METHODS IN NUMERICAL ANALYSIS (RCCSMM11)

TWO MARKS.

UNIT I

1. Define algebraic equation.
   ANS: An equation f(x) =0 is called an algebraic equation if the corresponding f(x) is a polynomial in x. **For example: $7x^2+x-8=0$.**
2. Define transcendental equation.
   ANS: An equation f(x) =0 is called a transcendental equation if the corresponding f(x) contains trigonometric, exponential or logarithmic functions.
   For example: log x – cos x =0; e^x+ tan x=0.
3. State the order of convergence and convergence conditions for Newton Raphson method.
   ANS: Order of convergence in Newton Raphson method is **2.**
   The convergence condition is $| f(x).f''(x) | = | (f'(x))^2 |$
4. What is the condition for convergence of iterative method for solving x = ϕ(x)?
   ANS: $|\phi'(x)| < 1$ is the condition for convergence of iterative method.
5. Define absolute error.
   ANS: Absolute error is the numerical difference between the true value of a quantity and its approximate value. Thus if X is the true value of a quantity, and Y is its approximate value, then the absolute error E_A is given by,
   $$E_A = X-Y = \delta.$$
6. What is the order of convergence for fixed point iteration?
   ANS: The convergence is linear and the convergence is of order 1.
7. State the fixed point iteration theorem. (or) If g(x) is continuous in [a,b], then under what condition in [a,b]?
   ANS: Let f(x) = 0 be the given equation whose exact root is α. The equation f(x) =0 can be rewritten as x = φ (x). Let I be the interval containing the root x = α. If φ ' (x) < 1 for all x in I, then the sequence of approximations x_1, x_2 ,.........x_n will converge to α , if the initial starting value x_0 is chosen in I.

8. What are truncation error?

ANS: These errors are caused by using approximate formula in computations such as the one that arises when the function f(x) is evaluated from an infinite series for x after truncating it at a certain stage.

9. Distinguish between direct and iterative (indirect) methods of solving simultaneous equations.

S.No.	Direct method	Iterative method
1.	We get exact solution	Approximate solution.
2.	Simple, take less time	Time consuming laborious

OR

ANS: Direct methods involve a certain amount of fixed computation and they are exact solutions. Iterative or indirect methods are those in which the solution is got by successive approximations. But the method of iteration is not applicable to all systems of equations.

10. What are absolute errors?

ANS: Absolute error is the numerical difference between the true value of a quantity and its approximate value. Thus if x is the true value of a quantity and x_1 is its approximate value, then the absolute error E_A is given by,

$$E_A = X - X_1 = \delta X.$$

11. **RELATIVE ERROR:** The relative error E_R is given by,

$$E_R = E_A / X = \delta / X.$$

12. **PERCENTAGE ERROR:** The percentage error E_P IS given by,

$$E_P = 100 E_R$$

13. Write Newton's formula to find the cube root of N.

ANS: Let $f(x) = x^3 - N$ and $f'(x) = 3x^2$. By Newton method

$$x_{n+1} = x_n - \frac{x_n^3 - N}{3x_n^2} = \frac{2x_n^3 + N}{3x_n^2}$$

14. How to reduce the number of iterations while finding the root of an equation by regular false method?

ANS: The number of iterations to get a good approximation to the real root can be reduced, if we start with a smaller interval for the root.

15. What are the merits of Newton's method of iteration?

ANS: Newton's method is successfully used to improve the result obtained by other methods. It is applicable to the solution of equations involving algebraical functions as well as transcendental functions.

UNIT II

1. State any two properties of divided differences.

ANS: The divided differences are symmetrical in all their arguments. The divided differences of sum or difference of two functions is equal to the sum or difference of the corresponding separate divided differences.

2. Prove that $\Delta = E - 1$

ANS: Consider $\Delta f(x) = f(x+h) - f(x) = Ef(x) - f(x) = (E-1)f(x) \Rightarrow \Delta = E - 1$

3. Prove that $\nabla = 1 - E^{-1}$

ANS: Consider $\nabla f(x) = f(x) - f(x-h) = f(x) - E^{-1}f(x) = (1 - E^{-1})f(x) \Rightarrow \nabla = 1 - E^{-1}$

4. Prove that $\delta = E^{1/2} - E^{-1/2}$

ANS: Consider $$\delta f(x) = f\left(x + \frac{h}{2}\right) - f\left(x - \frac{h}{2}\right) = E^{1/2} f(x) - E^{-1/2} f(x) = (E^{1/2} - E^{-1/2}) f(x)$$

$$\Rightarrow \delta = E^{1/2} - E^{-1/2}$$

5. What are the n^th^ divided difference of a polynomial of n^th^ degree?

ANS: The n^{th} divided differences of a polynomial of n^{th} degree are constant.

6. Show that the divided differences are symmetrical in their argument.

ANS: Consider $f(x_0, x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{f(x_0) - f(x_1)}{x_0 - x_1} = f(x_1, x_0)$. Hence, the divided differences are symmetrical in their arguments.

7. When do we apply Lagrange's Interpolation formula?

ANS: Lagrange's Interpolation formula can be used when the values of x are equally spaced or not. It is mainly used when the values are unevenly spaced.

8. Show that the divided difference operator Δ is linear.

ANS: Consider,

$$\Delta\,(f(x) \pm g(x)) = \frac{[f(x_1) \pm g(x_1)] - [f(x_0) - g(x_0)]}{x_1 - x_0} = \frac{f(x_1) - f(x_0)}{x_1 - x_0} \pm \frac{g(x_1) - g(x_0)}{x_1 - x_0} = \Delta f(x) \pm \Delta g(x).$$

Hence, the operator Δ is linear.

9. What are the advantages of Lagrange's Interpolation formula over Newton's Interpolation formula?

ANS: The forward and backward interpolation formula of Newton can be used only when the values of the independent variable x are equally spaced and can also be used when the differences of the dependent variable y become smaller ultimately. But Lagrange's Interpolation formula can be used whether the values are equally spaced or not and whether the differences of y become smaller or not.

10. What are the disadvantages in practice in applying Lagrange's Interpolation formula?

ANS: - It takes time.

- It is laborious.

11. When is Newton's backward interpolation formula used?

ANS: The Newton's backward interpolation formula is used mainly to interpolate the values of y near the end of a set of tabular values.

12. When is Newton's forward interpolation formula used?

ANS: The Newton's forward interpolation formula is used mainly to interpolate the values of y near the beginning of a set of tabular values.

13. When do we use Newton's divided difference formula?

ANS: The Newton's divided difference formula is used when the datas are unequally spaced.

14. A curve passes through (2, 8) (3, 27) (4, 64) and (5,125). Find the area of the curve between the x-axis and the line x=2, x=5.

ANS: h=1; $\frac{1}{2}[8 + 125 + 2(27 + 64)]$ = 315/2 = 157.5 units.

15. When does Simpson's rule give exact result?

ANS: Simpson's rule gives exact result if the entire function y=f(x) is itself a parabola.

16. Why is Trapezoidal rule so called?

ANS: The Trapezoidal rule is so called because it approximates the integral by the sum of n-trapezoids.

UNIT III

1. State the formula for Trapezoidal Rule of integration.
   ANS: $\int_{x_0}^{x_n} y\,dx = \frac{h}{2}[y_0 + 2(y_1 + \ldots\ldots + y_{n-1}) + y_n]$ is the equation for trapezoidal rule where $\boldsymbol{h = (x_n - x_0)/n}$ **.**
2. State the formula for Simpson's 1/3rd rule of integration.
   ANS: $\int_{x_0}^{x_n} y\,dx = \frac{h}{3}\{y_0 + 4(y_1 + y_3 + y_5 + \ldots\ldots + 2(y_2 + y_4 + y_6 + \cdots \ldots) + y_n\}$ is Simpson's 1/3rd rule formula. Here n should be **even.**
3. State the formula for Simpson's 3/8th rule of integration.
   ANS: $\int_{x_0}^{x_n} y\,dx = \frac{3h}{3}\{y_0 + 3(y_1 + y_2 + y_4 + y_5 \ldots\ldots + 2(y_3 + y_6 + \cdots) + y_n\}$ is Simpson's 3/8th rule formula. Here n should be **multiple of 3.**
4. What is the local error term in Trapezoidal Rule?
   ANS: Principal part of the error in the interval (x_1, x3) is - (h^2/12).y_1" where y_1 is the value of y and y" is the value of the second derivative of y at x=x_1.
5. Compare Trapezoidal rule and Simpson's 1/3rd rule of integration.
   ANS:

TRAPEZOIDAL RULE	***SIMPSON'S 1/3RD RULE***
Number of intervals may be even or odd.	Number of events should be even.
Order of error is h^2	Order of errors is h^4
Principal part of error in the interval (x_1,x_2) is **$-y^{II}$**	Principal part of error in the interval (x_1,x_2) is **$-y^{IV}$**

6. What is the local error term in Simpson's 1/3rd rule?
   ANS: Principal part of the error in the interval (x_1,x_3) is **$-(h^5/90).y^{IV}$** where y_1 is the value of y and y^{IV} is the value of the fourth derivative of y at x=x_1.
7. What is the order of error in Trapezoidal Rule?
   ANS: The order of error in Trapezoidal rule is **h^2.**
8. What is the order of error in Simpson's 1/3rd Rule?
   ANS: The order of error in Trapezoidal rule is **h^4**.
9. What is the order of error in Simpson's 3/8th Rule?
   ANS: The order of error in Trapezoidal rule is **h^5.**
10. Evaluate $\int_1^{1.4} e^{-x^2} dx$ taking h=0.1 by Simpson's 3/8th Rule.
11. Given f (0) = -1, f (1) = 1, f (2) = 4, Find $\int_0^2 f(x)dx$ taking h=1 by Trapezoidal Rule.
12. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by Trapezoidal Rule.
13. Evaluate $\int_0^6 \frac{\mathrm{dx}}{1+\mathrm{x}^2}$ by Simpson's 3/8th Rule.

14. Calculate $\int_{0.5}^{0.7} e^{-x}\sqrt{x}\,dx$ taking 5 ordinates in Trapezoidal Rule.
15. What is the error in Trapezoidal rule?

ANS: E=$\frac{h^3}{12}(y_0'' + y_1'' + \cdots + y_{n-1}'')$ is the error in Trapezoidal rule, Where E is the total error.

16. What is the error in Simpson's 1/3rd Rule?

ANS: E=$\frac{b-a}{180}h^4 y^{IV}(\bar{x})$, where $y^{IV}(\bar{x})$ is the largest value of fourth derivatives.

17. What is the error in Simpson's 3/8th Rule?

ANS: $E = \frac{-3}{80}h^5 y^{IV}(\bar{x})$ is the error in Simpson's 3/8th Rule?

UNIT IV

1. Write a sufficient condition for Gauss-seidal method to converge?

ANS: The process of iteration by Gauss-Jacobi method will converge if in each equation of the system, the absolute value of the largest coefficient is greater than the sum of the absolute values of the remaining coefficients.

2. Compare Gauss elimination and Gauss-seidel methods

ANS: **Gauss elimination method** has only a finite number of computation, since it is a direct method.

Gauss-seidel method converges only when the system is diagonally dominant. Iteration method is a self-correcting method. Errors made at any step in the computation are corrected in the subsequent iteration.

3. Compare Gauss Jacobi and Gauss Seidel methods

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Gauss Jacobi method	Gauss Seidel method
1.Convergence rate is slow	1. The rate of convergence of Gauss. Seidel method roughly twice that of Gauss-Jacobi method.
2.Indirect method	2.Indirect method
3. Condition for convergence is the coefficient matrix is diagonally dominant.	3. Condition for convergence is the coefficient matrix is diagonally dominant.

4. When Gauss-Elimination method fails?

ANS: This method fails if the element in the top of the first column is zero. We can rectify this by interchanging the rows of the matrix.

5. Why Guass Siedal method is better than Guass Jacobi method?

ANS: Since the current values of the unknowns at each stage of proceeding to the next stage of iteration, the convergence in Guass Siedal method will be more rapid than in Guass Jacobi method.

6. Is the iteration method a self-correcting method always?

ANS: In general, iteration is a self-correcting method since the round of error is smaller.

7. Give 2 indirect methods to solve the system of linear equations.
   - Guass Jacobi Method.
   - Guass Siedal Method.

8. Solve the following system of equations by Guass elimination method.
   2x+y=3 ; 7x-3y=4
   ANS: x=1 ; y=1
9. What is meant by diagonally dominant?
   ANS: We say a matrix is diagonally dominant if the numerical value of the leading diagonal element in each row is greater than or equal to the sum of the numerical values of the other elements in that row.
10. Find the inverse of the coefficient matrix by Guass elimination method.
   5x-2y=10; 3x-4y=12
   ANS: $A^{-1} = \begin{bmatrix} 2/13 & 1/13 \\ -3/26 & 5/26 \end{bmatrix}$

UNIT V

1. Formula for Taylor's series method.
   ANS: $\boldsymbol{y_{n+1} = y_1 + \left(\frac{h}{1!}\right) y'_n + \left(\frac{h}{2!}\right) y''_n + \left(\frac{h}{3!}\right) y'''_n} + \cdots \ldots \ldots \ldots$
2. Formula for Piscard's method.
   ANS: $\boldsymbol{y^{(n)} = y_0 + \int_{x_0}^{x} f(x, y^{(n-1)})\, dx}$ with $y^{(0)} = y_0$.
3. Formula for Euler's method.
   ANS: $y_{n+1} = y_n + h\, f(x_n, y_n) \quad n = 0,1,2,3, \ldots \ldots$
4. Formula for Modified Euler's method.
   ANS: $\boldsymbol{y}_1^{n+1} = y_0 + \left(\frac{h}{2}\right) [f(x_0, y_0) + f\ (x_1, y_1^n)]$ n= 0,1,2,3,……
5. Formula for Runge-Kutta 2nd order method.
   ANS: $y_1 = y_0 + \frac{1}{2}(k_1 + k_2)$
   where. $k_1 = h\, f(x_0, y_0)$
   $k_2 = h\, f(x_0 + h, y_0 + k_1)$
   and, $y_2 = y_1 + \frac{1}{2}(k_1 + k_2)$
   where, $k_1 = h\, f(x_1, y_1)$
   $k_2 = h\, f(x_1 + h, y_1 + k_1)$
   where $x_1 = x_0 + h$.
6. Formula for Runge-Kutta 4th order method.
   ANS: $y_1 = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$
   where, $k_1 = h\, f(x_0, y_0)$
   $k_2 = h\, f(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2})$
   $k_3 = h\, f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$
   $k_4 = h\, f(x_0 + h, y_0 + k_3)$
7. Adam's Bashforth Predictor Corrector formula.
   ANS: ADAM'S PREDICTOR FORMULA IS, $\boldsymbol{y_{4,p} = y_4 + \frac{h}{24}[55y'_3 - 59y'_2 + 37y'_1 - 9y'_0]}$
   ADAM'S CORRECTOR FORMULA IS, $\boldsymbol{y_{4,c} = y_3 + \frac{h}{24}[9y'_4 + 19y'_3 - 5y'_2 + y'_1]}$

8. Milne's predictor corrector formula.

ANS: MILNE'S PREDICTOR FORMULA IS, $y_4^p = y_0 + \frac{4h}{3}[2y_1' - y_2' + 2y_3']$

MILNE'S CORRECTOR FORMULA IS, $y_4^c = y_2 + \frac{h}{3}[y_2' + 4y_3' + y_4']$

9. State the disadvantage of Taylor's series method.

ANS: Consider the differential equation $\frac{dy}{dx} = f(x,y)$.

In this the function f(x,y) may have complicated algebraical structure so that the evaluation of higher order derivatives may become tedious. This is the disadvantage of Taylor's series method

10. Which is better? Taylor's series method or R-K Method?

ANS: R-K method is better than Taylor's series method since R-K methods do not require prior calculation of higher derivatives of y(x) as the Taylor's series does.

11. Limitations of Euler's method.

ANS: I. The attainable accuracy is limited by length of step h.

II. The method is slow and has limited accuracy.

12. What is the error of Euler's method?

ANS: The error at $X=X_1$ is $\frac{h^2}{2}\left[y''(x_1, y_1)\right]$

Hence, error is of order $h^{2.}$

13. What is the error in modified Euler's method?

ANS: The error in modified Euler's method is $\frac{h^2}{12} * constant$.

Error is of order h^3.

14. How many prior values are required to predict the next value in Milne's method and Adam's method?

ANS: Four prior values are required to predict the next value in Milne's method and Adam's method.

15. What is the error term in Milne's corrector formula?

ANS: The error term in Milne's corrector formula is $\frac{-h}{90}\Delta^4 yo$

16. What is a predicator corrector method of solving a differential equation?

ANS: The predictor corrector methods are the methods which require the value of y at X_n, X_{n-1},….. for computing the value of y at $X_{n+1.}$ We first use a formula to find the value of y at X_{n+1} and this is known as a predictor formula. The value of y so got is improved or corrected by another formula known as corrector formula.

17. Compare R-K methods and Predictor-Corrector methods.

ANS: The R-K methods are self-starting since they do not use information from previously calculated points whereas the predictor corrector methods require information about prior points and so, they are not self-starting. In R-K methods, it is not [possible to get any information about truncation errors and in predictor corrector methods, it is not possible to get a good estimate of the truncation error.

18. Is Euler's modified formula a particular case of 2^{nd} order R-K methods?

ANS: Yea. The modified Euler's method is a particular case of 2^{nd} order R-K methods.