**COMPLEX ANALYSIS**

**(16SCCMM13)**

**IMPORTANT TWO MARK QUESTIONS**

**(UNIT IV)**

1. **Taylor’s series of about the point**

Let be analytic in a region containing . Then can be represented as a power series in given by

The expansion is valid in the largest open disc with centre contained in .

**Eg**

The Tailor’s series for about is given by

We know that

Putting and

Now we proceed as follows

Hence the Taylor’s series expansion for about 1 is

1. **Maclaurin’s series of**

The Taylor’s series expansion of about the point zero is called the Maclaurin’s series. Thus the Maclaurin’s series of is given by

**Eg**

Let

Then for all and hence

Hence the Maclaurin’s series for is given by

And the expansion is valid in the entire complex plane.

1. Maclaurin’s series expansion of some of the standard functions
2. **Laurent’s theorem**

Let and denote respectively the concentric circle and with . Let be analytic in a region containing the circular annulus . Then can represented as convergent series of positive and negative powers of given by

Where and .

**Problem 1**

Find the Laurent’s series expansion of about .

**Solution:**

Since

Clearly is analytic at all points .

We know that

Putting

Now,

This is the required Laurent’s series expansion for at .

1. **Zeros of an analytic function**

Let be a function which is analytic in a region . Let . Then is said to be a zero of order (where is a positive integer) for if where is analytic at and .

**Eg:**

Consider

We know that

Where

Obviously is analytic and

is a zero of order 1 for .

1. **Singularity of a function**

A point is called a singular point or a singularity of a function if is not analytic at and is analytic at some point every disc .

**Eg: 1**

Consider the function

Then for all .

Thus is analytic except at .

is a singular point of .

**Eg: 2**

Consider the function .

0 and are singular point for .

1. **Isolated Singularity**

A point is called an isolated singularity for if

1. is not analytic at and
2. There exists such that is analytic in

(i.e) the neighbourhood contains no singularity of except .

**Eg: 1**

has three isolated singularities .

**Eg: 2**

Consider the principal branch of logarithm given by where .

All points on the negative real axis are singular points of this function. These singularities are not isolated.

1. **Classification of the isolated singularities of a function**

Let be an isolated singularity of a function . Let be such that is analytic in . In this domain the function can be represented as a Laurent series given by

Where and

The series consisting of the negative powers of in the above Laurent series expansion of is given by and is called the **principal part or singular part** of at .

The singular part of at determines the character of the singularity. There are three types of singularities. They are

1. Removable singularities.
2. Poles
3. Essential singularities.
4. **Removable singularities**

Let be an isolated singularity for . Then is called a removable singularity if the principal part of at has no terms.

**Eg:**

Let . Clearly 0 is an isolated singular point for . Now

Hence the principal part of at has no terms. Hence is a removable singularity.

1. **Pole**

Let be an isolated singularity of . The point is called pole if the principal part of at has a finite number of terms. If the principal part of at given by

Where , we say that is a pole of order for .

**Note:** Pole of order 1 is called a simple pole and pole of order two is called double pole.

**Eg:**

Consider

we know that

Hence the principal part of at has a single term . Hence is a simple pole of .

1. **Essential singularity**

Let be an isolated singularity of . The point is called an essential singularity of at if the principal part of at has an infinite number of terms.

**Eg:**

Let . Obviously is an isolated singularity for .

We know that

The principal part of has infinite number of terms. Hence has an essential singularity at .

1. **Riemann’s theorem**

Let be a function which is bounded and analytic throughout a domain . Then either is analytic at or else is a removable singular point of .

1. **Meromorphic function**

A function is said to be a meromorphic function if it is analytic except at a finite number of points and these finite set of points are poles.

**Eg: 1**

Let

is analytic except at and . Also 0 and 1 are poles of order 1 and 2 respectively. Hence is a meromorphic function.

**Eg: 2**

Let

is not a meromorphic function since is an essential singularity for .