ST. THERESA'S ARTS AND SCIENCE COLLEGE FOR WOMEN THARAGAMBADI

HOME TEST/

Semester IV

Sub: Linear Algebra(16SCCMM8) Class: II B.Sc Mathematics

Part A(5 x 2= 1 0)

Answer All the questions

1. Define eigen values and eigen vectors of linear transformation?
2. Define unitary matrix and give an example
3. Define vector space V over a field of F.
4. Define homomorphism of a vector space.
5. Define inner product space
6. Let $T \in A(V) and\ \lambda \epsilon F$. If λ is an eigen value of T. Prove that $\lambda I - T$ is an singular.
7. Show that the matrix $\begin{pmatrix} Cos\theta & Sin\theta \\ -Sin\theta & Cos\theta \end{pmatrix}$
8. Define basis of a vector space.
9. Define skew symmetric matrix
10. Define kernel of vector space.

Part B(5 x 5= 25)

Answer All the questions

11. (a) prove that the intersection of two subspaces of a vector space is a subspace. **(OR)**

    (b). $T: R^2 \to R^2$ defined by $T(a, b) = (2a - 3b, a + b)$. prove that T is a linear transformation.

12. (a). S.T a square matrix A is orthogonal , iff $A^{-1} = A^T$. **(OR)**

    (b).Reduce the matrix A=$\begin{bmatrix} 1 & 2 & -1 \\ 1 & 1 & 2 \\ 2 & 4 & -2 \end{bmatrix}$ to the canonical form.

13. (a). state and prove cauley Hamiltan theorem. **(OR)**

    (b) Find the sum of square matrix of the eigen values of A=$\begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$

14. (a).Verify the Cayley Hamilton theorem A=$\begin{pmatrix} 1 & 3 & 2 \\ 0 & 1 & -1 \\ 0 & -1 & -2 \end{pmatrix}$. **(OR)**

    (b). Let V be the vector space of dimension n, and let $\{v_1, v_2, \ldots . v_r\}$ be the linearly independent vectors in V. P.T. there exist n-r new vectors $v_{r+1},\ v_n\ in\ V$ such that $v_1, v_2, \ldots . v_n$ basis of V.

15. (a). P.T. the characteristic root of a Hermitan matrix are all real. **(OR)**

(b). Let A and B are two square matrix of the same order . Prove That AB and BA have the same eigen value.

Part C(3 x 10= 30)

Answer THREE the questions

16. Let V be the vector space over F and W be the subspace of V. Let V/W is a vector space over F under the following operation. (a). $(W + v_1) + (W + v_2) = W + v_1 + v_2$ (b). $\alpha(W + v_1) = W + \alpha v_1$.

17. Let V be the vector space over a field F. Let $S = \{v_1, v_{2,\ldots} v_n\} \subseteq V$. Prove that the following are equivalent. (a). S is basis for V (b). S is maximal linearly independent set. (C). S is minimal generating set.

18. Find the eigen values and eigen vectors of the Matrix A=$\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$

19. Prove that V is a vector space of finite dimension and W is a subspace of V. then $dim \frac{V}{W} = \dim V - \dim W$.

20. If A and B the subspace of a vector space on V onto a field F. Show that $\frac{A+B}{B} = \frac{A}{A \cap B}$