Electricity and Magnetism

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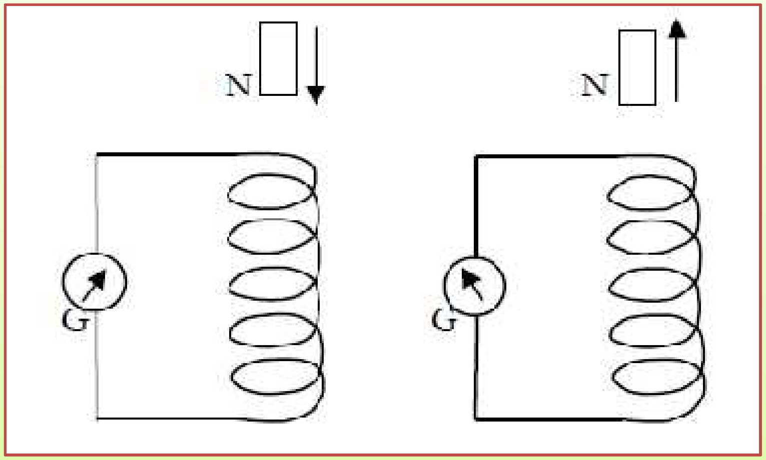
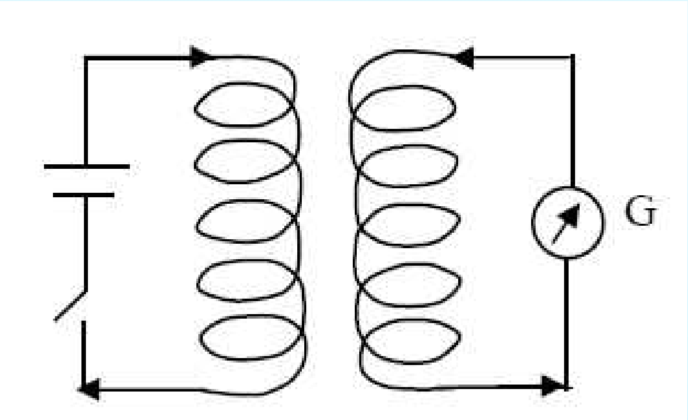
Electromagnetic induction

The current produced in the conductor in this way is called the induced current and the e.m.f that causes this current is known as induced e.m.f. This phenomenon is known as electromagnetic induction.

The Electromagnetic induction is the property of the coil by virtue of which it possess the growth and decay of current





LAWS OF ELECTROMAGNETIC INDUCTION

Faraday's First Law

Whenever the magnetic flux associated with any closed circuit changes, an induced current flows through the circuit which lasts only so long as the change lasts.

Faraday's Second Law

The magnitude of the induced e.m.f produced in a coil is directly proportional to the rate of change of the magnetic flux through the coil.

$$e \propto \frac{d\phi}{dt}$$

Here ϕ is the magnetic flux through the circuit.

$$\therefore \text{Induced e.m.f,} \quad e = -\frac{d\phi}{dt}.$$

Fleming's Right Hand Rule

The direction of the induced e.m.f. and current can be determined by Fleming's right hand rule. *Stretch the thumb, the fore finger and the central finger mutually perpendicular to one another. If the thumb represents the direction of the motion of the conductor and the forefinger the direction of the magnetic field, then central finger points the direction in which current is induced in the circuit.*

3.3. SELF INDUCTION

When a current flows through a coil, a magnetic field is produced around the coil. If there is a change in current flowing through the coil, there is a change in magnetic flux linked with the coil. Due to this an induced e.m.f. is produced. *Thus a varying current flowing through a coil, induces an e.m.f in the same coil. This phenomenon is known as* ***self induction.***

Self inductance

The magnetic flux ϕ linked with a coil is proportional to the current I flowing through the coil.

$$\phi \propto i$$

Or $$\phi = L\, i \qquad \text{----- (1)}$$

Where L is a constant known as self inductance of the coil.

The self inductance or coefficient of self induction of a coil is defined as the magnetic flux linked with the coil when unit current flows through it.

From equation (1), $$\frac{d\phi}{dt} = L\frac{di}{dt}$$

But induced e.m.f. $$e = -\frac{d\phi}{dt}$$

$$\therefore \qquad e = -L\frac{di}{dt} \qquad \text{----- (2)}$$

3.4. SELF INDUCTANCE OF A LONG SOLENOID

Consider a long air-cored solenoid of length l, total number of turns N, and area of cross-section A. The number of turns per unit length is N/l. Let a current I flow through it. Then the magnetic field inside the solenoid,

$$B = \mu_0 \left(\frac{N}{l}\right) I$$

Therefore flux through each turn $= \phi_B = BA = \frac{\mu_0 NIA}{l}$

Total flux through N turns

$$\phi = N\phi_B = N \times \frac{\mu_0 NIA}{l} = \frac{\mu_0 N^2 IA}{l}$$

Self-inductance of the solenoid, $L = \frac{\phi}{I} = \frac{\mu_0 N^2 A}{l}$

If the solenoid is wound on a core of constant permeability μ, then

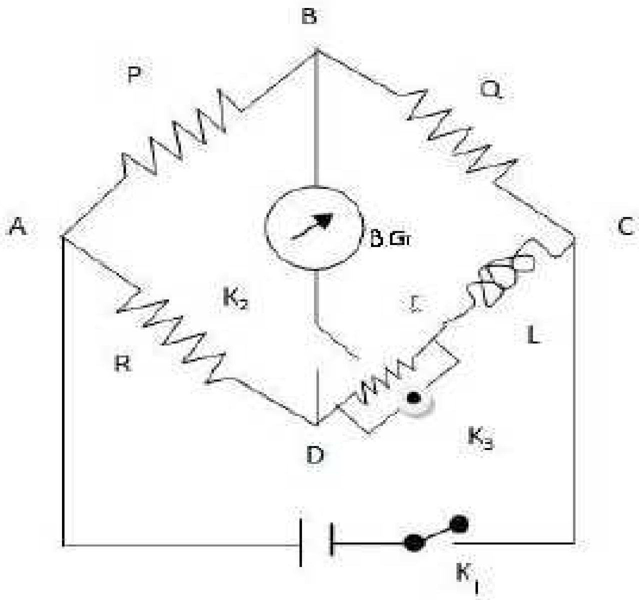
$$L = \frac{\mu N^2 A}{l}$$

Where $\mu = \mu_0\mu_r$

In general, if there is a core consisting of number of media of relative permeabilities, μ_{r1}, μ_{r2}, ...etc., and areas of cross-section, A_1, A_2,.......etc., then,

$$L = \frac{\mu_0 N^2}{l} [\mu_{r1}A_1 + \mu_{r2}A_2 + ...]$$

3.8. DETERMINATION OF SELF INDUCTANCE – RALEIGH'S METHOD



(a) Initially K_3 is kept closed. The ohmic resistance S of the inductance coil alone is included in the fourth arm. P is made equal to Q. Then R is adjusted for no deflection in the B.G., by first pressing battery key K_1 and then galvanometer key K_2. Under this condition, no current flows through the galvanometer.

(b) If now the galvanometer key K_2 is closed first and then the battery key K_1, then a throw θ_1 is observed in the galvanometer. This throw arises due to an extra emf $L\frac{di}{dt}$ induced in the coil while the current is growing.

If G is galvanometer resistance, then current through it due to induced emf is

$$i' = \frac{kL}{G}\frac{di}{dt}$$

where k is a constant which depends upon the relative resistance in the circuit.

Hence the total charge passing through the galvanometer, as the current in the coil grows from zero to a steady maximum value i_0 is given by

$$q = \int_0^{i_0} i'\,dt = \frac{kL}{G}\int_0^{i_0}\frac{di}{dt}\,dt = \frac{kL}{G}i_0 \qquad \text{----- (1)}$$

If θ_1 be the first throw of the galvanometer, then

$$q = K\,\theta_1\left(1+\frac{\lambda}{2}\right) \qquad \text{----- (2)}$$

$$\therefore \quad \frac{kL}{G}i_0 = K\,\theta_1\left(1+\frac{\lambda}{2}\right) \qquad \text{(since from Eqn. 1 and 2)}$$

$$\frac{kL}{G}i_0 = \left(\frac{T}{2\pi}\right)\left(\frac{c}{NBA}\right)\theta_1\left(1+\frac{\lambda}{2}\right) \qquad \text{----- (3)}$$

(c) To eliminate k and i_0, the key K_3 is opened and the resistance r is included in the arm CD. As r is small, it does not affect the current i_0 in the arm CD appreciably. But it will introduce an additional emf ri_0 in the arm CD. This causes a steady current $(kr/G)i_0$ through the galvanometer. K_1 is closed first and then K_2. The steady deflection ϕ in the galvanometer is noted. Then,

$$\frac{kL}{G} i_0 = \left(\frac{c}{NBA}\right)\phi \qquad \text{----- (4)}$$

Here, $\left(\frac{c}{NBA}\right)$ is the current reduction factor of the galvanometer.

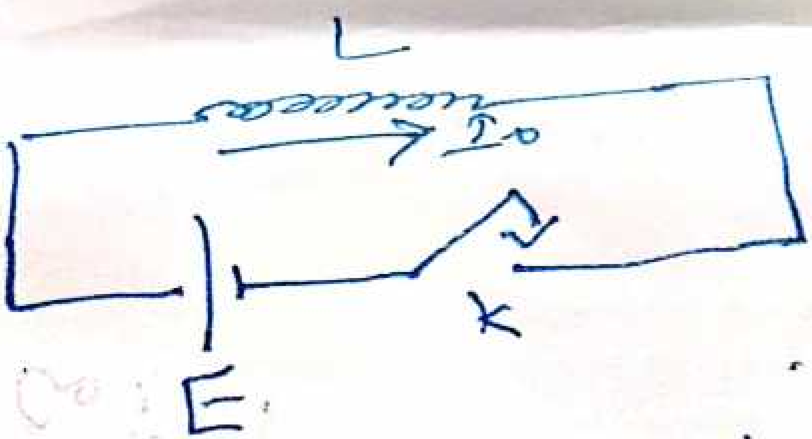
Dividing Eqn. (3) by (4), we get

$$\frac{L}{r} = \left(\frac{T}{2\pi}\right)\frac{\theta_1}{\phi}\left(1+\frac{\lambda}{2}\right)$$

$$L = \left(\frac{Tr}{2\pi}\right)\frac{\theta_1}{\phi}\left(1+\frac{\lambda}{2}\right) \qquad \text{----- (5)}$$

Knowing the values of T, r, θ_1 and ϕ we have to find out self inductance L value.

Energy Stored in Inductance coil..



Let a coil of self inductance L be connected ~~which~~ in series with a cell of emf 'E' and a tap key K.

When K is closed, the current is started from cell and passing through 'L', while current flowing through 'L', the magnetic flux through the coil grows from zero to maximum value I_0 at the given time i.e.

... the coil at any instant

The emf of the [illegible]

$\cancel{e}$ $e = -L \cdot \frac{dI}{dt}$

L - inductance $\frac{dI}{dt}$ - current flow

the current flow through coil at given

time t $\Rightarrow I \cdot dt = -L \cdot \frac{dI}{dt} \cdot I \cdot dt$

$= -L \cdot I \cdot dI$

this work is supplied from the cell.
Hence total work done when the current in the coil increases from zero to I_0 is

$$W = \int_0^{I_0} L \cdot I \cdot dI = \frac{1}{2} L \cdot I_0^2$$

Hence work done is amount of energy transformed $W = \Delta E$

$$\Delta E = W$$

$$\Delta E = \frac{1}{2} L \cdot I_0^2$$

Therefore the work done to increase the current in the coil is the amount of energy stored in the coil.

3.5. MUTUAL INDUCTION

Consider two coils placed close to one another. When a current is passed through the coil-1, magnetic flux is produced. A part of the flux passed through the coil-2. If the current through the coil-1 is changed, the flux through the coil-2 also changes. Due to this change in flux, an e.m.f is induced in coil-2. *The phenomenon of the production of an e.m.f in one coil when the current changes in another coil is called mutual induction.*

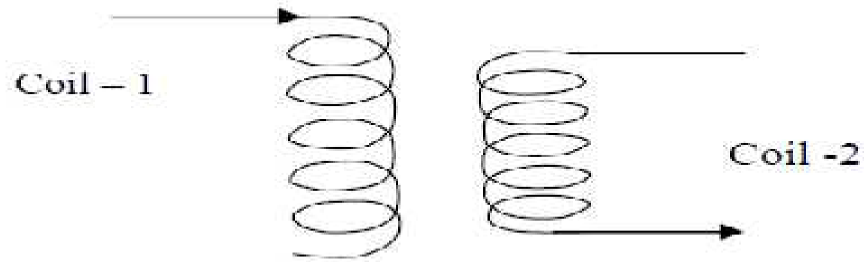


Fig. 3.3

The magnetic flux ϕ linked with the coil-2 is directly proportional to the current i flowing through the coil-1.

$$\phi \propto i$$

Or $$\phi = Mi \qquad \text{----- (1)}$$

where M is a constant called coefficient of mutual inductance.

If i=1 ampere, then ϕ = M. From this we can define the mutual inductance. *The mutual inductance of a gives pair of coil is numerically equal to the magnetic flux linked with one coil when the current through the other coil is unity.*

From equation (1), $\frac{d\phi}{dt} = M\frac{di}{dt}$

But induced e.m.f, $e = -\frac{d\phi}{dt}$

$$\therefore \quad e = -M\frac{di}{dt} \qquad \text{----- (2)}$$

Equation (2) is the relation between induced e.m.f and mutual inductance.

If d*i*/dt=1 amp/sec, then e= - M. From this we can define the mutual inductance. The mutual inductance of given pair of coil is numerically equal to the e.m.f induced in one coil when the current through the other coil changes at the rate of one ampere per second.

Mutual inductance is measured in the unit of Henry. In equation (2) if di/dt=1 ampere/second and e=1 volt, then M=1 Henry.

The mutual inductance of the given pair of coil is 1 Henry, if an e.m.f of 1 volt is induced in one coil when the current in the other coil changes at the rate of 1 ampere per second.

3.6. MUTUAL INDUCTANCE OF A PAIR OF SOLENOIDS

Consider a long air-cored solenoid with primary and secondary. Let N_1 and N_2 number of turns in primary and secondary respectively. A is the area of cross-section, l is the length of the primary and I be the current in the primary.

Then the magnetic field at any point inside the primary,

$$B = \frac{\mu_0 N_1 I}{l}$$

Magnetic flux through each turn of the primary = BA = $\frac{\mu_0 N_1 I}{l} A$

Since the secondary is wound closely over the central portion of the primary, the same flux is also linked with each turn of the secondary.

Magnetic flux through each turn of the secondary = $\frac{\mu_0 N_1 I}{l} A$

Total magnetic flux through N_2 turns of the secondary is

$$\phi = \frac{\mu_0 N_1 N_2 A I}{l}$$

Therefore mutual-inductance of the solenoid, $M = \frac{\phi}{I} = \frac{\mu_0 N_1 N_2 A}{l}$

If the core is material of permeability μ_r, then

$$M = \frac{\mu_0 \mu_r N_1 N_2 A}{l}$$

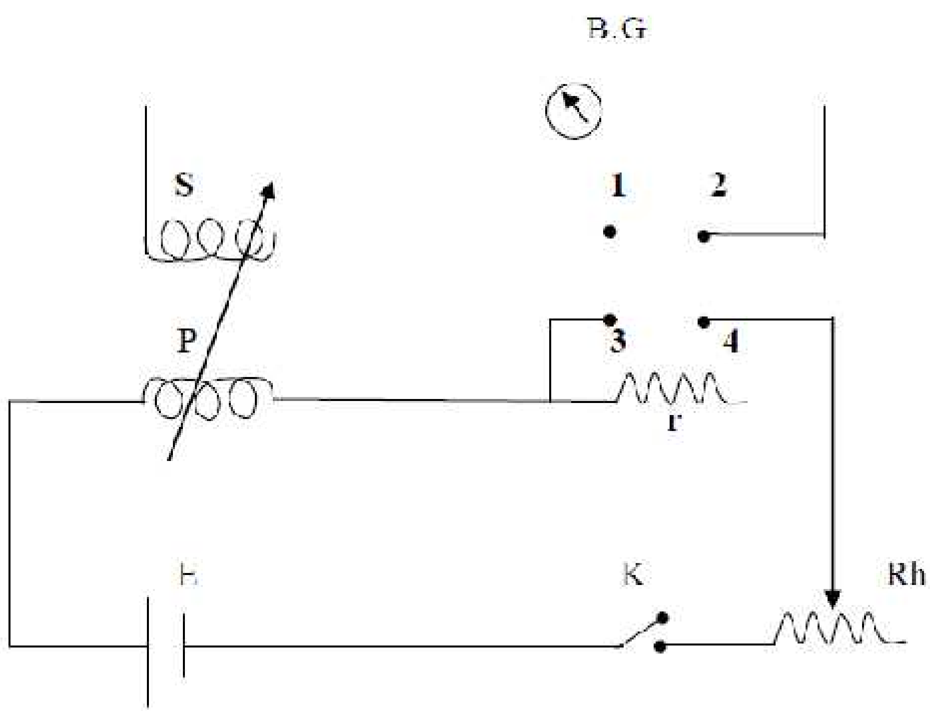
If there are a number of cores of areas of cross-section A_1, A_2,.......etc., and relative permeabilities, μ_{r1}, μ_{r2}, ...etc.

$$M = \frac{\mu_0 N_1 N_2}{l}[\mu_{r1} A_1 + \mu_{r2} A_2 + \text{..........}]$$

3.9. DETERMINATION OF MUTUAL INDUCTANCE

The circuital arrangement for the determination of mutual inductance between two coils P and S is shown in figure. 3.6. The primary P of the pair of coils is joined in series with a battery, a rheostat, a low resistance r and a key K. The secondary S of the pair of coils is joined to the terminal of a B.G through the segment 1 and 2 of a quadrant key C. The ends of r are connected to the segments 3 and 4 of the quadrant key.

First the segment 1 and 2 are connected together so that the B.G and the secondary coil S may form closed circuit. The segment 3 and 4 are connected together to short circuit the resistance r. The rheostat is adjusted so that a suitable current passes through the primary on pressing the key K.



When the key K is pressed, the current in the primary takes some time to reach maximum value. During this time, the flux linked with the secondary coil changes. Hence an induced e.m.f is produced in the secondary and a momentary current flows in the secondary due to which B.G gives throw.

If i is the instantaneous current in the primary, the e.m.f induced in the secondary is given by

$$e = -M\frac{di}{dt} \qquad \text{----- (1)}$$

If R is the total resistance of the secondary circuit, then the instantaneous current in the secondary is given by

$$i' = \frac{e}{R} = \frac{M}{R}\frac{di}{dt} \qquad \text{----- (2)}$$

This current flow through the B.G connected in the secondary circuit. The current in the primary grows from zero to steady value i_o. During this time interval t, the total charge passes through B.G is given by

$$q = \int_0^t i'\,dt = \frac{M}{R}\int_0^t \frac{di}{dt}\,dt = \frac{M}{R}\int_0^{i_0} di$$

$$\therefore \quad q = \frac{Mi_0}{R} \qquad \text{----- (3)}$$

Due to this charge, there is a throw in the B.G. Let it be θ_1

$$\therefore \quad q = \left(\frac{T}{2\pi}\right)\left(\frac{c}{NBA}\right)\theta_1\left(1+\frac{\lambda}{2}\right) \qquad \text{----- (4)}$$

Comparing equation (3) and (4), we get

$$\frac{Mi_0}{R} = \left(\frac{T}{2\pi}\right)\left(\frac{c}{NBA}\right)\theta_1\left(1+\frac{\lambda}{2}\right) \qquad \text{----- (5)}$$

To determine M, we have to eliminate i_o and C/NBA from equation (5). For this the contact between 1 and 2, and 3 and 4 is broken and the segments 1 and 3, and 2 and 4 are connected. Now the resistance r is included in the primary. The same steady current i_o is now passed in the primary circuit. As the value of resistance r is very small, it does not affect appreciable the current i_o in the primary circuit. The p.d across the resistance r is $i_o r$. This extra p.d sends a steady current $i_o r/R$ through the B.G. If θ_0 is the steady deflection of the galvanometer due to this current, then

$$\frac{r i_0}{R} = \left(\frac{c}{NBA}\right) \theta_0 \qquad \text{----- (6)}$$

Dividing Eqn.(5) by (6), we get

$$\frac{M}{r} = \left(\frac{T}{2\pi}\right) \frac{\theta_1}{\theta_0} \left(1 + \frac{\lambda}{2}\right)$$

$$\therefore \qquad M = \left(\frac{Tr}{2\pi}\right) \frac{\theta_1}{\theta_0} \left(1 + \frac{\lambda}{2}\right) \qquad \text{----- (7)}$$

Using equation (7), mutual inductance M can be calculated.