Stochastic Processes

M.Sc., Mathematics

Code: P16MAE 2A

Unit-I

1. Time dependent poisson process.

Let assume that λ is a non-random function of time 't' and $N(t)$ be the number of times an event 'E' occurs in $[0,t]$.

2. Decomposition of the poisson process

A random selection from a poisson process yields a poisson process suppose that $N(t)$ the number of occurance of an event 'E' in an interval of length 't' is a poisson process with parameter λ.

Suppose that each occurence of 'E' has a constant probability 'p' of being recorded, and that the recording of an occurance is independent of that of other occurence and also of $N(t)$.

3. Poisson cluster process [compound or cumulative poisson process].

Suppose that several event can happen simultaneously at such an instant. Then a cluster at a point is take as .

(i) The number of $N(t)$ of clusters in time t That is the point poisson process with mean rate λ.

(ii) Each cluster has a random number of occurance.

$$P_r\{X_i = k\} = P_k \quad , \quad \begin{matrix} k=1,2,3 \\ i=1,2,3 \end{matrix}$$

The probability generating function $P(s) = \sum^{\infty} P_k s^k$

4. Postulates for poisson process

(i) Independence:

$N(t)$ is independence of the number of occurrances in the event E is an interval Prior to $(0, t)$. ie, further changes in $N(t)$ are independent of the passed changes.

(ii) Homogenity in Time:

$P_n(t)$ depends only on the length 't' the interval and is independent of whose this interval is situated.

(iii) Regularity:

In an interval of infinitesimal length h, the probability of exactly one occurrene is $\lambda h + o(h)$.

5. Markov process.

If $\{x(t), t \in T\}$ is a stochastic process Given the value $x(s)$, the value of $x(t), t>s$ do not depend on the value of $x(u)$ $u<s$ Then the process is Said to be a Markov process.

ie, If for ~~$t_1 t$~~ $t_1 < t_2 < \dots < t_n < t$

Probability of $P_r\{a \leq x(t) \leq b / x(t_1) = x_1, \dots x(t_n) = x_n\}$

$$= P_r\{a \leq x(t) \leq b / x(t_n) = x_n\}.$$

The process $\{x(t), t \in T\}$ is a Markov process A discrete parameter Markov process is known as Markov chain.

6. Stationary process

Second order processes.

A Stochastic process $\{X(t), t \in T\}$ is called a Second order process. If ~~a~~ expectation of $E[X(t)]^2 < \infty$. It is a collection of second order random variable.

ie, Random variables with finite second order moments.

7. Renewal Theorem (Blackwell's and Smith's)

For X_i non lattice and for fixed $h \geq 0$

$$M(t) - M(t-h) \to h/\mu \text{ as } t \to \infty$$

and for lattice X_i with period d,

$$\lim_{h \to \infty} P_r\{\text{renewal at } nd\} \to d/\mu.$$

8. Smith's theorem or Key Renewal Theorem

Let $H(t)$ be Directly Rienmann integrable and $H(t) = 0$ for $t < 0$. if X_i is non-lattice, then

$$\int_0^t H(t-x)\,dM(x) \to \frac{1}{\mu}\int_0^\infty H(t)\,dt \text{ as } t \to \infty$$

The limit being interpreted as zero when $\mu = \infty$

If X_i is lattice with period d, then

$$H(c+nd) \to \frac{d}{\mu}\sum_{t=0}^{\infty} h(c+kd).$$

9. Elementary Renewal Theorem.

Let $\frac{M(t)}{t} \to \frac{1}{\mu}$ as $t \to \infty$, where $\mu = E(X_n) < \infty$, the limit being interpreted as 0 (zero) when $\mu = \infty$.

10. Stopping Time.

An integer valued random variable N said to be a stopping time for the sequence $\{X_t\}$ if the event $\{N=n\}$ is independent of $X_{n+1}, X_{n+2}, X_{n+3}, \ldots$ $\forall\, n=1,2,3..$

11. Renewal process and theory.

Renewal process was introdue as a generalization of poission process it was consider as a process in continuous time Renewal theory and renewal therotical arugments. law beans advanced in a vavity of situation such as demographic man power studies.

12. Renewal period.

The interval between occurences of two successive renewals is called a renewal peerid. of the process.

13. Periodic.

The renewal event E^* is said to be periodic if there exist an integer m ∋; E^* can occures only at trial number m, 2m, ... property the greatest 'M' with this said to be periodic of E^*. It is said to be a periodic if ~~...~~ no such 'M' exist.

14. Pure Death Process

$\lambda_n = 0 \ \forall n$, That is an individual can't give birth to a new individual and the probability of death of an individual in $(t, t+h)$ is $\mu h + o(h)$. Then, if n individuals are present at time t, The probability of one death in $(t, t+h)$ is $n\mu h + o(h)$.

15. when we say that two states are communicate?

If two states i and j are such that each is accessible from the other then, the two states Communicate, it is denoted by $i \leftrightarrow j$: Then There exist integer m and n $\ni$:

$$P_{ij}^{(n)} > 0 \text{ and } p_{ji}^{(m)} > 0.$$

16. Define a Stationary distribution.

A Markov chain with transition probabilities p_{jk} and t.p.m. $P = (p_{jk})$. A probability distribution $\{v_j\}$ is called Stationary for the given chain if

$$v_k = \sum_j v_j p_{jk} \quad \ni; \ v_j \geq 0, \ \sum_j v_j = 1.$$

and general

$$v_k = \sum_i v_i p_{ik}^{(n)}, \ n \geq 1.$$

17. Define class property.

All states of a chain is a class property if its possession by one state in a class implies its possession by all states of the same class. One such property is the periodicity of a state.

18. State Wald's Equation.

Let $\{X_i\}$ be a sequence of independent random variables. having the same expectation and let N be a stopping time for $\{X_i\}$ and $E(N) < \infty$ Then

$$E\left\{\sum_{i=1}^{N} X_i\right\} = E(X_i)\,E(N).$$

19. Random variables.

A random exprimment have some Sample space S. A random variable X is a function that assigns a real value to each out come occure in S.

20. Gamma distribution.

Let X have a two parameter gamma distribution with parameters λ, K ($\lambda > 0$ is the Scale parameter and $K > 0$ is the shape parameter.).

The density function of the Random variable X is

$$f_{\lambda,K}(x) = \frac{\lambda^K x^{K-1} \exp(-\lambda x)}{\Gamma(x)}, \quad x > 0.$$

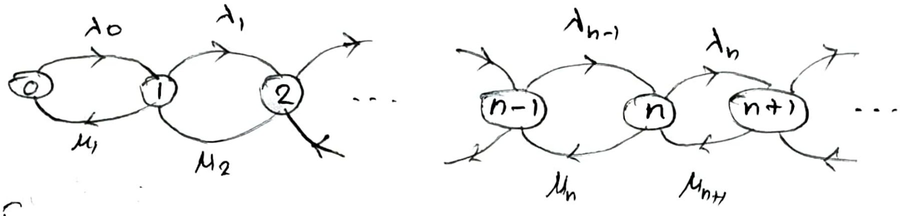
$$= 0, \quad x < 0.$$

21. Mean Recurrence Time.

For a persistent and aperiodic renewal event (patten) $F'(1) = \sum n f_n = E(T)$ is The mean recurrence time. (ie, mean time between two consecutive renewals or mean waiting time between two consecutive renewals).

$F'(1)$ is may be finite or infinite.

22. Draw a state-transition rate diagram.



For $n=0$, $\lambda_0 P_0 = \mu_1 P_1$

for $n>0$. $(\lambda_n + \mu_n) P_n = \lambda_{n-1} P_{n-1} + \mu_{n+1} P_{n+1}$.

23. Erlang's Second formula.

The probability that an arriving unit has to wait is given by

$$C(s, \lambda/\mu) = P_r\{N \geq s\} = \sum_{n=s}^{\infty} P_n$$

$$= \frac{(\lambda/\mu)^s}{s!\,(1-\rho)} P_0 = \frac{P_s}{1-\rho}.$$

24. Write down the Little's formula.

$L = \lambda W$, where L is the expected number of units in the system, λ is the arrival rate, and W is the expected waiting time in the system in steady state.

25. Renewal function.

The function $M(t) = E\{N(t)\}$ is called the renewal function of the process with distribution F. It is clear that

$\{N(t) \geq n\} \Leftrightarrow \{S_n \leq t\}$

26. Renewal process

The Random variable $N(t) = \sup\{n : S_n \leq t\}$.

The process $\{N(t), t \geq 0\}$ is called a renewal process with distribution F.