**COMPLEX ANALYSIS**

**(16SCCMM13)**

**IMPORTANT TWO MARK QUESTIONS**

**(UNIT V)**

1. **Residue of**

Let be an isolated singularity for . Then the residue of at is defined to be the coefficient of in the Laurent’s series expansion of about and is denoted by .

Thus where C is a circle such that is analytic in .

**Eg:**

Consider

We know that

has double pole at .

coefficient of

1. **Methods for calculation of residues**

**Lemma1:** If is a simple pole for then

**Proof:** Since is a simple pole for the Laurent’s series expansion for about is given by

**Problem 1:** Calculate the residue of at its pole.

**Solution:** Let

and are simple poles for

**Lemma 2:** If is a simple pole for and where is analytic at and then

**Proof:** by lemma 1,

**Problem 2:** Find theresidue of at its pole

**Solution:** Let . is a simple pole for

Let then .

**Lemma 3:** If is a simple pole for and if is of the form where and k(z) are analytic at and and then

**Proof**

**Problem 3:** Find the residue of at

**Solution:**  is a simple pole for .

Let

**Lemma 4:** Let be a pole of order for and let where is analytic at and . Then

**Proof:** we know by theorem of higher derivatives that

Where is a circle such that is analytic in

**Problem 4:** Find the residue of at its pole.

**Solution:** Let .

is a pole of order 3 for .

We know that

put and

Let so that and then

1. **Cauchy’s Residue Theorem**

Let be a function which is analytic inside and on a simple closed curve C except for a finite number of finite number of singular points inside C. Then

**Eg:** Evaluate where C is the circle .

Let

and are simple poles for and both of them lie inside .

Now,

By Residue theorem

1. **Argument theorem**

Let be a function which is analytic inside and on a simple closed curve C except for a finite number of poles inside C. Also let have no zeros on C. Then where is the number of zeros of inside C and P is the number of poles of inside C.

1. **Rouche’s Theorem**

If and are analytic inside and on a simple closed curve C and if on C then and have the same number of zeros inside C.

1. **Fundamental theorem of algebra**

A polynomial of degree with complex coefficients has zeros in C.