Mathematical Modeling M.Sc., Mathematics
Code: P16MAE1B

Unit - 1

1. what is meant by Radio-Active decay?

The Radio-active decay at a rate proportional to the amount of the radioactive substance present at any time and each of them has a half-life period.

Example: Uranium 238 it is 4.55 billion years

2. write down the Fick's law

The flux is directly proportional to the concentration gradient.

$$J = -D\frac{dc}{dt}$$ Negative sign indicate decrease in concentration.

or $\frac{dc}{dt} = k(a-c)$, $c(0) = c_0$ $c_0 < a$.

$\Rightarrow$ $a - c(t) = (a - c(0))e^{-kt}$

3. what is meant by Rate of Dissolution?

$x(t)$ be the amount of undissolved solute in a solvent at time t and let c_0 be the maximum concentration or saturation concentration. That is The maximum amount of the solute that can be dissolved in a unit volume of the solvent.

4. Define an Orthogonal Trajectories.

Any curve, which cuts every member of a given family of curves at right angles, is called an Orthogonal Trajectories of the family. But if they intersect the angle between their tangents at every point of intersection is $90°$.

5. Define a Simple harmonic Motion?

Simple harmonic motion arises when we consider the motion of a particle whose acceleration points towards a fixed point O and is proportional to the distance of the particle from O [so the acceleration increases as the distance from the fixed point increases].

Mathematical Modeling.

Unit - 2

1. Write down the any two assumptions for pre-predator model.

i) The prey population will grow exponentially when the predator is absent.

ii) The predator population will starve in the absence of the prey population.

2. Define Domar Macro Model.

Let $S(t)$, $I(t)$, $Y(t)$ be the Savings, Investment and National Income at time t, then it is assumed that,

(i) Savings are proportional to national income so that, $S(t) = \alpha Y(t)$, $\alpha > 0$

(ii) Investment is proportional to the rate of increase of national income so that

$I(t) = \beta Y'(t)$, $\beta > 0$

iii) All Savings are invested, so that

$S(t) = I(t)$

3. Write short Note on Model for Diabetes Mellitus.

Let $x(t)$, $y(t)$ be the blood sugar and insulin levels in the blood stream at time t. The rate of change dy/dt for insulin level is proportional to three possible ways gives

$$\frac{dy}{dt} = a_1(x - x_0) H(x - x_0) - a_2 y + a_3 d(t).$$

4. What is meant by Modelling in dynamics.

If a particle moves in two dimensional space then determine $x(t)$, $y(t)$, its coordinates at any time t and $u(t)$, $v(t)$ its velocity components at the same time. The three dimensional, so determine $x(t)$, $y(t)$, $z(t)$, $u(t)$, $v(t)$, $w(t)$

5. What is meant by Susceptible-Infected-Susceptible (SIS) Model.

A Susceptible person can become infected at a rate proportional to SI and an infected person can recover and become susceptible again at a rate γI, So that

$$\frac{dS}{dt} = -\beta SI + \gamma I, \quad \frac{dI}{dt} = \beta SI - \gamma I$$

$$\Rightarrow \frac{dI}{dt} = (\beta(n+1) - \gamma)I - \beta I^2.$$

Mathematical Modelling

Unit-3.

1. Write down Kepler's laws of planetary Motions

i) Every planet describes an ellipse with the sun at one focus.

ii) The radius vector from the sun to a planet describes equal areas in equal intervals of time.

iii) The squares of periodic time of planets are proportional to the cubes of the semi major axes of the orbits of the planets.

2. Define Circular Motion.

A particle moves in a circle of radius a so that $r = a$, the radial component of velocity $= r' = 0$, the transverse component of velocity $= r\theta' = a\theta$ the radial component of acceleration $= r'' - r\theta'^2 = -a\theta'^2$, the transverse component of acceleration is equal to $\frac{1}{r}\frac{d}{dt}(r^2\theta') = \frac{1}{a}\frac{d}{dt}(a^2\theta') = a\theta''$

3. When will you say the motion is critically damped?

Rectilinear Motion $m\lambda^2 + c\lambda + k = 0$. if $c^2 = 4km$ the roots are real and equal and

$x(t) = (A_1 + A_2 t)\exp\left(-\frac{c}{2m}t\right)$, and again $x(t) \to 0$ as $t \to \infty$.

4. Define Circular Motion.

A particle moves in a circle of radius a so that $r = a$ the radial component of velocity $= r' = 0$, the transverse component of velocity $r\theta' = a\theta'$ the radial component of acceleration $= r'' - r\theta'^2 = -a\theta'^2$.

5. Write down any two assumption for Phillip's Stabilization Model.

(i) The producers adjust the national production Y demand D. if $D > Y$ is increase production $D < Y$, is decrease production

$$dY/dt = \alpha(D - Y), \alpha = 0.$$

(ii) Aggregate demand D is the sum of private demand government demand G and an exogenous disturbance u. The private demand is proportional to the national income or output so that

$$D = (1 - L)Y + G - u.$$

Mathematical Modelling.

Unit-4

1. State the Cobweb Model.

Let p_t is price of a commodity in the years t and q_t is amount of the commodity available in the market in year t,

it gives $q_t = \alpha + \beta p_{t-1}$, $\beta > 0$

and $p_t = \gamma + \delta q_t$, $\delta \leq 0$

$\Rightarrow p_t - \beta \delta p_{t-1} = \gamma + \alpha \delta$.

2. State the Samuelson's Interaction Models.

The basic equations for the first interaction model

$Y(t) = C(t) + I(t)$, $C(t) = \alpha Y(t-1)$, $I(t) = \beta [C(t) - C(t-1)]$.

α is positive marginal propensity respect to income of previous years

β is relation given by the accelartation principle.

3. Define Two-Period fixed points.

A point is called a two-period fixed point if it repeats itself after two periods if $y_{t+2} = y_t$

if $y_{t+2} = m y_{t+1} (1 - y_{t+1}) = m^2 y_t (1 - y_t)(1 - m y_t + m y_t^2) = y_t$

4. State Hardy-weinberg Law.

The Hardy-weinberg Law state that the genotype frequencies in a population will remain constant from generation to generation in the absence of other evolutionary influences.

5. what do you mean by Markov chain.

The experiences transitions from one state to another according to certain probabilistic rules. The defining characteristic of a Markov chain is that no matter how the process arrived at its present state the possible future States are fixed.

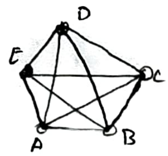
Mathematical Modelling

Unit-5

1. Define a complete graph.

A graph is called complete if every pair of its vertices is joined by an edge

example



2. Mentions any two facts about Nature of Models in Terms of Graphs.

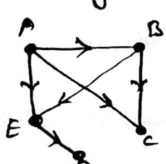
(The length of the edge joining two vertices will not be relevant.

(a) Which edges are joined

(b) which edges are directed and in which direction.

3. Define Representing Results of Tournaments.

The graph



i) Team A has defeated teams B, C, E

ii) Team B has defeated teams C, E

iii) Team E has defeated D.

4. General Communication Networks.

a) For Communication of messages where the directed edge represents the channal and the weight represent the capacity, per second in bit.

b) Communication roads where the weight are the capacities is cars per hour.

5. Lumped Mechanical systems

If the linear graph represent a lumped mechanical system with the vertices representiting rigid bodies, Matrices A and B aries for Neton's force and displacement equations respectively and $v-1$ and $e-v+1$ represent the number of linearly Independent force and displacement equation.