PART C — $(3 \times 10 = 30)$

Answer any THREE questions.

16. If V is a finite-dimensional vector space, then prove that any two bases of V have the same number of elements.

17. Let V be an n-dimensional vector space over the field F, and let W be an m-dimensional vector space over F. Then prove that the space $L(V,W)$ is finite-dimensional and have dimension mn.

18. If F is a field, then prove that a non-scalar monic polynomial in $F[x]$ can be factored as a product of monic primes in $F[x]$ in one and, except for order, only on way.

19. Let T be the linear operator on R^3 which is represented in the standard ordered basis by the matrix $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$.

    Prove that T is diagonalizable by exhibiting a basis for R^3, each vector of which is a characteristic vector of T.

20. State and prove primary decomposition theorem.

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(For candidates admitted from 2016-2017 onwards)

M.Sc. DEGREE EXAMINATION, APRIL 2018.

Mathematics

LINEAR ALGEBRA

Time : Three hours   Maximum : 75 marks

PART A — $(10 \times 2 = 20)$

Answer ALL questions.

1. Define the product of two matrices.

2. Let F be a subfield of the complex numbers. Prove that, in F^3, the vectors $(3,0,-3)$, $(-1,1,2)$, $(4,2,-2)$ and $(2,1,1)$ are linearly dependent.

3. Define the rank and nullity of a linear transformation.

4. Let V be a vector space over the field F. Let U, T and T_2 be linear operators on V, then prove that $U(T_1 + T_2) = UT_1 + UT_2$.

5. When we say that two algebras are isomorphic?

6. Define an ideal in $F[x]$.

7. Define a characteristic vector of a linear operator on a vector space over a field.

8. Give an example for the linear operator T may not have any characteristic values.

9. Let $A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$. Then find A^3.

10. Define a projection of a vector space.

PART B — ($5 \times 5 = 25$)

Answer ALL questions, choosing either (a) or (b) in each.

11. (a) Let e be an elementary row operation and let E be the $m \times n$ elementary matrix $E = e(I)$. Then prove that for every $m \times n$ matrix A, $e(A) = EA$.

Or

(b) If W is a subspace of a finite-dimensional vector space V, then prove that every linearly independent subset of W is finite and is part of a finite basis for W.

12. (a) Let V and W be vector space over the field F and let T be a linear transformation from V into W. If T is invertible, then prove that the inverse function T^{-1} is a linear transformation from W onto V.

Or

(b) Let V be a finite-dimensional vector space over the field F, and let W be a subspace of V. Then prove that $\dim W + \dim W^0 = \dim V$.

13. (a) Let f and g be non-zero polynomials over F. Then prove that fg is a non-zero polynomial.

Or

(b) If f, d are polynomials over a field E and d is different from 0 then prove that there exist polynomials q, r in $f[x]$ such that

(i) $f = dq + r$

(ii) either $r = 0$ or $\deg r < \deg d$.

14. (a) Suppose that $T\alpha = c\alpha$. If f is any polynomial, then prove that $f(T)\alpha = f(c)\alpha$.

Or

(b) Let K be a commutative ring with identify, let A and B be $n \times n$ matrices over K. Then prove that $\det(AB) = (\det A)(\det B)$.

15. (a) Let T be a linear operator on an n-dimensional vector space V. Prove that the characteristic and minimal polynomial for T have the same roots, except for multiplicities.

Or

(b) Let V be a finite-dimensional vector space over the field F and let T be a linear operator on V. Then prove that T is diagonalizable if and only if the minimal polynomial for T has the form $p = (x - c_1) \dots (x - c_k)$ where $c_1, \dots, c_k$ are distinct elements of F.

17. Let $T:V \to W$ be a linear transformation between vector spaces V and W. If V is finite dimensional then prove that rank (T)+ nullify $(T)=\dim(V)$.

18. State and prove Taylor's formula.

19. Diagonalise the matrix $\begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$.

20. State and prove Primary Decomposition theorem.

(For candidates admitted from 2008–2009 onwards)

M.Sc. DEGREE EXAMINATION, APRIL 2013.

Mathematics — Elective

LINEAR ALGEBRA

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20)

Answer ALL questions.

1. Show that every m × n matrix A is row-equivalent to a row-reduced echelon matrix.

2. Define basis of a vector space.

3. Define the rank and nullity of linear transformation.

4. Show that trace of A is a linear functional.

5. State Legranges interpolation formula.

6. Show that a polynomial of degree n over a field F has atmost 'n' roots in F.

7. Show that similar matrices have the same characteristic polynomial.

8. Find the characteristic values of $A = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$.

9. Define nilpotent linear transformation.

10. State Cyclic Decomposition Theorem.

PART B — $(5 \times 5 = 25)$

Answer ALL questions.

11. (a) Prove that every m × n matrix over the field F is row - equivalent to a row-reduced matrix.

Or

(b) Show that a non-empty subset W of V is a subspace of V if and only if for each pair of vectors α, β in W and each scalar C in F the vector $C\alpha+\beta$ is again in W.

12. (a) If A is an m × n matrix with entries in the field F then prove that row rank (A) = column rank (A).

Or

(b) Prove that every n-dimensional vector space over the field F is isomorphic to the space F^n.

13. (a) If F is a field then show that a non-scalar monic polynomial in $F[x]$ can be factored as a product of monic primes in $F[x]$ in one and only one way.

Or

(b) Find B^{-1} if $B = \begin{bmatrix} x^2-1 & x+2 \\ x^2-2x & x \end{bmatrix}$.

14. (a) Show that the characteristic and minimal polynomials for T have the same roots except for multiplicities where T is a linear operator on an n-dimensional vector space V.

Or

(b) Show that the linear transformation T is triangulable if and only if the minimal polynomial for T is a product of linear polynomials over F.

15. (a) Show that $\begin{bmatrix} 2 & 0 & 0 \\ a & 2 & 0 \\ b & c & -1 \end{bmatrix}$ is similar to a diagonal matrix if and only if $a=0$.

Or

(b) Let M be an m × n matrix with entries in the polynomial algebra $F[x]$, then prove that M is equivalent to a matrix N which is in the normal form.

PART C — $(3 \times 10 = 30)$

Answer any THREE questions.

16. If W_1 and W_2 are finite - dimensional subspaces of a vector space V then prove that W_1+W_2 is finite dimensional and $\dim(W_1)+\dim(W_2) = (W_1 \cap W_2) + \dim(W_1+W_2)$.