \dots + C_n x_n \text{ is equivalent}$$

$$\text{to } \text{max } h = -z = -C_1 x_1 - C_2 x_2 - \dots - C_n x_n$$

$$z = -h.$$

② The linear constraint (\leq type).

$$a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n \geq b_1 \text{ is}$$

equivalent to

$$a_{11} x_1 - a_{12} x_2 - \dots - a_{1n} x_n \leq -b_1$$

The eqn may be replaced by two

work in equivalently in opposite direction.

$$a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n = b_1 \text{ is}$$

equivalent to

$$a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n \leq b_1$$

$$a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n \geq b_1$$

③ All the variables also non-negative

a variable which is ~~variable~~ unrestricted in sign

(+ve, -ve, (or) 0) the difference between 2 non-negative variable x_j is unrestricted can be replaced by $(x_j' - x_j'')$ both are non-negative that is $(x_j' - x_j'')$

$$x_j' \geq 0, x_j'' \geq 0$$

The standard form (2m)

① The general LPP in the form maximize and minimize $Z = C_1x_1 + C_2x_2 + \dots + C_nx_n$

subject to,

$$a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n \geq 0$$

$$i = 1, 2, \dots, m$$

is known standard form all the constraints

are in the form of equation for the

non-negative.

② Right hand side each constraint equation is non-negative max (a); min $Z =$

or subj to $A \times b$

$$x \geq 0.$$

① max $Z = x_1 + 2x_2$

subject to:

$$2x_1 + x_2 \leq 1000$$

$$x_1 + x_2 \leq 800$$

$$x_1 \leq 200$$

$$x_2 \leq 700$$

$$x_1 \geq 0$$

$$x_2 \geq 0$$

Handwritten calculations for the constraints:

- $2x_1 + x_2 = 1000$
- $2(400) + x_2 = 1000 \Rightarrow x_2 = 1000 - 800 = 200$
- $2x_1 + x_2 = 1000$
- $x_1 + x_2 = 800 \Rightarrow x_1 = 200$

sol:-

$$2x_1 + x_2 \leq 1000 \quad \text{--- (1)}$$

$$x_1 + x_2 \leq 800 \quad \text{--- (2)}$$

$$2x_1 + x_2 = 1000 \rightarrow \text{--- (1)}$$

$x_1 = 0$ sub in \rightarrow (1)

$$\text{--- (1)} \Rightarrow 2(0) + x_2 = 1000$$

$$\boxed{x_2 = 1000} \quad (0, 1000)$$

$x_2 = 0$ sub in eqn \rightarrow (1)

$$\text{--- (1)} \Rightarrow 2x_1 + 0 = 1000$$

$$2x_1 = 1000$$

$$x_1 = \frac{1000}{2}$$

$$\boxed{x_1 = 500} \quad (500, 0)$$

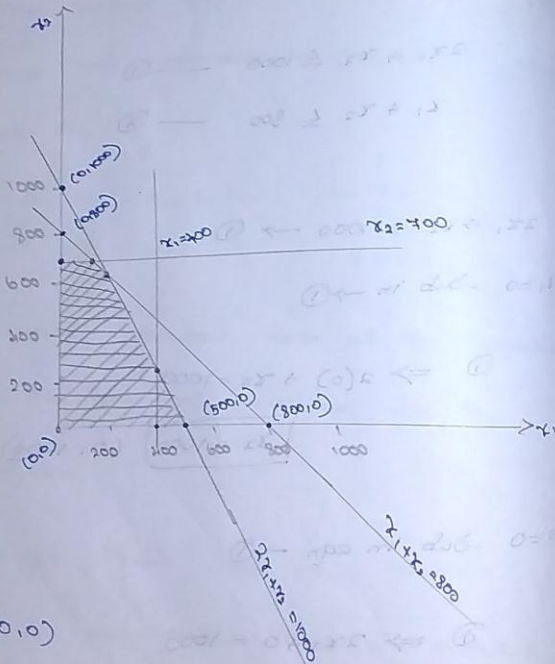
$$x_1 + x_2 = 800$$

$$x_1 = 0 \text{ sub in eqn (2)}$$

$$x_2 = 800 \text{ (0, 800)}$$

$$x_2 = 0 \Rightarrow \text{sub in eqn (2)}$$

$$x_1 = 800 \text{ (800, 0)}$$



$$1) (0,0)$$

$$2) (200,0)$$

$$3) 2x_1 + x_2 = 1000$$

$$800 + x_2 = 1000$$

$$x_2 = 200$$

$$(200, 200)$$

5/2/16

$$2x_1 + x_2 = 1000$$

$$x_1 + x_2 = 800$$

$$\begin{array}{r} \text{c) } \text{c) } \\ \hline \text{c) } \end{array}$$

$$x_1 = 200$$

$$x_1 = 200 \text{ sub in } x_1 + x_2 = 800$$

$$200 + x_2 = 800$$

$$x_2 = 600$$

$$(200, 600)$$

$$b) x_1 + x_2 = 800$$

$$x_2 = 700$$

$$x_1 + 700 = 800$$

$$x_1 = 100$$

$$(100, 700)$$

$$b) (0, 700)$$

$$\text{max } z = 4x_1 + 3x_2$$

$$1) (0,0) \Rightarrow \text{max } z = 4(0) + 3(0) = 0$$

$$2) (200,0) \Rightarrow \text{max } z = 4(200) + 3(0) = \underline{800}$$

$$3) (200, 200) \Rightarrow \text{max } z = 4(200) + 3(200)$$

$$= 1600 + 600$$

$$= \underline{2200}$$

$$\begin{aligned} \text{At } (200, 600) \Rightarrow \max Z &= 4(200) + 3(600) \\ &= 800 + 1800 \\ &= \underline{2600} \end{aligned}$$

$$\text{At } (100, 700) \Rightarrow \max Z = 4(100) + 3(700) = \underline{2500}$$

$$\text{At } (0, 700) \Rightarrow \max Z = 4(0) + 3(700) = \underline{2100}$$

$\therefore \max Z = 2600$
at point $(200, 600)$.

② $\max Z = 5x_1 + 3x_2$

sub to;

$$x_1 + x_2 \leq 6$$

$$2x_1 + 3x_2 \geq 3$$

$0 \leq x_1 \leq 3, 0 \leq x_2 \leq 3$ using Graphical

method.

Sol:-

$$x_1 + x_2 = 6 \quad \text{--- (1)}$$

$$2x_1 + 3x_2 = 3 \quad \text{--- (2)}$$

$$\begin{aligned} x_1 &= 3 - x_2 \quad \text{--- (3)} \\ x_2 &= 3 - 2x_1 \quad \text{--- (4)} \end{aligned}$$

$$x_1 + x_2 = 6$$

$$x_1 \geq 0 \Rightarrow x_2 = 6 \quad (0, 6)$$

$$x_2 = 0 \Rightarrow x_1 = 6 \quad (6, 0)$$

$$2x_1 + 3x_2 = 3$$

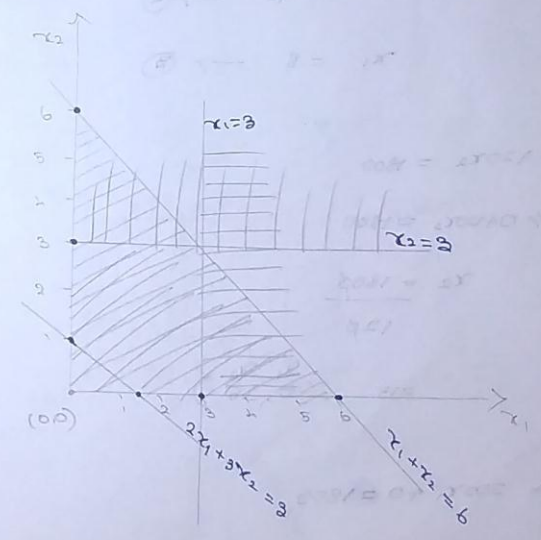
$$x_1 = 0 \Rightarrow 3x_2 = 3$$

$$x_2 = 1 \quad (0, 1)$$

$$x_2 = 0 \Rightarrow 2x_1 = 3$$

$$x_1 = 3/2$$

$$x_1 = 1.5 \quad (1.5, 0)$$



Ans:- That is infeasible solution.

$$\textcircled{2} \text{ max } z = 8 \times 50x_1 + 80 \times 15x_2$$

sub to :-

$$8 \times 25x_1 + 8 \times 15x_2 \geq 1800$$

$$5x_1 + 3x_2 \geq 45$$

$$x_1 \leq 8$$

$$x_2 \leq 0, x_1 \geq 0,$$

using Graphical method.

sol:-

$$8 \times 25x_1 + 8 \times 15x_2 = 1800$$

$$200x_1 + 120x_2 = 1800 \rightarrow \textcircled{1}$$

$$5x_1 + 3x_2 = 45 \rightarrow \textcircled{2}$$

$$x_1 = 8 \rightarrow \textcircled{3}$$

$$200x_1 + 120x_2 = 1800$$

$$x_2 = 0 \Rightarrow 0 + 120x_2 = 1800$$

$$x_2 = \frac{1800}{120}$$

$$= 15 \quad (0, 15)$$

$$x_2 = 0 \Rightarrow 200x_1 + 0 = 1800$$

$$x_1 = \frac{1800}{200}$$

$$= 9$$

$$(9, 0)$$

$$5x_1 + 3x_2 = 45$$

$$x_1 = 0 \Rightarrow 0 + 3x_2 = 45$$

$$x_2 = \frac{45}{3}$$

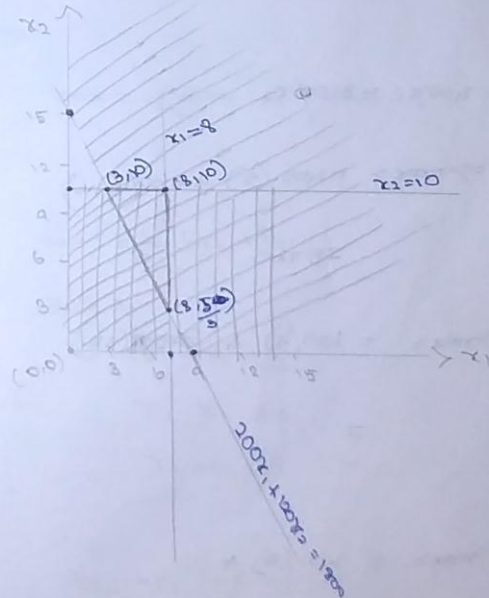
$$= 15 \quad (0, 15)$$

$$x_2 = 0 \Rightarrow 5x_1 + 0 = 45$$

$$x_1 = \frac{45}{5}$$

$$= 9 \quad (9, 0)$$

$$x_1 = 8 \quad (8, 0)$$



$$5x_1 + 3x_2 = 15$$

$$x_1 = 3$$

$$5(3) + 3x_2 = 15$$

$$3x_2 = 15 - 15$$

$$3x_2 = 0$$

$$x_2 = 0 \quad (3, 0) \quad (8, 1.6)$$

$$\rightarrow (8, 1.6)$$

$$\rightarrow 5x_1 + 3x_2 = 15$$

$$5x_1 + 3x_2 = 15$$

$$x_2 = 0$$

$$5x_1 = 15$$

$$x_1 = 15/5 = 3 \quad (3, 0)$$

$$\max z = 400x_1 + 360x_2$$

$$\rightarrow (8, 1.6) \Rightarrow \max z = 400(8) + 360(1.6)$$

$$= 5760$$

$$\rightarrow (3, 0) \Rightarrow \max z = 400(3) + 360(0)$$

=

$$\rightarrow (3, 0) \Rightarrow \max z = 400(3) +$$

$$\max z = 400x_1 + 360x_2$$

$$\rightarrow (8, 5/3) \Rightarrow \max z = 400(8) + 360(5/3)$$

$$= 3200 + 360 \times 5/3$$

$$= 3800$$

$$\rightarrow (8, 1.6) \Rightarrow \max z = 400(8) + 360(1.6)$$

$$= 5760$$

$$\rightarrow (3, 0) \Rightarrow \max z = 400(3) + 360(0)$$

$$= 1200$$

Ans: -

$$\max z = 5760 \text{ at point } (8, 1.6)$$

$$\textcircled{5} \text{ mini } z = 6x_1 + x_2$$

Sub to

$$2x_1 + x_2 \geq 3$$

$$x_2 - x_1 \geq 0$$

$$x_1, x_2 \geq 0$$

Ans: -

$$2x_1 + x_2 = 3 \Rightarrow \textcircled{1}$$

$$x_2 - x_1 = 0 \Rightarrow \textcircled{2}$$

$$2x_1 + x_2 = 3$$

$$x_1 = 0 \Rightarrow x_2 = 3 \quad (0, 3)$$

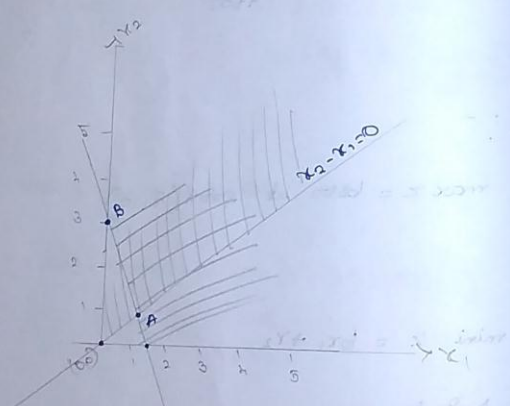
$$x_2 = 0 \Rightarrow 2x_1 = 3$$

$$x_1 = \frac{3}{2} = 1.5 \quad (1.5, 0)$$

$$x_2 - x_1 = 0$$

$$x_1 = 0 \Rightarrow x_2 = 0 \quad (0, 0)$$

$$x_2 = 0 \Rightarrow x_1 = 0 \quad (0, 0)$$



$$A = (1, 1)$$

$$B = (0, 3)$$

$$\text{min } z = 6(1) + (1) = 7$$

$$\text{min } z = 0 + 3 = 3$$

Ans:- That is infeasible solution.

4) solve Graphical method the following
 LP max $z = 3x_1 + 2x_2$
 Sub to :-
 $-2x_1 + x_2 \leq 1 ; x_1 \leq 2$
 $x_1 + x_2 \leq 3 ; x_1, x_2 \geq 0$

Ans:-

$$\textcircled{1} \rightarrow -2x_1 + x_2 = 1$$

$$x_1 + x_2 = 3$$

$$x_1 = 2$$

$$\textcircled{1} \Rightarrow -2x_1 + x_2 = 1$$

$$\underline{x_1 = 0} \Rightarrow x_2 = 1 \quad (0, 1)$$

$$\underline{x_2 = 0} \Rightarrow -2x_1 = 1$$

$$x_1 = -\frac{1}{2} = -0.5 \quad (-0.5, 0)$$

$$\textcircled{2} \Rightarrow x_1 + x_2 = 3$$

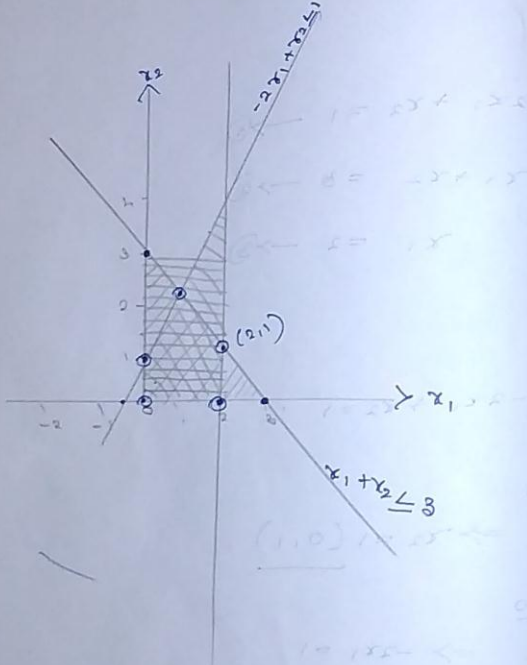
$$\underline{x_1 = 0} \Rightarrow x_2 = 3 \quad (0, 3)$$

$$\underline{x_2 = 0}$$

$$\Rightarrow x_1 = 3 \quad (3, 0)$$

② \Rightarrow

$$x_1 = 2 \quad (2, 0)$$



$$1) (0, 0)$$

$$2) (2, 0)$$

$$3) x_1 + x_2 = 3$$

$$x_1 = 2$$

$$4) 2x_1 + x_2 = 3$$

$$x_2 = 1$$

$$(2, 1)$$

$$4) -2x_1 + x_2 = 1$$

$$\begin{array}{r} x_1 + x_2 = 3 \\ (-) \quad (-) \quad (-) \\ \hline -3x_1 = -4 \end{array}$$

$$+3x_1 = +4$$

$$x_1 = \frac{4}{3}$$

$$\frac{4}{3} + x_2 = 3$$

$$x_2 = 3 - \frac{4}{3}$$

$$x_2 = \frac{9}{3} - \frac{4}{3} = \frac{5}{3}$$

$$\left(\frac{4}{3}, \frac{5}{3} \right)$$

$$5) (0, 1)$$

9/2/15

The approximation mathematical formulation of the given LPP is.

$$\min Z = 2x_1 + 4x_2$$

Sub 2b

$$3x_1 + 6x_2 \geq 108$$

$$3x_1 + 12x_2 \geq 36$$

$$20x_1 + 10x_2 \geq 100 \text{ and } x_1 \geq 0, x_2 \geq 0$$

$$3x_1 + 6x_2 = 108 \rightarrow \textcircled{1}$$

$$3x_1 + 12x_2 = 36 \rightarrow \textcircled{2}$$

$$20x_1 + 10x_2 = 100 \rightarrow \textcircled{3}$$

$$3x_1 + 6x_2 = 108$$

$$\textcircled{1} \Rightarrow x_1 = 0$$

$$36(0) + 6(x_2) = 108$$

$$6x_2 = 108$$

$$x_2 = \frac{108}{6}$$

$$x_2 = 18$$

(0, 18)

$x_2 = 0$

$$3x_1 + 6(0) = 108$$

$$3x_1 = 108$$

$$x_1 = \frac{108}{3} = 36 \quad (36, 0)$$

$$3x_1 + 2x_2 = 36$$

$$\textcircled{2} \Rightarrow x_1 = 0$$

$$3(0) + 2x_2 = 36$$

$$2x_2 = 36$$

$$x_2 = \frac{36}{2}$$

$$x_2 = 18 \quad (0, 18)$$

$x_2 = 0$

$$3x_1 + 12(0) = 36$$

$$3x_1 = 36$$

$$x_1 = \frac{36}{3} = 12 \quad (12, 0)$$

$$20x_1 + 10x_2 = 100$$

$$\textcircled{3} \Rightarrow x_1 = 0$$

$$20(0) + 10x_2 = 100$$

$$10x_2 = 100$$

$$x_2 = \frac{100}{10}$$

$$x_2 = 10 \quad (0, 10)$$

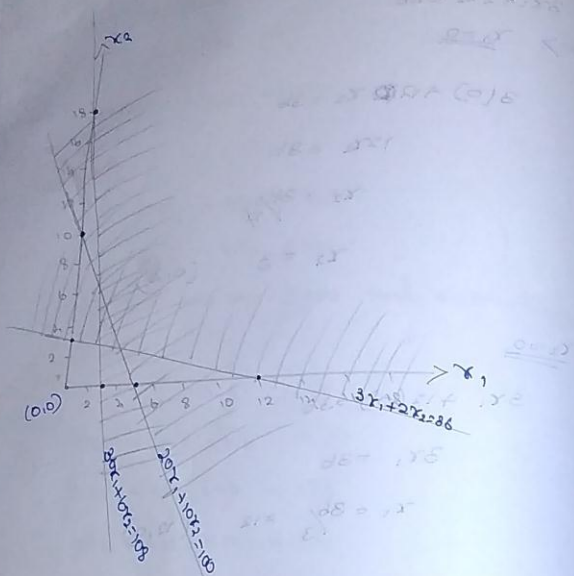
$x_2 = 0$

$$20x_1 + 10(0) = 100$$

$$20x_1 = 100$$

$$x_1 = \frac{100}{20}$$

$$x_1 = 5 \quad (5, 0)$$



That is unfeasible solution.

⑧ Max $Z = 2x_1 + 4x_2$

sub to:-

$$x_1 + 2x_2 \leq 5$$

$$x_1 + x_2 \leq 4 \text{ and } x_1, x_2 \geq 0$$

$$\textcircled{1} \rightarrow x_1 + 2x_2 = 5$$

$$\textcircled{2} \rightarrow x_1 + x_2 = 4$$

$$x_1 + 2x_2 = 5$$

$$x_1 = 0$$

$$0 + 2x_2 = 5$$

$$x_2 = 5/2$$

$$x_2 = 2.5 \quad (0, 2.5)$$

$$x_2 = 0$$

$$x_1 + 2(0) = 5$$

$$x_1 = 5 \quad (5, 0)$$

$$x_1 + x_2 = 4$$

$$x_1 = 0$$

$$0 + x_2 = 4$$

$$x_2 = 4 \quad (0, 4)$$

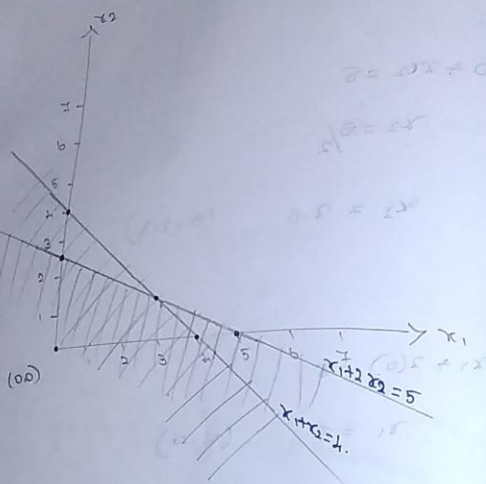
$$x_2 = 0$$

$$x_1 + 0 = 4$$

$$x_1 = 4 \quad (4, 0)$$

$$(4, 0)$$

$$(0, 4)$$



i) (0,0)

ii) (4,0)

iii) $x_1 + 2x_2 = 5$

$x_1 + x_2 = 4$

$x_1 = 4 - x_2$

$x_2 = 1$

$x_1 + x_2 = 4$

$x_1 = 4 - 1$

$= 3$

(3,0)

iv) (0, 2.5)

max $Z = 2x_1 + 4x_2$

(0,0)

$= 2(0) + 4(0)$

$= 0$

ii) $\Rightarrow (4,0)$

$= 2(4) + 4(0)$

$= 8$

iii) $\Rightarrow (3,0)$

$= 2(3) + 4(0)$

$= 6$

iv) $\Rightarrow (0, 2.5)$

$= 2(0) + 4(2.5)$

$= 10$

(0, 2.5)

Home Work

① max $Z = 10x_1 + 6x_2$

Sub to :-

$5x_1 + 3x_2 \leq 30$

$x_1 + 2x_2 \leq 18$ and $x_1, x_2 \geq 0$

Sol:-

$5x_1 + 3x_2 = 30$ — (1)

$x_1 + 2x_2 = 18$ — (2)

$5x_1 + 3x_2 = 30$

$x_1 = 0$

$0 + 3x_2 = 30$

$x_2 = \frac{30}{3}$

$x_2 = 10$

$(0, 10)$

$x_2 = 0$

$5x_1 + 0 = 30$

$5x_1 = 30$

$x_1 = \frac{30}{5}$

$x_1 = 6$

$(6, 0)$

$x_1 + 2x_2 = 18$

$x_1 = 0$

$0 + 2x_2 = 18$

$x_2 = \frac{18}{2}$

$x_2 = 9$

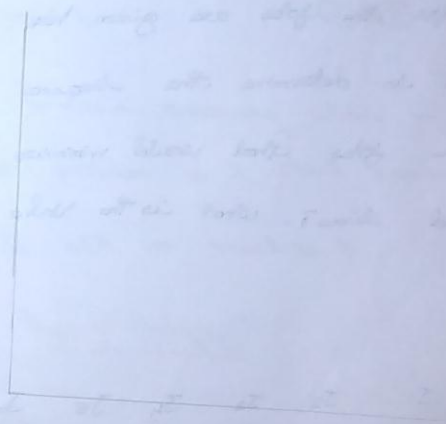
$(0, 9)$

$x_2 = 0$

$x_1 + 0 = 18$

$x_1 = 18$

$(18, 0)$



10/2/16

Unit IV

Sequencing Problem

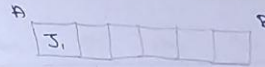
Processing (n) jobs through 2 machines

Problem: 1

There are six jobs to perform each of which go through two machines A and B in the order A, B. The processing (in hours) for the jobs are given here. You are required to determine the sequence for performing the jobs that would minimize the total elapsed time. What is the value of T?

Job :	J ₁	J ₂	J ₃	J ₄	J ₅	J ₆
Machine A :	1	3	8	5	6	3
Machine B :	8	6	3	2	2	10

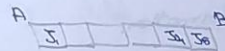
The smallest processing time in the given problem is 1 on machine A.



The reduced set of processing times

Job :	J ₂	J ₃	J ₄	J ₅	J ₆
mach A :	3	8	5	6	3
mach B :	6	3	2	2	10

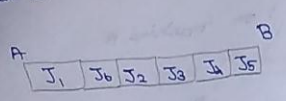
The minimum processing time in this reduced problem is 2 which corresponds to J₄ and J₅ both on machine B.



Job :	J ₂	J ₃	J ₆
mach A :	3	8	3
mach B :	6	3	10

The minimum processing time in this reduced problem is 3 which corresponds to

J_1 and J_6 on machine A. and J_6 on machine B.



Jobs	Machine A		Machine B		idle time on B
	in	Out	in	Out	
J_1	0	1	1	6	-
J_6	1	2	6	16	-
J_2	2	7	16	22	-
J_3	4	15	22	25	-
J_4	15	20	25	27	-
J_5	20	26	27	29	-

From the above information,

we get = 29 hours.

Idle time of machine A = $(29 - 26) = 3$ hrs.

idle time on B = 1 hr. //

② Book

Book	b_1	b_2	b_3	b_4	b_5	b_6
Printing time (hrs)	30	120	80	20	90	100
Binding time (hrs)	80	100	90	60	30	70

Job: -

The minimum elapsed time from the start of the first book to the completion of the last corresponding the optimal sequence is computed as shown in the following table :-

not opt = without printing
not opt = without printing

not opt = without printing
not opt = without printing

Book	Printing Machine		Binding machine		Idle time of binding machine
	Time in	Time Out	Time in	Time out	
b ₁	0	20	20	80	20
b ₂	20	80	80	160	-
b ₃	80	160	160	280	-
b ₄	160	220	280	350	-
b ₅	220	310	350	380	-
b ₆	310	410	380	390	-

Idle time of Printing machine is

$$410 - 390 = 20 \text{ hrs.}$$

$$T = 410 \text{ hrs}$$

$$\text{Binding machine} = 390 \text{ hrs}$$

$$\text{Printing machine} = 410 \text{ hrs} //$$

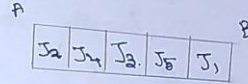
$$\left. \begin{array}{l} \text{Idle time of binding} \\ \text{machine} \end{array} \right\} = 20 \text{ hrs.}$$

Job : J₁ J₂ J₃ J₄ J₅

Machine A : 5 1 9 3 10

Machine B : 2 6 7 8 4

Seq. :-



Job	Machine A		Machine B		Idle time of Job.
	Time in	Time Out	Time in	Time Out	
J ₂	0	1	1	7	1
J ₄	1	4	7	15	-
J ₃	4	13	15	22	-
J ₅	7	23	22	26	-
J ₁	10	28	21	28	-

Total hours = 28 hrs

machine B = 28 hrs

Idle time = 28 - 28
= 0

Processing n jobs through k machines:-

① Determine the optimal sequence of jobs that minimizes the total elapsed time based on the following information: processing time on machines is given in hours and preemption is not allowed.

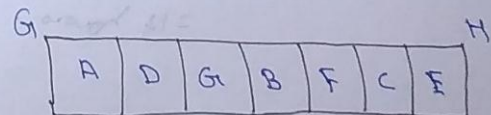
job :	A	B	C	D	E	F	G
machine M_1 :	3	8	7	4	9	8	4
" M_2 :	4	3	2	5	1	4	3
" M_3 :	6	4	8	11	5	6	12

Ans:-

$$G_i = M_1 + M_2 = A_i + B_i$$

$$H_i = M_2 + M_3 = B_i + C_i$$

job :	A	B	C	D	E	F	G
G_i :	7	11	9	9	10	12	10
H_i :	10	10	7	16	6	10	15



Jobs	Machine m ₁		M ₂		M ₃	
	In	Out	In	Out	In	Out
A	0	04	03	4	4	4
D	0	4	4	4	4	0
G	4	5	4	22	25	0
B	4	8	22	30	34	0
F	8	30	30	34	44	0
C	8	24	34	36	38	0
E	34	46	44	47	55	60

Total Time = 60 hours

Idle time of machine 3 : 7 hours

" " " " : 60 - 47 = 13 hours

" " " " : 60 - 46 = 14 hours

= 14 hours

7	2	7	5	2	1	1
---	---	---	---	---	---	---

2) job : 1 2 3 4 5 6
 m.A : 2 10 6 3 10 12
 m.B : 9 4 8 4 2 2
 m.C : 14 6 10 13 9 14

Find Total Time,

idle time of m.A, m.B, m.C

n job 2 machine

① Job: J₁ J₂ J₃ J₄ J₅ J₆ J₇ J₈

m.A:	8	6	10	6	12	1	3	9
m.B:	4	4	3	9	11	6	4	2

Find the total elapsed time, idle time of machine A & B.

Sol:-

3

J₆ J₄ J₅ J₁ J₂ J₇ J₃ J₈

Job	m.A		m.B		Idle time of m.B
	In	Out	In	Out	
J ₆	0	1	1	4	1
J ₄	1	7	4	10	0
J ₅	7	19	19	30	3
J ₁	19	27	30	37	0
J ₂	27	37	37	41	0
J ₇	37	42	42	46	1
J ₃	42	52	52	55	6
J ₈	52	61	61	63	6
					17

Idle time of machine A

$$= 0 + (1-1) + (7-7) + (19-19) + (27-27) + (37-37) + (42-42) + (52-52) + (63-61)$$

$$= 2 \text{ hrs.}$$

Total elapsed time T = 63 hrs.

Idle time of machine B is = 17 hrs.

n job through 3 machine

② Job: 1 2 3 4 5 6

m(A):	4	13	6	3	10	12
m(B):	9	7	3	7	2	2
m(C):	14	15	10	13	9	14

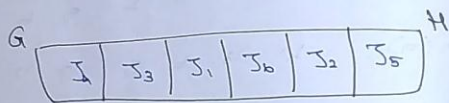
Find total elapsed time & idle time of m(A), m(B), m(C)

Sol:-

$$G_i = t_{ij} + t_{2j}$$

$$H_i = t_{2j} + t_{3j}$$

Job	$G_i = B_i + B_i$	$H_i = B_i + G_i$
		23
J ₁	13	22
J ₂	20	15
J ₃	11	20
J ₄	10	18
J ₅	14	16
J ₆	12	



Job	m(A)		m(B)		m(C)		Idle time of m(C)
	In	Out	In	Out	In	Out	
J ₄	0	3	3	10	10	23	10
J ₅	3	9	10	15	23	33	0
J ₁	9	13	15	24	33	47	0
J ₆	13	25	25	27	47	61	0
J ₂	25	38	38	45	61	76	0
J ₃	37	48	48	52	76	85	0
							10 hrs

Idle time = 10 hrs.

Idle time = 10 hrs.

Total elapsed time = 85 hrs.

Idle time of m(A) = 10 hrs

$$= 10 + (23 - 23) + (33 - 33) + (47 - 47) + (61 - 61) + (76 - 76)$$

Idle time of m(B) = 3 + (10 - 10) + (15 - 15) +

$$(25 - 25) + (38 - 38) + (48 - 48) + (85 - 85)$$

$$= 51 \text{ hrs.}$$

9/8/16

Processing any job is through m machines

Job :	1	2	3	4
m(A) :	9	8	7	10
m(B) :	7	8	6	5
m(C) :	4	6	4	5
m(D) :	8	7	8	4
m(E) :	12	10	8	

Sol:-

mini $B_i = 7$

maxi $B_i = 8$

maxi $C_i = 7$

maxi $D_i = 8$

maxi $E_i \geq (\text{maxi } B_i \text{ and } D_i)$

$12 \geq (8, 8)$

We can solve this problem consider the following table jobs.

Job	$x = A_i + B_i + C_i + D_i$	$y = B_i + C_i + D_i + E_i$
1	28	27
2	29	33
3	28	31
4	27	22

$x = [1 \ 3 \ 2 \ 4]$

Job	m(A)		m(B)		m(C)		m(D)		m(E)		[Objective of m(E)]
	1	0	1	0	1	0	1	0	1	0	
1	0	9	9	16	16	20	20	25	25	36	
2	9	16	16	22	22	29	29	37	37	47	
3	16	24	24	32	32	38	38	45	47	59	
4	24	32	32	39	39	44	45	49	59	67	

Total elapsed time = 67 hrs.

Idle time of m(E) = 26 hrs.

$$m(D) = 20 + 1 + 1 + 0 + (67 - 42) = 43 \text{ hrs.}$$

$$m(C) = 16 + 2 + 3 + 1 + (67 - 42) = 45 \text{ hrs.}$$

$$m(B) = 41 \text{ hrs.}$$

$$m(A) = 33 \text{ hrs.}$$

②

Job : 1 2 3 4

m(A) : 58 20 28 64

m(B) : 12 10 12 16

m(C) : 12 18 16 12

m(D) : 28 32 42 22

35:-

$$\text{mini } A_i = 28$$

$$\text{maxi } B_i = 16$$

$$\text{maxi } C_i = 18$$

$$\text{maxi } D_i = 28$$

~~maxi~~ ~~fit~~

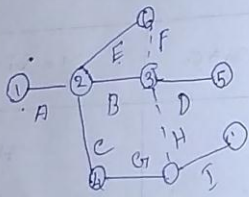
We can solve this problem Consider the following table jobs.

Job	$x = A_i + B_i + C_i$	$y = B_i + C_i + D_i$
1	80	46
2	58	60
3	56	32
4	92	40

x y

3	2	1	4
---	---	---	---

Job	m(A)		m(B)		m(C)		m(D)		Idle time of m(D)
	I	O	I	O	I	O	I	O	



11/3/16

UNIT - II

CPM - Critical path Method.

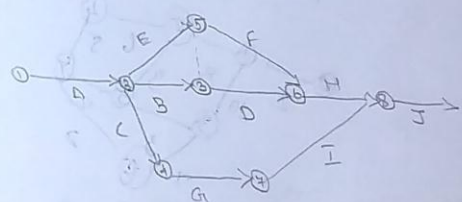
① Draw a Network diagram for the following data.

Activity : A B C D E F

Preceding activities : A A B A B E

G H I J
C D F G H I

Ans:-

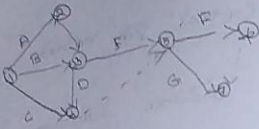


② Draw a Network diagram for the following data:-

Activity : A B C D E F G

Preceding activity : - - - AB A/B C/D E C,D,E

1 1 1 1 1



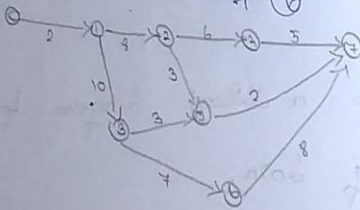
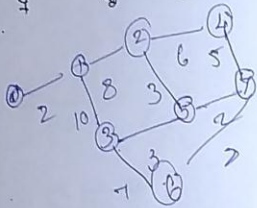
8) Draw a network diagram for the following data:

Activity : 0-1 1-2 1-3 2-4 2-5 3-5

days : 2 8 10 6 3 3

3-6 4-7 5-7 6-7

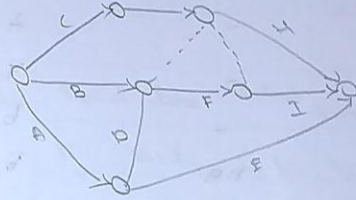
4 5 2 8



9) Activity : A B C D E F G H I

P.D : - - - A A B D C B G F G

Sol: -

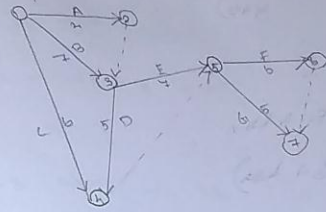


Small project of 7 activities for which data are given.

Activity	Preceding Activity	Activity duration
A	-	4
B	-	7
C	-	6
D	AB	5
E	AB	4
F	C, D, E	6
G	C, D, E	5

i) Draw the network and find the project completion time.

ii) Calculate total float for each of the activities and highlight the critical path.



Activity (i,j)	duration tij	Earliest time		Latest time		Total float
		Ei	Ei+tij	Lj-tij	Lj	
1-2	4	0	4	3	4	3
1-3	7	0	7	0	7	0
1-4	6	0	6	3	9	3
3-5	5	7	12	9	14	2
3-6	4	7	11	4	11	0
5-6	6	14	20	14	20	0
5-7	5	14	19	16	20	1

$L_j - (E_i + t_{ij})$

Forward calculation.

$$E_1 = 0$$

$$E_2 = E_1 + t_{12} = 0 + 4 = 4$$

$$E_3 = \max(E_1 + t_{13}, E_2 + t_{23})$$

$$= \max(0+7, 4+0)$$

$$= 7$$

$$E_4 = \max(E_1 + t_{14}, E_3 + t_{34})$$

$$\max(0+6, 7+0) = 7$$

$$E_5 = \max(E_3 + t_{35}, E_4 + t_{45})$$

$$\max(7+7, 12+0) = 14$$

$$E_6 = E_5 + t_{56} = 14 + 5 = 19$$

$$E_7 = \max(E_5 + t_{57}, E_6 + t_{67})$$

$$= \max(14+6, 19+0) = 20$$

Backward calculations:-

$$L_7 = 20$$

$$L_6 = L_7 - t_{67}$$

$$= 20 - 0 = 20$$

$$L_5 = \min(L_4 - t_{54}, L_6 - t_{56})$$

$$= \min(20 - 6, 20 - 5)$$

$$= \min(14, 15)$$

$$L_5 = 14$$

$$L_4 = L_5 - t_{45} = 14 - 0$$

$$= 14$$

$$L_3 = \min(L_4 - t_{34}, L_5 - t_{35})$$

$$= \min(14 - 7, 14 - 7)$$

$$= \min(7, 7)$$

$$L_3 = 7$$

$$L_2 = L_3 - t_{23}$$

$$= 7 - 0$$

$$= 7$$

$$L_1 = \min(L_2 - t_{12}, L_3 - t_{13}, L_4 - t_{14})$$

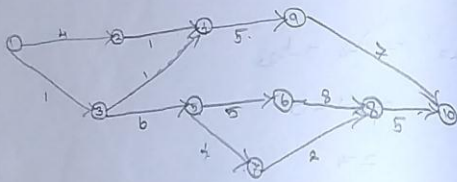
$$= \min(7 - 4, 7 - 7, 14 - 6)$$

$$= 0$$

∴ The Critical path = 1-3-5-6-7

Activity	1-2	1-3	2-4	3-4	3-6	4-9
time	4	1	1	1	6	8
	5-6	6-7	6-8	7-8	8-10	7-10
	5	4	8	2	5	4

- i) Draw a network
 ii) Find the CP.



Forward Calculations:-

Let $E_1 = 0$

$E_2 = E_1 + t_{12}$
 $= 0 + 4$

$E_2 = 4$

$E_3 = E_1 + t_{13}$
 $= 0 + 1$

$E_3 = E_1 + t_{13}$
 $= 0 + 1$

$E_3 = 1$

$E_4 = \max(E_2 + t_{24}, E_3 + t_{34})$
 $= \max(4+1, 1+1)$

$E_4 = 5$

$E_5 = E_3 + t_{35}$
 $= 1 + 6$

$= 7$

$E_6 = E_5 + t_{56}$
 $= 7 + 5$

$= 12$

$E_7 = E_5 + t_{57}$
 $= 7 + 4$

$= 11$

$E_8 = \max(E_6 + t_{68}, E_7 + t_{78})$

$= \max(12+8, 11+2)$

$E_8 = 20$

$E_9 = E_4 + t_{49}$

$= 5 + 5$

$= 10$

$E_{10} = \max(E_8 + t_{810}, E_9 + t_{910})$

$= \max(20+5, 10+4)$

$E_{10} = 25$

Backward calculations :-

$$L_{10} = E_{10} = 25$$

$$L_9 = L_{10} - t_{9,10} \\ = 25 - 7 \\ = 18$$

$$L_8 = L_{10} - t_{8,10} \\ = 25 - 5 \\ = 20$$

$$L_7 = L_8 - t_{7,8} \\ = 20 - 2 \\ = 18$$

$$L_6 = L_8 - t_{6,8} \\ = 20 - 8 \\ = 12$$

$$L_5 = \min(L_6 - t_{5,6}, L_7 - t_{5,7}) \\ = \min(12 - 6, 18 - 4) \\ = 6$$

$$L_4 = L_5 - t_{4,5} \\ = 6 - 5 \\ = 1$$

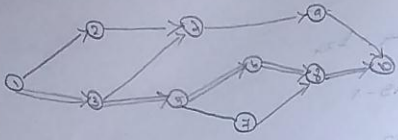
$$L_3 = \min(L_4 - t_{3,4}, L_5 - t_{3,5}) \\ = \min(1 - 1, 6 - 0) \\ = 1$$

$$L_2 = L_4 - t_{2,4} \\ = 1 - 1 \\ = 0$$

$$L_1 = \min(L_2 - t_{1,2}, L_3 - t_{1,3}) \\ = \min(0 - 1, 1 - 1) \\ = 0$$

Activities (i,j)	duration t _{ij}	Earliest Time		Latest Time		Total float L _j - E _i + t _{ij}
		E _i	E _i +t _{ij}	L _i -t _{ij}	L _i	
1-2	1	0	1	0	0	0
1-3	1	0	1	0	0	0
2-4	1	1	2	1	1	0
3-4	1	1	2	1	1	0
3-5	6	1	7	1	7	0
4-9	5	5	10	5	10	0
5-6	6	4	10	4	10	0
5-7	1	4	5	4	5	0
6-8	8	12	20	12	20	0
7-8	2	11	13	11	13	0
8-10	5	20	25	20	25	0
7-10	7	10	17	10	17	0

The CP is 1-3, 3-8, 5-6, 6-8, 8-10



Home Work

① Activity : 0-1 1-2 1-3 2-4 2-5 3-5 3-6 4-7 5-7 6-8

Time : 2 8 10 6 3 3 7 4 5 2 8

A < D, E

B, D < F

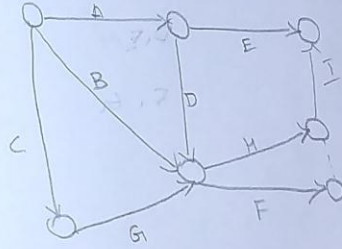
C < G

B, G < H

F, H < I

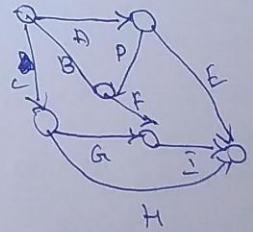
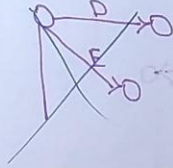
Draw a Diagram.

:-



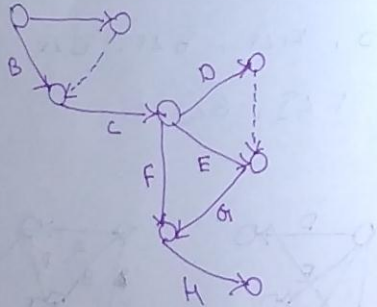
② A < D, A < E, B < F, D < F, C < G, C < H, F < I, G < I

:-



Activity	Preceding Activity
A	-
B	-
C	A, B
D	C
E	C
F	C
G	D, E
H	F, G

Sol:-



Activity	Preceding Activity
A	-
B	-
C	A
D	A
E	A
F	B, C
G	D
H	E, G

Sol:-



Simplex Method

① Min $Z = x_1^2 - 3x_2 + 2x_3$

Sub to: $3x_2 - x_3 + 2x_5 \leq 7$

$-2x_2 + 4x_3 \leq 12$

$-4x_1 + 2x_3 + 8x_5 \leq 10; x_1, x_2, x_3, x_5 \geq 0$

Sol:-

Step: 1

$3x_2 - x_3 + 2x_5 + s_1 = 7$

$-2x_2 + 4x_3 + 0x_5 + s_2 = 12$

$-4x_1 + 2x_3 + 8x_5 + s_3 = 10$

Step: 2

$A \cdot X = B$

$$\begin{bmatrix} 3 & -1 & 2 & 1 & 0 & 0 \\ -2 & 4 & 0 & 0 & 1 & 0 \\ -4 & 3 & 8 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 12 \\ 10 \end{bmatrix}$$

Step 3

initial iteration

C_B	Y_B	X_B	x_1	x_2	x_3	x_5	s_1	s_2	s_3	
0	s_1	7	3	-1	2	1	0	0		$\frac{7}{3} = 2.33$
0	s_2	12	-2	4	0	0	1	0		$\frac{12}{4} = 3$
0	s_3	10	-4	3	8	0	0	1		$\frac{10}{8} = 1.25$
$Z_j - C_j$	$Z = 0$		1	-3	2	0	0	0		

Step 4

C_B	Y_B	X_B	x_1	x_2	x_3	x_5	s_1	s_2	s_3	
0	s_1	10	5/2	0	2	1	1/2	0		
3	x_2	3	-1/2	1	0	0	1/2	0		
0	s_3	1	-5/2	0	2	0	-1/2	1		
$Z_j - C_j$	$Z = 9$		-1/2	0	2	0	3/2	0		

Step 5

C_B	Y_B	X_B	x_1	x_2	x_3	x_5	s_1	s_2	s_3	
-1	x_1	4	1	0	2/5	2/5	1/10	0		
3	x_2	5	0	1	2/5	1/5	3/10	0		
0	s_3	11	0	0	10	1	-1/2	1		
$Z_j - C_j$	$Z = 11$		0	0	12/5	1/5	8/10	0		

ans:-

$$\min Z = -\max Z$$

$$= -11 \text{ with}$$

$$x_2 = 4; x_3 = 5 \text{ and } x_5 = 0.$$

② $\max z = 3x_1 + 4x_2$

sub to:

$$2x_1 + x_2 \leq 1$$

$$x_1 \leq 2$$

$$x_1 + x_2 \leq 3$$

sol:-

step 1

$$-2x_1 + x_2 + s_1 = 1$$

$$x_1 + 0x_2 + s_2 = 2$$

$$x_1 + x_2 + s_3 = 3$$

Ans.

step 2

$$Ax = B$$

$$\begin{bmatrix} -2 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

step 3

CB	YB	XB	3	4	0	0	0
			x_1	x_2	s_1	s_2	s_3
0	s_1	1	-2	1	1	0	0
0	s_2	2	1	0	0	1	0
0	s_3	3	1	1	0	0	1
Zj-Cj	Z=0		+3	-2	0	0	0

$1/-2 = -0.5$
 $2/1 = 2$
 $3/1 = 3$

$$0 \times -2 = 0$$

$$0 \times 1 = 0$$

$$0 \times 1 = 0$$

step 4

C_B	Y_B	X_B	x_1	x_2	s_1	s_2	s_3
0	s_1	5	0	0	1	2	0
3	x_1	2	1	0	0	1	0
0	s_3	1	0	1	0	-1	1
$Z_j - C_j$	$Z = 6$		0	-2	0	3	0

step 5

C_B	Y_B	X_B	x_1	x_2	s_1	s_2	s_3
0	s_1	4	0	0	1	3	-1
3	x_1	2	1	0	0	1	0
2	x_2	1	0	1	0	-1	1
$Z_j - C_j$	$Z = 8$		0	0	0	1	2

$\max Z = 8$

$x_1 = 2$

$x_2 = 1$

$3x_1 + 2x_2$

$3(2) + 2(1)$

$6 + 2 = 8 //$

step 1

1	2	3	4	6
4	3	2	0	8
0	2	2	1	10
4	6	8	6	

$\sum b_j$

$\sum b_i$

$\sum a_i = 24$

$\sum b_j = 24$

$\sum a_i = \sum b_j$

step 2

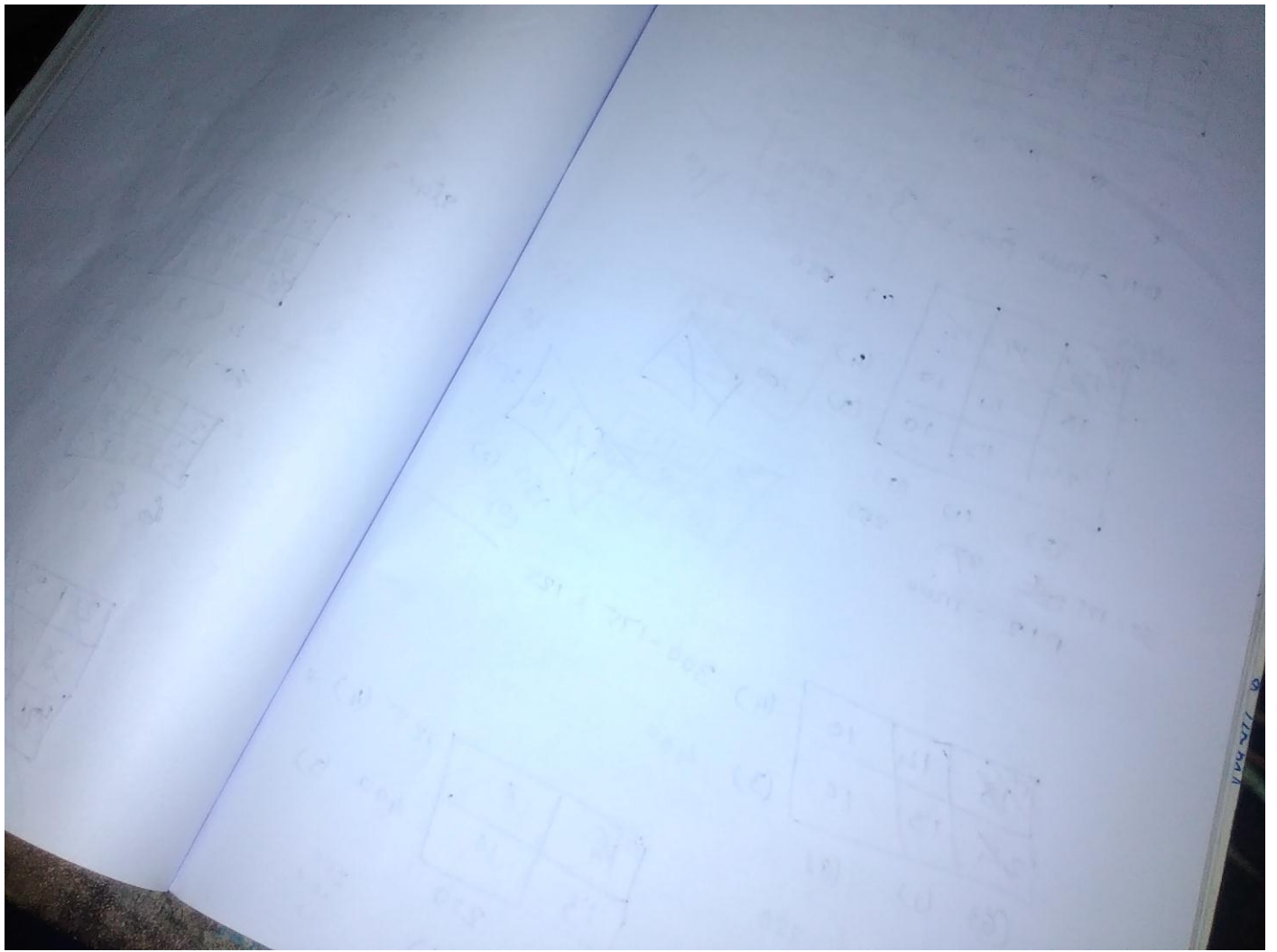
1	2	3	4	6
4	3	2	0	8
0	2	2	1	10-4
4	6	8	6	

$\sum b_j = 10-4 = 4$

$\sum a_i = 10-4 = 4$

2	3	4	6
3	2	0	8-6=2
2	2	1	4
6	8	6	

2	3	6
3	2	2
2	2	4
6	8	8



WPM methods :-

13	17	14
18	14	10
24	15	10

250 (3)
300 (4)
400 (5)

200 (5)
235 (5)
275 (0)

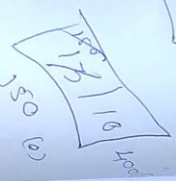
n11 = min(150, 200) = 200

100

steps:

18	14	10
18	14	10
24	15	10

(1) 950
(4) 900
(5) 400



50 = 175 - 200
n12 = min

18	14	10
24	13	10

(6) (1) (0)
(3) 400
(4) 300 - 175 = 125

175 275 250

275 - 125 = 150

14	18
13	10

125 (4)
400 (5)
275 (1)
250 (0)
250
150
400

Assignment

8	7	6
5	7	8
6	8	7

2	1	0
0	2	3
0	2	1

2	1	3
0	2	3
0	2	1

2	10	8
10	2	8
8	1	10

WR

1	2	3	4	6
4	3	2	0	8
0	2	2	1	10

Step 1:
 $\sum a_i = 2+3+10 = 24$
 $\sum b_j = 4+6+8+10 = 24$
 $\sum a_i = \sum b_j$

4	2	3	3
4	3	4	0
0	2	4	6

$m+n-1 = 0$
 $7-6 = 1 = 0$
 $3+4 = 7 = 1+6$

Step 2

4	2	3	4
4	3	2	0
0	2	2	1

$x_{11} = (6, 4) = 4$

$1 \times 4 \quad 2 \times 2 \quad 4 \times 3 \quad 1 \times 2$

4	3	4
3	2	0
2	2	1

$4 = 2 - 6 \quad 8 \quad 6$
 $x_{12} = (6, 2) = 2$

$2 \times 2 \quad 6 \times 1$
 $2 + 4 + 10 + 8 + 8 + 6 = 42 //$

4	0
2	2

$x_{22} = \min(8, 4) = 4$

4	0
2	1

$4 = 8 \quad 6$

4	1
---	---

1

ZCM

1	2	3	4	6
4	3	2	0	8
0	2	2	1	10

$\sum a_i = 6+8+10 = 24$
 $\sum b_j = 4+6+8+6 = 24$
 $\sum a_i = \sum b_j$

Step 1

1	2	3	4	6
4	3	2	0	8
0	2	2	1	10

$8-6 = 2$
 $\neq 2, 4, 2 \notin (8, 6)$
 $= 6$

4	3
3	2
2	2

Step 2

1	2	3	6
4	3	2	2
4	2	2	10-4 = 6

$2 = 6 - 8 \quad 6$
 $6 - 4 = 6$

3	2
2	2

$2 - 0$
 $= 6$

$$x_1 + x_2 = 8 \rightarrow (1)$$

$$3x_1 + 4x_2 = 3 \rightarrow (2)$$

$$6x_1 + 7x_2 = 5 \rightarrow (3)$$

$$x_1 + x_2 = 0 \rightarrow (4)$$

$x_1 = 0$ sup in (1)

$$0 + x_2 = 8$$

$$x_2 = 8$$

(0, 8)

$$x_2 = 0$$

(0, 8) 2, 4, 8

18	26	17	11
13	28	14	26
38	19	18	15
19	26	24	10

$$\frac{26}{11}$$

$$\frac{28}{13}$$

$$\frac{26}{13}$$

$$\frac{38}{15}$$

7	15	65	10
0	15	40	13
23	4	32	8
9	16	14	8

7	15	6	0
0	15	1	13
23	14	3	0
9	16	14	0

7	11	5	10
0	11	8	13
23	0	2	8
9	12	13	8

7	11	3	10
0	11	8	13
23	0	8	8
9	12	11	8

E F G H-N				
A	7	11	3	10
B	8	11	10	13
C	23	10	8	8
D	9	12	11	8

$$\frac{26}{11}$$

$$A \rightarrow H + B \rightarrow G + C \rightarrow F + D \rightarrow E$$

$$= 11 + 14 + 19 + 19$$

$$= 63$$

$$x_1 + x_2 = 6 \rightarrow (1)$$

$$2x_1 + 3x_2 = 3 \rightarrow (2)$$

$$x_1 + x_2 = 0 \rightarrow (3)$$

$$x_1 = 0 \text{ Sub} \rightarrow (1)$$

$$0 + x_2 = 6$$

$$x_2 = 6$$

$$(0, 6)$$

$x_1 = 0, x_2 = 6$

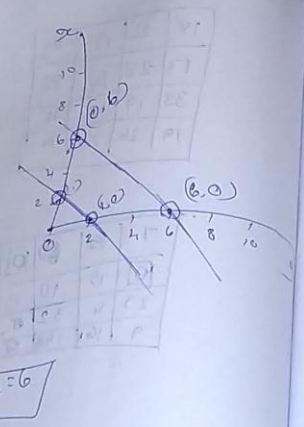
$$x_2 = 0 \text{ Sub} \rightarrow (2)$$

$$2x_1 + 0 = 3$$

$$x_1 = 3/2$$

$$x_1 = 1.5$$

$x_1 = 1.5, x_2 = 0$



$$2x_1 + 3x_2 = 3$$

$$x_2 = 0 \text{ Sub} \rightarrow (2)$$

$$2x_1 + 0 = 3$$

$$= 3/2$$

$$= 1$$

$x_1 = 1, x_2 = 0$

$$2x_1 + 3x_2 = 3$$

$$x_1 = 0 \text{ Sub} \rightarrow (1)$$

$$0 + 3x_2 = 3$$

$$x_2 = 3/3 = 1$$

$x_1 = 0, x_2 = 1$

6	1	9	3	70 (2)
11	5	2	8	55 (3)
10	12	4	7	70 (3)

85 35 50 45
(4) (4) (2) (4)
*

11	8	5	8	70
11	5	2	8	55
10	12	4	7	70

85 35 50 45
15

11	8	55 (3)
10	7	70 (3)
15	45	11

11	8	55 (3)
10	7	70 (3)
15	35	50 45

11	8	55 (6) *
10	7	70 (3)
15	45	(4)

11	8	5 (3) *
10	7	70 (3)
15	45	(4)

20	20
4	20
50	

11	8	5
10	7	70
15	45	

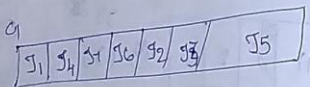
11	8	15
10	7	20
15	50	45
(4)	(2)	(1)

4	20
50	25-5
(4)	(7)
45	20
25	

4	20
50	45-25
(4)	(7)
20	(3)

4	2
50	
10	

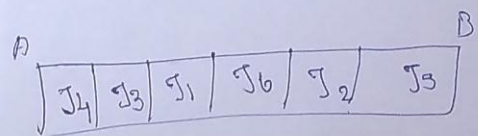
Job	$A = M_1 + M_2$	$B = M_2 + M_3$
J ₁	7	10
J ₂	11	10
J ₃	9	7
J ₄	9	16
J ₅	10	6
J ₆	12	10
J ₇	10	15



	J ₁	J ₂	J ₃	J ₄	J ₅	J ₆	J ₇
A	7	11	9	9	10	12	10
B	10	10	7	16	6	10	15

Job	m.A		m.B		idle time
	input	out	in	out	
J ₁	0	7	7	17	
J ₂	7	16	17	33	
J ₆	16	28	33		
J ₂	28	39			
J ₃	39	48			
J ₅	48	158			

Job	$G = A + B_i$	$B = B_i + C_i$
J ₁	13	23
J ₂	20	22
J ₃	11	15
J ₄	10	20
J ₅	14	13
J ₆	14	16



Job	m.A		m.B		m.C		Idle line
J ₄	0	3	3	10	10	23	70
J ₃	3	9	10	15	23	43	70
J ₄	9	13	15	24	43	57	70
J ₆	13	23	24	26	47	62	70
J ₂	25	38	26	33	61	76	70
J ₅	38	48	33	35	76	85	70

Idle line = 10 hrs

~~(10-10)~~ ~~(15-15)~~

Idle line = 85

$$\text{Machin B} = (10-10) + (15-15) + (24+24) + (26+26) + (33+33) + (85-76)$$

$$\begin{array}{r} 7^{15} \\ 85 \\ \underline{76} \\ 9 \end{array}$$

In 1642 at the age of 23

French m. mathematician Blaise

Pascal.

Faint handwritten notes and diagrams on the right page, including a grid of numbers and some mathematical symbols.

①

5	9	11	6
8	5	9	6
4	7	10	7
10	4	3	3

②

7	10	5	13	15	16
1	3	9	18	3	6
2	10	7	2	9	2
5	11	9	7	7	12
4	9	10	4	19	

3d

0	2	6	1
3	0	4	3
0	3	6	3
1	1	5	0

2	2	1
3	2	3
3	3	3
7	1	10

2	1	1
3	3	3
3	3	3
7	1	10

2	2	1
3	2	3
3	3	3
7	1	10

$S_1 \rightarrow M_1 + 9 \rightarrow M_2 + 3 \rightarrow M_3 + 9 \rightarrow M_4$

28

A \rightarrow II + B \rightarrow I + C \rightarrow III
D \rightarrow IV + E \rightarrow V

5 + 3 + 2 + 7 + 12

29

③

1	2	3	4	5	6
12	10	15	22	18	8
10	18	25	15	16	12
11	10	3	8	5	11
6	14	10	13	15	19
12	11	7	13	10	10

1 5 2 7 14 10 10
2 15 5 6 2
7 5 2 6
8 4 9 7 6
1 5 4 6 3

4 2 7 10 10 0
0 0 0 0

4	3	14	4
6	11	5	1
5	10	5	3
8	16	3	1

4 3 14 4
6 11 5 1
5 10 5 3
8 16 3 1
1 3 10 2 3

A 0 7 14 10 0
0 6 5 5 6 2
8 5 0 5 2 6
0 6 4 7 7 6
1 3 4 0 6 3

1	14	10	4
6	15	5	6
5	10	5	6
6	3	3	6
1	3	4	6

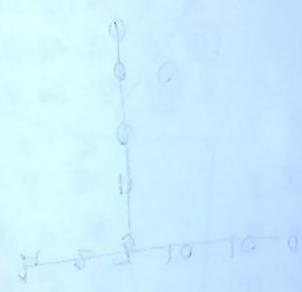
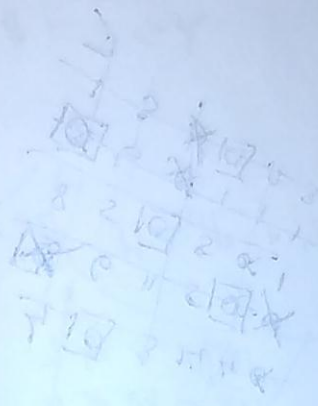
4 7 14 8
6 15 5 4 2
8 5 10 5 6 6
6 4 7 6 6
1 3 4 6 3

4 7 14 4
6 15 5 4
8 5 10 5 4
6 4 7 1 4
1 3 4 0 2 3

A \rightarrow 2 + B \rightarrow 5 + C \rightarrow 3 + D \rightarrow 1
E \rightarrow 4
10 + 16 + 3 + 6 + 7



$1 \rightarrow 5, B \rightarrow 2 + C \rightarrow 2 + D \rightarrow 1$
 $1 \rightarrow 5, B \rightarrow 2 + C \rightarrow 2 + D \rightarrow 1$
 $1 \rightarrow 5, B \rightarrow 2 + C \rightarrow 2 + D \rightarrow 1$
 $1 \rightarrow 5, B \rightarrow 2 + C \rightarrow 2 + D \rightarrow 1$



1 2 3 4 5
 6 7 8 9 10
 11 12 13 14 15
 16 17 18 19 20
 21 22 23 24 25
 26 27 28 29 30
 31 32 33 34 35
 36 37 38 39 40
 41 42 43 44 45
 46 47 48 49 50
 51 52 53 54 55
 56 57 58 59 60
 61 62 63 64 65
 66 67 68 69 70
 71 72 73 74 75
 76 77 78 79 80
 81 82 83 84 85
 86 87 88 89 90
 91 92 93 94 95
 96 97 98 99 100

8

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
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$1 \rightarrow 5, B \rightarrow 2 + C \rightarrow 2 + D \rightarrow 1$
 $1 \rightarrow 5, B \rightarrow 2 + C \rightarrow 2 + D \rightarrow 1$
 $1 \rightarrow 5, B \rightarrow 2 + C \rightarrow 2 + D \rightarrow 1$
 $1 \rightarrow 5, B \rightarrow 2 + C \rightarrow 2 + D \rightarrow 1$

1 2 3 4 5
 6 7 8 9 10
 11 12 13 14 15
 16 17 18 19 20
 21 22 23 24 25
 26 27 28 29 30
 31 32 33 34 35
 36 37 38 39 40
 41 42 43 44 45
 46 47 48 49 50
 51 52 53 54 55
 56 57 58 59 60
 61 62 63 64 65
 66 67 68 69 70
 71 72 73 74 75
 76 77 78 79 80
 81 82 83 84 85
 86 87 88 89 90
 91 92 93 94 95
 96 97 98 99 100

1 2 3 4 5
 6 7 8 9 10
 11 12 13 14 15
 16 17 18 19 20
 21 22 23 24 25
 26 27 28 29 30
 31 32 33 34 35
 36 37 38 39 40
 41 42 43 44 45
 46 47 48 49 50
 51 52 53 54 55
 56 57 58 59 60
 61 62 63 64 65
 66 67 68 69 70
 71 72 73 74 75
 76 77 78 79 80
 81 82 83 84 85
 86 87 88 89 90
 91 92 93 94 95
 96 97 98 99 100

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
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OR

April - 2013

PART - A

10 × 2 = 20

- ① write any two applications of OR.
- ② what are the methods for deriving the solution to an OR model?
- ③ Define slack Variable
- ④ Define feasible ~~var~~ solution.
- ⑤ when does degeneracy happen in transportation problem?
- ⑥ what is the objective of the assignment problem.
- ⑦ what is no passing rule in sequencing algorithm?
- ⑧ what is Gantt chart?
- ⑨ Define interference flood.
- ⑩ Define optimistic time.

PART - B

5 × 5 = 25

- ① Q2 Explain the applications of OR.
(OR)

b) solve graphically the following LPP.

$$\text{maximize } Z = 3x_1 + 2x_2$$

$$\text{sub to: } -2x_1 + x_2 \leq 1$$

$$x_1 \leq 2$$

$$x_1 + x_2 \leq 3, \quad x_1, x_2 \geq 0.$$

10) a) obtain and the basic solution to the following system linear eq.

$$x_1 + 2x_2 + x_3 = 2$$

$$2x_1 + x_2 + 5x_3 = 8.$$

b) Use simplex method to solve the following LPP.

$$\text{maximize } z = 3x_1 + 2x_2$$

$$\text{subject to } x_1 + x_2 \leq 4, \quad x_1 - x_2 \leq 2,$$

$$x_1, x_2 \geq 0.$$

15) a) obtain an IBFS to the following transportation problem using major minima

Comer side.

	D ₁	D ₂	D ₃	D ₄	
O ₁	1	2	3	4	6
O ₂	2	3	2	0	8
O ₃	0	2	2	1	10
Demand	4	6	8	6	

b) solve the following assignment problem.

	A	B	C	D
1	18	24	28	32
2	4	18	17	19
3	10	18	19	22

14) a) Explain problems with 2 jobs and R machine.

b) Determine the optimal sequence, minimum total elapsed time and idle time of machine A and B for which the times (in hours) are as follows.

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A Grade Paper
Non-

Job	J ₁	J ₂	J ₃	J ₄	J ₅	J ₆
machine	1	3	8	5	6	3
A						
B	5	6	3	2	2	10

18) a) Construct the network diagram comprising activities a, b, c, d and e such that the following constraints are satisfied.

B < E, F ; C < G, I ; E < G, H ; I < H, L ;

L < M ; H, M < N ; H < S ;

I, J < P ; P < Q ; (1)

b) Discuss the advantages of PERT / CPM.

PERT-C

19) Explain the uses and limitations of OR.

19) Use two phase simplex method.

to maximize $Z = 5x_1 - 2x_2 + 3x_3$

sub to :

$$2x_1 + 4x_2 - 6x_3 = 20$$

$$6x_1 + 5x_2 + 10x_3 \leq 76$$

$$8x_1 - 3x_2 + 6x_3 \leq 50$$

$$x_1, x_2, x_3 \geq 0$$

20) Solve the following assignment problem.

	E	F	G	H
A	18	26	4	11
B	13	28	17	26
C	33	19	18	15
D	19	26	23	10

21) Determine the optimal sequence

of jobs that minimize as the total elapsed time based on the following information processing time on machine is given in hours and passing is not allowed.

Job	A	B	C	D	E	F	G
Machine	1	3	6	7	4	9	8
M ₂	1	3	2	5	1	3	3
M ₃	6	4	5	1	5	6-12	

20) A small project is composed of 7 activities whose time estimates are listed in the following table.

Activity Estimated duration

i	j	optimistic	most likely	pessimistic
1	2	1	1	1
1	3	1	2	7
1	4	2	2	4
2	5	1	1	8
3	6	2	5	14
4	6	2	5	18
5	6	3	6	15