

This is the relation between the three vectors  $\mathbf{B}$ ,  $\mathbf{H}$  and  $\mathbf{M}$ .  $\mathbf{H}$  points in the same direction as that of  $\mathbf{B}$  or  $\mathbf{M}$ . Its unit is  $\text{Am}^{-1}$ .

$$\text{Eq. (1) can be written as } H = \frac{Ni_0}{l} = ni_0$$

where  $n$  is the number of turns per unit length. Thus the value of  $H$  depends only on the free current and is independent of the core material.

When no magnetic material is present in the core of the Rowland ring, *i.e.*, there is vacuum in the core,  $\mathbf{M} = 0$ . Therefore Eq. (2) becomes

$$B_0 = \mu_0 H \quad \dots(3)$$

In vacuum, the magnetic field strength  $\mathbf{H}$  is related to the magnetic induction  $\mathbf{B}_0$  by the above relation.

When a magnetic material is placed in an external magnetic field, the specimen is magnetised by producing (or reorienting) magnetic dipoles in the specimen. This will produce additional field. Thus the resultant field  $\mathbf{B}$  is greater than  $\mathbf{B}_0$ . In such a case,  $\mathbf{H}$  is related to  $\mathbf{B}$  by the relation,

$$\mathbf{H} = (\mathbf{B}/\mu_0) - \mathbf{M}.$$

#### 14.4 MAGNETIC SUSCEPTIBILITY

For isotropic linear para- and diamagnetic materials it is found experimentally that the magnetisation  $\mathbf{M}$  is proportional to the magnetic field intensity  $\mathbf{H}$ . That is,

$$\mathbf{M} \propto \mathbf{H} \quad \text{or} \quad \mathbf{M} = \chi_m \mathbf{H}$$

The constant  $\chi_m$  is called the *magnetic susceptibility* of the material. It may be defined as the ratio of the magnetisation  $\mathbf{M}$  to the magnetic field intensity  $\mathbf{H}$ .

$$\chi_m = \mathbf{M}/\mathbf{H}$$

Therefore, the magnetic susceptibility of a material is defined as the intensity of magnetisation acquired by the material for unit field strength.

**Definition.** *Magnetic susceptibility ( $\chi_m$ ) of a material is the ratio of the intensity of magnetisation ( $M$ ) produced in the sample to the magnetic field intensity ( $H$ ) which produces the magnetisation.*

$$\chi_m = \frac{M}{H}$$

- It has no units

We can classify magnetic materials in terms of susceptibility  $\chi_m$ . If  $\chi_m$  is positive, the material is called *paramagnetic*. If  $\chi_m$  is negative, the material is *diamagnetic*. The characteristic of ferromagnetic materials is that  $\chi_m$  is positive and very large. However, in ferromagnetic materials,  $\mathbf{M}$  is not exactly proportional to  $\mathbf{H}$ , and so  $\chi_m$  is not a constant.  $\mathbf{M}$  may even be finite when  $\mathbf{H} = 0$ .

In most hand books on physical data  $\chi_m$  is not listed directly, but instead is given as the *mass susceptibility*  $\chi_{m, \text{mass}}$  or *molar susceptibility*,  $\chi_{m, \text{molar}}$ . These are defined by,

$$\chi_m = \chi_{m, \text{mass}} \rho$$

$$\chi_m = \chi_{m, \text{molar}} \frac{\rho}{A}$$

Here,  $\rho$  and  $A$  are the density and molecular weight of the material respectively. Here  $\chi_{m, \text{mass}}$ , and  $\chi_{m, \text{mol}}$  known as volume, mass and molar susceptibilities respectively.

### 14.5 MAGNETIC PERMEABILITY

Consider the relation

$$\begin{aligned} \mathbf{B} &= \mu_0(\mathbf{H} + \mathbf{M}) \\ &= \mu_0(\mathbf{H} + \chi_m \mathbf{H}) \\ &= \mu_0(1 + \chi_m) \mathbf{H} \\ &= \mu \mathbf{H} \end{aligned}$$

Here,  $\mu = \mu_0(1 + \chi_m)$ , is called the *magnetic permeability* of the material.

$\therefore$  *Magnetic permeability ( $\mu$ ) of a medium is defined as the ratio of magnetic induction to the intensity of the magnetising field.*

$$\mu = \frac{B}{H}$$

For vacuum  $\chi_m = 0$  and  $\mu = \mu_0$

Hence magnetic induction in vacuum is  $B_0 = \mu_0 \mathbf{H}$ .

The ratio 
$$\frac{B}{B_0} = \frac{\mu}{\mu_0}$$

is called the *relative permeability*  $\mu_r$ . Obviously  $\mu_r = 1 + \chi_m$ .

**Establish the relation  $\mu_r = 1 + \chi_m$**

When a specimen is placed in a magnetising field  $H$ , it is magnetised due to alignment of current loops. The magnetic flux density within the specimen is the resultant of magnetising field and the magnetic field induced (*i.e.*, intensity of magnetisation).

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) \quad \dots(1)$$

We have

$$\mathbf{B} = \mu \mathbf{H} \quad \dots(2)$$

Here,  $\mu$  is called the magnetic permeability of the material.

$$\therefore \mu \mathbf{H} = \mu_0(\mathbf{H} + \mathbf{M})$$

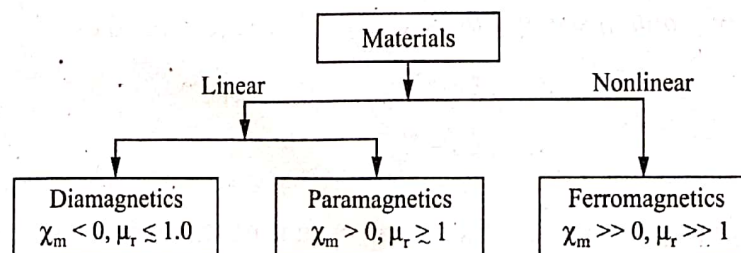
or 
$$\mu = \mu_0 \left( 1 + \frac{\mathbf{M}}{\mathbf{H}} \right)$$

or 
$$\mu = \mu_0(1 + \chi_m)$$

or 
$$\mu/\mu_0 = 1 + \chi_m$$

*i.e.*, 
$$\mu_r = 1 + \chi_m \quad \dots(3)$$

We use the magnetic susceptibility  $\chi_m$  or the relative permeability,  $\mu_r$  to classify materials in terms of their magnetic property or behaviour (Fig. 14.3).



**FIG. 14.3** Classification of Materials.



A material is said to be *nonmagnetic* if  $\chi_m = 0$  (or  $\mu_r = 1$ ); it is magnetic otherwise. Free space and air are regarded as nonmagnetic.

We may also classify magnetic materials in terms of the relative permeability  $\mu_r$ ,

*Diamagnetism:*  $\mu_r < 1$  (For Bi,  $\mu_r = 1 - 0.00017$ )

*Paramagnetism:*  $\mu_r > 1$  (For Al,  $\mu_r = 1 + 0.00002$ )

*Ferromagnetism:*  $\mu_r \gg 1$  (For Fe,  $\mu_r = 200,000$ )

**Example 1.** A rod of magnetic material, 0.5 m in length has a coil of 200 turns wound over it uniformly. If a current of 2 ampere is sent through it, calculate (a) the magnetising field  $H$ , (b) the intensity of magnetisation  $M$ , (c) the magnetic induction  $B$  and (d) the relative permeability  $\mu_r$  of the material. Given  $\chi_m = 6 \times 10^{-3}$

**Solution.** Here,  $N = 200$ ,  $l = 0.5$  m,  $i = 2$  A.

$$(a) H = \frac{Ni}{l} = \frac{200 \times 2}{0.5} = 800 \text{ Am}^{-1}.$$

$$(b) M = \chi_m H = (6 \times 10^{-3}) \times 800 = 4.8 \text{ Am}^{-1}.$$

$$(c) B = \mu_0 (H + M) = 4\pi \times 10^{-7} (800 + 4.8) = 1.08 \times 10^{-3} \text{ Wb m}^{-2}$$

$$(d) \mu_r = \frac{B}{\mu_0 H} = \frac{1.08 \times 10^{-3}}{(4\pi \times 10^{-7}) \times 800} = 1.006$$

**Example 2.** An iron rod 0.2 m long, 10 mm in diameter and of relative permeability 1000 is placed inside a long solenoid wound with 300 turns/metre. If a current of 0.5 ampere is passed through the rod, find the magnetic moment of the rod.

**Solution.**  $n =$  number of turns/metre = 300,  $i = 0.5$  A

$\therefore$  magnetic field intensity =  $H = ni = 300 \times 0.5 = 150$  ampere turns/metre

$$M = \frac{B}{\mu_0} - H = \frac{\mu H}{\mu_0} - H = \mu_r H - H = (\mu_r - 1) H$$

$$= (1000 - 1) 150 = 149850 \text{ ampere turns/metre}$$

Volume of the rod =  $V = \pi r^2 l$

$$= 3.142 \times (5 \times 10^{-3})^2 \times 0.2 \text{ m}^3 = 1.57 \times 10^{-5} \text{ m}^3$$

Magnetic dipole moment of the rod

$$m = M \times V = 149850 \times (1.57 \times 10^{-5}) = 2.353 \text{ ampere meter}^2$$

## 14.6 DIAMAGNETISM

- The individual atoms of a diamagnetic material do not possess a permanent magnetic moment (Fig. 14.4)
- When an external magnetic field  $H_0$  is applied, the atoms acquire a small induced magnetic moment in a direction opposite to the direction of applied field (Fig. 14.5). The strength of the induced magnetic moment is directly proportional to the applied field  $H_0$ . The induced dipoles and magnetization vanish as soon as the applied magnetic field is removed.
- Diamagnetism is a property of all atoms because of the influence of an applied magnetic field on the motion of electrons in their orbits.

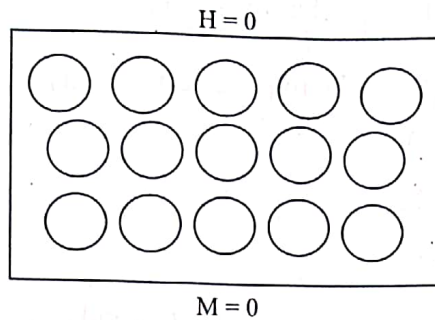


FIG. 14.4

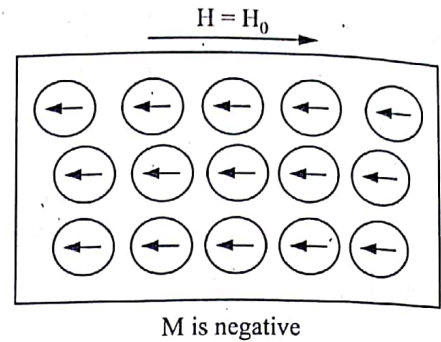


FIG. 14.5

**Properties of diamagnetic materials**

- (1) Permanent dipoles are absent. There is no permanent dipole moment.
  - Antimony, bismuth, mercury, gold and copper are some examples of diamagnetic substances.
- (2) Figure 14.6 shows a bar of diamagnetic material placed in an external magnetic field. The magnetic lines of force are repelled or expelled. The field inside the material is reduced.

Figure 14.7 shows the behaviour of a perfect diamagnetic material in the presence of the magnetic field.

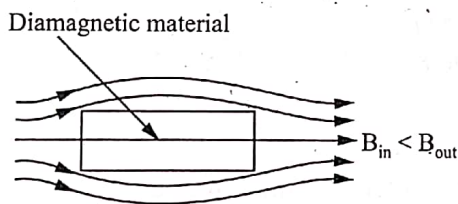


FIG. 14.6

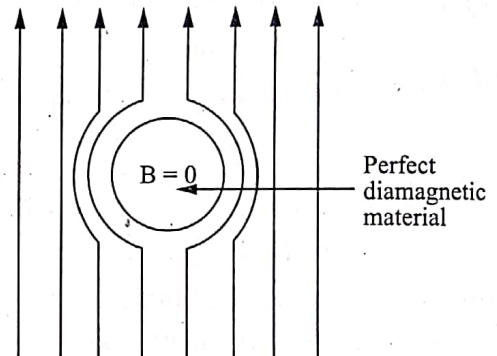


FIG. 14.7

- Diamagnetic materials repel the magnetic lines of force.
  - The magnetic flux density  $B$  is less inside than outside.
- (3) The magnetic susceptibility is negative ( $\chi_m < 0$ ), i.e., magnetisation opposes the applied field. Magnetic susceptibility is independent of temperature and applied magnetic field strength.
  - (4) Relative permeability is slightly less than unity ( $\mu_r < 1$ ).
  - (5) If suspended freely, they set themselves  $\perp$  to the field (Fig. 14.8).

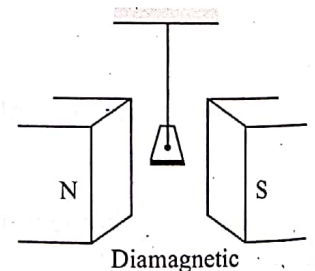


FIG. 14.8

**14.7 PARAMAGNETISM**

- (i) In the absence of external magnetic field. The individual atoms of paramagnetic material possess a permanent magnetic dipole moment of their own. (Fig. 14.9).
  - Each atom possesses a permanent magnetic moment.
  - When  $H = 0$ , all the magnetic moments are randomly oriented because of the ceaseless random thermal motion of the atoms. So the net magnetization  $M = 0$ .

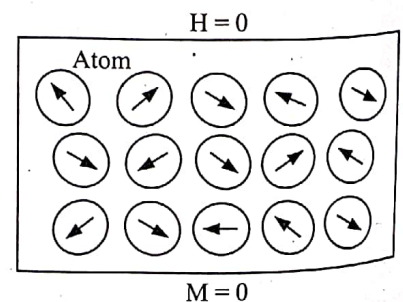


FIG. 14.9



(ii) **When an external magnetic field is applied.** When an external magnetic field  $H_0$  is applied, the magnetic dipoles tend to align themselves in the direction of the magnetic field (Fig. 14.10). The individual atomic dipole moments point in the same direction. The material becomes magnetized. This effect is called *paramagnetism*.

- Magnetisation  $M$  and magnetic field  $H$  are in the same direction. Since  $\chi_m = M/H$ , the susceptibility  $\chi_m$  is positive.
- **Examples of paramagnetic materials:** Platinum, aluminium, ferric oxide, ferrous sulphate, nickel sulphate, etc.

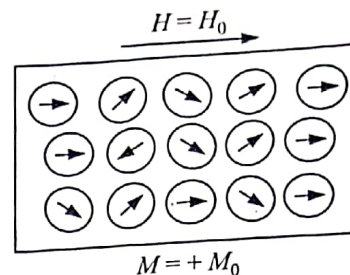


FIG. 14.10

### Properties of Paramagnetic Materials

- (1) Paramagnetic materials possess permanent magnetic dipoles.
- (2) In the absence of an external applied field, the dipoles are randomly oriented. Hence the net magnetization in any given direction is zero.
- (3) When placed inside a magnetic field, it attracts the magnetic lines of force (Fig. 14.11). The field lines get concentrated inside the material, and the field inside is enhanced. This enhancement is slight, being one part in  $10^5$ .
- (4) Paramagnetic susceptibility is positive and depends on temperature.

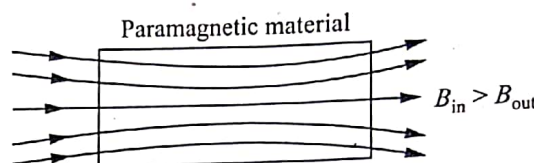


FIG. 14.11

- $\chi_m = \frac{C}{T}$  is Curie's law.
- $\chi_m = \frac{C}{T - \theta}$  is called *Curie-Weiss Law*

Here,  $C$  is *Curie constant* and  $\theta$  is a constant called *paramagnetic Curie temperature*.

- (5) The value of the paramagnetic susceptibility is independent of the applied magnetic field strength.
- (6) Spin alignment is random (Fig. 14.12).
- (7) When a rod is suspended in a magnetic field, the rod becomes parallel to the field if it is paramagnetic or ferromagnetic (Fig. 14.13).

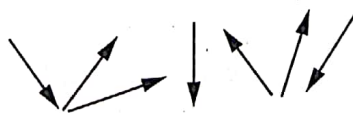


FIG. 14.12

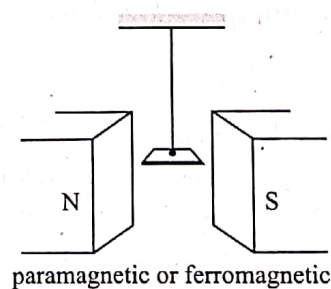


FIG. 14.13

## 14.8 FERROMAGNETISM

- Ferromagnetism is the existence of a *spontaneous magnetization*, even in zero applied field. Ferromagnetic materials have a small amount of magnetisation event in the absence of an external magnetic field. This indicates that there is a strong internal field within the material which makes the atomic magnetic moments align with each other.
- When placed in a magnetic field, ferromagnetic materials become strongly magnetized in the direction of the applied field. The direction of magnetization is the same as that of the external field.

**Origin of Ferromagnetism.** Ferromagnetism arises due to permanent magnetic moment in the atoms or molecules of the material. When an external field is applied, the magnetic moments line up in the same direction as that of the applied field.

**Examples of ferromagnetic materials.** Iron (Fe), Cobalt (Co), Nickel (Ni), and Gadolinium (Gd).

**Properties of Ferromagnetic Materials**

1. All the dipoles are aligned parallel to each other due to the magnetic interaction between any two dipoles. Figure 14.14 shows the dipole alignment.
2. Ferromagnetic materials have permanent dipole moment.
3. When placed inside a magnetic field, a ferromagnetic material attracts the magnetic lines of force very strongly (Fig. 14.15).

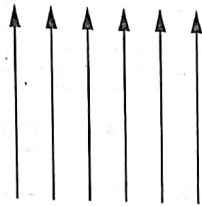


FIG. 14.14

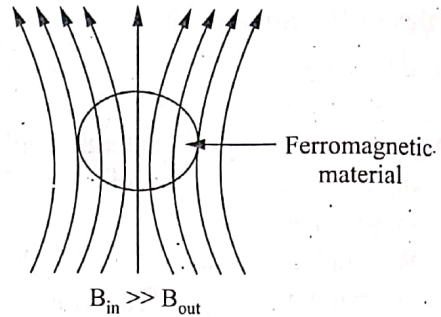


FIG. 14.15

4. They exhibit magnetisation even in the absence of a magnetic field. This property of ferromagnetic materials is called **spontaneous magnetisation**.
5. Ferromagnetic materials exhibit the phenomenon of hysteresis.
6. On heating, they lose their magnetisation slowly.
7. The magnetic susceptibility ( $\chi_m$ ) of ferromagnetic materials is very high and depends on **temperature (T)**.
  - As temperature increases, the value of susceptibility decreases.
  - The ferromagnetic property depends on temperature. At high enough temperatures, a ferromagnet becomes a paramagnet.

The temperature of transition from ferromagnetism to paramagnetism is called the *paramagnetic Curie temperature*  $\theta$ .

The susceptibility above the Curie temperature, *i.e.*, in the paramagnetic phase is described by,

$$\chi_m = \frac{C}{T - \theta} \quad (T > \theta).$$

Here, C is the Curie constant.

For  $T > \theta$ , paramagnetic behaviour.

For  $T < \theta$ , ferromagnetic behaviour.

Figure 14.16 shows the variation of susceptibility with temperature for ferromagnetic materials.

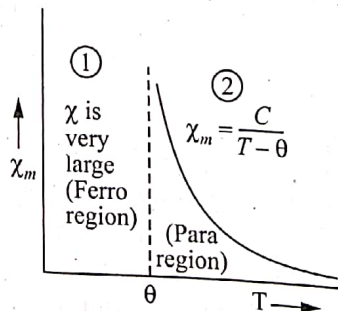
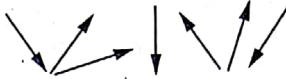

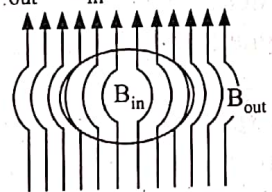
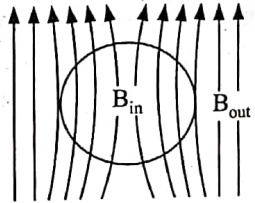
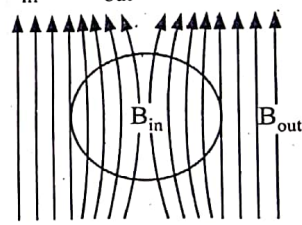


FIG. 14.16

8. The relative permeability  $\mu_r$  of ferromagnetic materials is very high. The relative magnetic permeability is  $> 1000!$



**Table 14.1** Comparison of dia, para and ferromagnetic materials

	<i>Diamagnetic material</i>	<i>Paramagnetic material</i>	<i>Ferromagnetic material</i>
1. Magnetic moment	There is no permanent dipole moment (or) magnetic moment in each atom.	There is permanent dipole moment (or) magnetic moment in each atom.	There is enormous permanent dipole moment (or) magnetic moment in each atom.
2. Spin alignment	No spin.	All spins are randomly oriented. 	Spin alignment is parallel in the same direction. 
3. Susceptibility and its temperature dependence	Susceptibility is always negative. It is independent of the temperature and strength of applied magnetic field.	It is always positive and small. It is inversely proportional to absolute temperature of the material. $\chi \propto \frac{1}{T}$ or $\chi = \frac{C}{T}$ .	It is always positive and very large $\chi_m = \frac{C}{T - \theta}$ (Curie-Weiss law) (i) For $T > \theta$ , paramagnetic behaviour. (ii) For $T < \theta$ , ferromagnetic behaviour.
4. Behaviour of material in the presence of magnetic field	When the material is placed in the magnetic field, the magnetic lines of force are repelled away from the material. $B_{out} > B_{in}$ . 	The magnetic lines of force are attracted towards the centre of the material. $B_{in} > B_{out}$ . 	The magnetic lines of force are highly attracted towards the centre of the material. $B_{in} \gg B_{out}$ . 
5. Relative magnetic permeability ( $\mu_r$ )	$\mu_r$ is slightly less than 1.	$\mu_r$ is slightly greater than 1.	$\mu_r$ is very much greater than 1. $\mu_r \gg 1$
6. Examples	Hydrogen, bismuth, antimony, gold and superconducting materials like Niobium.	Aluminium, platinum, sodium, titanium, zirconium and chromium	Iron, nickel, cobalt, gadolinium.

## 14.9 THE ELECTRON THEORY OF MAGNETISM

The paramagnetic, diamagnetic and ferromagnetic behaviour of substances can be explained in an elementary way in terms of the electron theory of matter.

Each electron is supposed to be revolving in an orbit around the nucleus. Each moving electron behaves like a tiny current loop and therefore possesses orbital magnetic dipole moment. Furthermore, each electron is spinning about an axis through itself. This spin also gives rise to a magnetic dipole moment. In general, the resultant magnetic dipole moment of an atom is the vector sum of the orbital and spin magnetic dipole moments of its electrons.

### Explanation of Diamagnetism

Diamagnetism occurs in those substances whose atoms consist of an *even* number of electrons. The electrons of such atoms are paired. The electrons in each pair have orbital motions as well as



### 14.10 LANGEVIN'S THEORY OF DIAMAGNETISM

Consider an electron (mass =  $m$ , charge =  $e$ ) rotating about the nucleus (charge =  $Ze$ ) in a circular orbit of radius  $r$ . Let  $\omega_0$  be the angular velocity of the electron. Then

$$F_0 = m\omega_0^2 r = Ze^2 / (4\pi \epsilon_0 r^2)$$

or

$$\omega_0 = \sqrt{\frac{Ze^2}{4\pi\epsilon_0 mr^3}} \quad \dots(1)$$

The magnetic moment of the electron is

$$\bar{m} = \text{current} \times \text{area} = \frac{e\omega_0}{2\pi} \times \pi r^2 = \frac{e}{2} \omega_0 r^2 \quad \dots(2)$$

Let a magnetic field of induction  $\mathbf{B}$  be now applied.  $\mathbf{B}$  is normal to and into the page (Fig. 14.19).

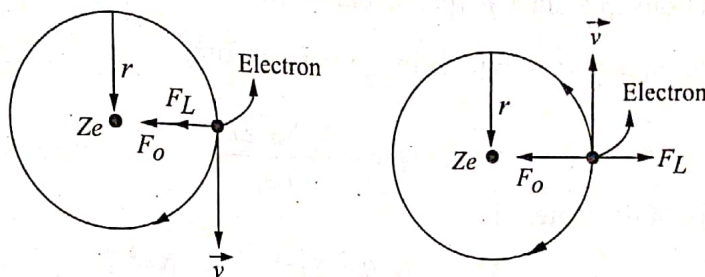


FIG. 14.19

An additional force  $F_L$  called Lorentz force acts on the electron.

$$F_L = -e(\mathbf{v} \times \mathbf{B}) = -eBr\omega$$

The condition of stable motion is now given by

$$mr\omega^2 = \frac{Ze^2}{4\pi\epsilon_0 r^2} - eBr\omega \quad \dots(3)$$

or

$$\omega^2 + \frac{eB}{m} \omega - \frac{Ze^2}{4\pi\epsilon_0 mr^3} = 0$$

Solving the quadratic equation in  $\omega$

$$\omega = \frac{\frac{-eB}{m} \pm \sqrt{\left(\frac{eB}{m}\right)^2 + 4\left(\frac{Ze^2}{4\pi\epsilon_0 mr^3}\right)}}{2}$$

$$= \pm \sqrt{\omega_0^2 + \left(\frac{eB}{2m}\right)^2} - \frac{eB}{2m}$$

or

$$\omega = \pm \omega_0 - \frac{eB}{2m} \quad \left( \because \frac{eB}{2m} \ll \omega_0 \right) \quad \dots(4)$$

Thus the angular frequency is now different from  $\omega_0$ . The result of establishing a field of flux density  $\mathbf{B}$  is to set up a precessional motion of the electronic orbits with angular velocity  $-(e/2m)\mathbf{B}$ . This is called Larmor theorem. Then

$$\left. \begin{array}{l} \text{change in frequency of} \\ \text{revolution of the electron} \end{array} \right\} = \delta n = -\frac{eB}{4\pi m}$$



The corresponding change in the magnetic moment of the electron is

$$\Delta m = \text{current} \times \text{area} = \left\{ e \times \left( \frac{-eB}{4\pi m} \right) \right\} \times \pi r^2 = -\frac{Be^2 r^2}{4m} \quad \dots(5)$$

On summing over all electrons in the atom, the induced moment per atom becomes

$$\Delta m_{\text{atom}} = -\frac{Be^2 \Sigma r^2}{4m}$$

Let  $N$  be the number of atoms per unit volume. Then the magnetisation  $M$  is given by

$$M = -\frac{N Be^2 \Sigma r^2}{4m} \quad \dots(6)$$

All the electron orbits are not oriented normal to the magnetic field.

Hence  $r^2$  in Eq. (6) should be replaced by the average of the square of the projection of orbit radii for various electrons in a plane perpendicular to  $B$ .

Hence we should replace  $r^2$  in Eq. (6) by  $\frac{2}{3} r^2$ . Therefore,

$$M = -\frac{N Be^2 \Sigma r^2}{6m}$$

Volume susceptibility of the material

$$\chi_m = \frac{M}{H} = -\frac{N Be^2 \Sigma r^2}{6mH} = -\frac{\mu_0 Ne^2 \Sigma r^2}{6m} \quad (\because B = \mu_0 H)$$

$$\therefore \chi_m = -\frac{\mu_0 Ne^2 \Sigma r^2}{6m} = -\frac{\mu_0 e^2}{6m} NZ \langle r^2 \rangle \quad \dots(7)$$

Here,  $Z$  is the atomic number of the atom.

Eq. (7) shows that  $\chi_m$  is independent of the field strength and temperature. This is in accord with Curie's experimental results.

## 14.11 LANGEVIN'S THEORY OF PARAMAGNETISM

He assumes that each atom has a permanent magnetic moment  $m$ .

The only force acting on the atom is that due to the external field  $\mathbf{B}$ . Let  $\theta$  be the angle of inclination of the axis of the atomic dipole with the direction of the applied field  $\mathbf{B}$ . Then magnetic potential energy of the atomic dipole is

$$U = -mB \cos \theta$$

Now, on classical statistics, the number of atoms making an angle between  $\theta$  and  $\theta + d\theta$  is

$$dn = Ce^{mB \cos \theta / kT} \sin \theta d\theta$$

where  $k$  is Boltzmann's constant and  $T$  is the absolute temperature. Put  $mB/kT = \alpha$ . Then

$$dn = Ce^{\alpha \cos \theta} \sin \theta d\theta \quad \dots(1)$$

Hence the total number of atomic magnets in unit volume of the paramagnetic material

$$n = \int_0^\pi dn = \int_0^\pi Ce^{\alpha \cos \theta} \sin \theta d\theta \quad \dots(2)$$

Put  $\cos \theta = x$ . Then,  $-\sin \theta d\theta = dx$ .

$$n = \int_{+1}^{-1} -Ce^{\alpha x} dx = C \int_{-1}^{+1} e^{\alpha x} dx$$



$$C = \frac{n\alpha}{e^\alpha - e^{-\alpha}} \quad \dots(3)$$

The component of each dipole moment parallel to  $B$  is  $m \cos\theta$ . The total magnetic moment of all the  $n$  atoms contained in unit volume of the gas is the magnetisation  $M$ . It is given by

$$M = \int_0^\pi m \cos \theta \, dn = \int_0^\pi m \cos \theta \, C e^{\alpha \cos \theta} \sin \theta \, d\theta \quad \dots(4)$$

Put  $\cos \theta = x$ . Then,  $-\sin \theta \, d\theta = dx$ . Therefore, we get

$$M = \int_{+1}^{-1} -mx C e^{\alpha x} \, dx = Cm \int_{-1}^{+1} x e^{\alpha x} \, dx$$

Evaluating this integral and substituting the value of  $C$  from (3), we get

$$\begin{aligned} M &= mn \left[ \frac{e^\alpha + e^{-\alpha}}{e^\alpha - e^{-\alpha}} - \frac{1}{\alpha} \right] \\ &= mn \left[ \coth \alpha - \frac{1}{\alpha} \right] \\ &= mn L(\alpha) \end{aligned} \quad \dots(5)$$

where  $L(\alpha) = \left[ \coth \alpha - \frac{1}{\alpha} \right]$  is called the *Langevin function*.

The variation of  $M$  with  $\alpha$  is shown in Fig. 14.20.

**Case (i)** At low temperatures or large applied field,  $L(\alpha) \rightarrow 1$ . Hence, magnetization  $M$  in this case will be

$$M = mn \quad \dots(6)$$

So saturation is reached when all the atomic dipoles are parallel to  $B$ .

**Case (ii)** Under normal conditions  $\alpha$  is very small. Then,

$$L(\alpha) = \coth \alpha - \frac{1}{\alpha} \approx \frac{\alpha}{3} \quad \dots(7)$$

$$M = mn \frac{\alpha}{3} = \frac{nm^2 B}{3kT} = \frac{nm^2 \mu_0 H}{3kT} \quad \dots(8)$$

$$\chi_m = \frac{M}{H} = \frac{\mu_0 nm^2}{3kT} = \frac{C}{T} \quad \dots(9)$$

Here  $C = \mu_0 nm^2/3k$  is called the Curie constant.

Eq. (9) shows that  $\chi_m \propto \frac{1}{T}$ , which is Curie's law.

**Failure of Langevin Theory.** (i) Langevin's theory was unable to explain a more complicated dependence of susceptibility upon temperature exhibited by several paramagnetics such as highly compressed and cooled gases, very concentrated solutions of salts etc.

(ii) Langevin's theory could not account for the intimate relation between para- and ferro-magnetism.

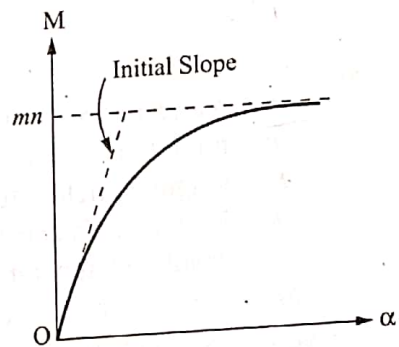


FIG. 14.20



### 14.15 WEISS'S THEORY OF FERROMAGNETISM

Langevin's theory of paramagnetism was extended by Weiss to give a theoretical explanation of the behaviour of ferromagnetics. He made the following two assumptions:

- (i) Weiss assumed that a ferromagnetic specimen contains a number of small regions (domains) which are spontaneously magnetised. The total spontaneous magnetisation is the vector sum of the magnetic moments of the individual domains.
- (ii) The spontaneous magnetisation of each domain is due to the existence of an internal molecular field. This tends to produce a parallel alignment of the atomic dipoles.

Weiss also assumed that the internal molecular field  $H_i$  is proportional to the magnetisation  $M$ , i.e.,  $H_i = \gamma M$  where  $\gamma$  is a constant called *Weiss constant*. If now an external field  $H$  acts on the dipole, then the effective field  $H_{eff}$  is given by

$$H_{eff} = H + H_i = H + \gamma M \quad \dots(1)$$

According to Langevin's theory of paramagnetism, at high temperatures,

$$M = \frac{nm^2 \mu_0 H}{3kT}$$

Weiss suggested that the corresponding result for ferromagnetics could be obtained by replacing  $H$  by  $H_{eff}$ . Hence for ferromagnetics,

$$M = \frac{nm^2 \mu_0}{3kT} [H + \gamma M]$$

or

$$M = \frac{nm^2 \mu_0 H}{3k \left( T - \frac{nm^2 \gamma \mu_0}{3k} \right)} \quad \dots(2)$$

The susceptibility  $\chi_{ferro}$  is,

$$\chi_{ferro} = \frac{M}{H} = \frac{nm^2 \mu_0}{3k \left( T - \frac{nm^2 \gamma \mu_0}{3k} \right)} = \frac{C}{(T - \theta)} \quad \dots(3)$$

Here,  $\frac{nm^2 \mu_0}{3k}$  is called the Curie constant.

$\theta = \frac{nm^2 \gamma \mu_0}{3k}$  is called the Curie temperature.

It is the temperature below which the material shows ferromagnetic behaviour.

For values of temperature above  $\theta$ , the ferromagnetic substance behaves like a paramagnetic substance.

Eq. (3) is called the *Curie-Weiss law* for ferromagnetics.

### 14.16 EXPERIMENT TO DRAW M-H CURVE (HORIZONTAL MODEL)

The lagging of the intensity of magnetisation  $M$  (or magnetic induction  $B$ ) behind the magnetising field  $H$  is known as *hysteresis*.

The variation of  $M$  with  $H$  can be studied by the following experiment. A deflection magnetometer can be used for drawing the  $M-H$  curve for iron in the form of a rod.

**Description.** Fig. 14.28 shows the experimental set up. A solenoid  $SS$  and compensating coil  $(C)$  are connected in series through a commutator to a battery, ammeter and rheostat. The solenoid is



mounted horizontally in the east-west direction. The sample rod  $AB$  is placed well within the solenoid along its axis. A magnetometer  $M'$  set in end-on position [Tan A] is placed in level with the axes of  $SS$  and  $C$ . The magnetometer is adjusted to read zero-zero.

**Working.** The sample rod  $AB$  is removed from  $SS$ . The compensating coil is placed on the other side of the magnetometer in such a way that its magnetic field is opposite to the solenoid's field at the magnetometer. A current is passed through the solenoid. The position of the compensating coil  $C$  is adjusted until the magnetometer deflection becomes zero. The position of  $C$  is then fixed throughout the experiment. The current is switched off.

The sample rod  $AB$  is then introduced in  $SS$ . A current  $i$  is passed through it. To get the hysteresis loop we need a series of values of  $M$  and  $H$  (or  $\theta$  and  $i$ ). The current is gradually increased to 0.5 A. The readings of the two ends of the pointer of the magnetometer are noted and the average deflection  $\theta$  is calculated. The current through the solenoid is gradually increased in steps of 0.5 A, noting the deflections in each case upto the maximum current of 3A. Then the current is gradually decreased in steps of 0.5 A to zero. Now the current is reversed in its direction by changing the commutator and increased upto  $-3A$ . Then the current is brought to zero in steps of 0.5, reversed to the original direction and increased upto 3A. The deflections are noted in each case. Thus the specimen has been taken through a complete cycle of magnetisation.

**Theory.** The magnetising field  $H$  is calculated from the equation

$$H = ni$$

where  $i$  is the current and  $n$  is the number of turns per unit length of the solenoid.

The value of  $M$  is calculated as follows. The deflection in  $M'$  is now due to the magnetism acquired by the sample alone. Let it be measured as  $\theta$ . The field at  $M'$  is

$$B = B_0 \tan \theta \quad \dots(1)$$

where  $B_0$  is the horizontal induction of earth's field.

Let  $d$  = distance between the centre of the rod  $AB$  and the centre of the magnetometer  $M'$   
 $2L$  = length of the rod  $AB$ ,

$m$  = magnetic dipole moment developed in the rod.

Then, the field at  $M'$  producing the deflection  $\theta$  is given by

$$B = \frac{\mu_0}{4\pi} \frac{2md}{(d^2 - L^2)^2} \quad \dots(2)$$

From Eqs. (1) and (2),

$$\frac{\mu_0}{4\pi} \frac{2md}{(d^2 - L^2)^2} = B_0 \tan \theta$$

$\therefore$

$$m = \frac{4\pi (d^2 - L^2)^2}{\mu_0 2d} B_0 \tan \theta$$

Let  $r$  be the radius of the rod. Then,

volume of the rod =  $V = (\pi r^2) 2L$

$\therefore$  intensity of magnetisation of the rod

$$M = m/V$$

$\therefore$

$$M = \left( \frac{4\pi}{\mu_0} \right) \frac{(d^2 - L^2)^2}{2d (\pi r^2) 2L} B_0 \tan \theta$$

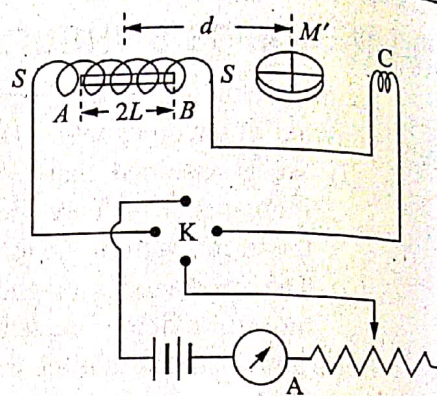


FIG. 14.28



Knowing  $B_0$ , the intensity of magnetisation  $M$  is calculated for various deflections  $\theta$  corresponding to different values of current  $i$ .

A graph is drawn plotting  $H$  along the  $X$ -axis and  $M$  along the  $Y$ -axis. The graph obtained is in the form of a loop  $ABCDEF$  (Fig. 14.29). The loop is called the *hysteresis loop*. From the graph, the following properties of the material can be calculated.

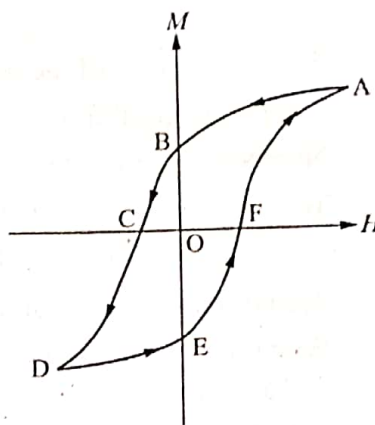


FIG. 14.29

- (i) **Retentivity.** When the magnetising field becomes zero, the magnetisation does not become zero but retains a value  $OB$ . This value of  $M$  is called the *retentivity* of the material. Retentivity =  $OB$
- (ii) **Coercivity.** The field  $OC$  which should be applied to the material, in the direction opposite to its magnetisation, to remove its residual magnetism and make the magnetisation zero is called *coercivity* (or *coercive force*).

Coercivity =  $OC$

The chief sources of error in this method are (i) uncertainty in the location of the poles of the magnetised rod, and (ii) insensitiveness of the deflection magnetometer.

### 14.17 EXPERIMENT TO DRAW B-H CURVE (BALLISTIC METHOD)

**Circuit Description.** A specimen of the given ferromagnetic material is taken in the form of a ring (Rowland ring). The experimental arrangement is shown in Fig. 14.30.

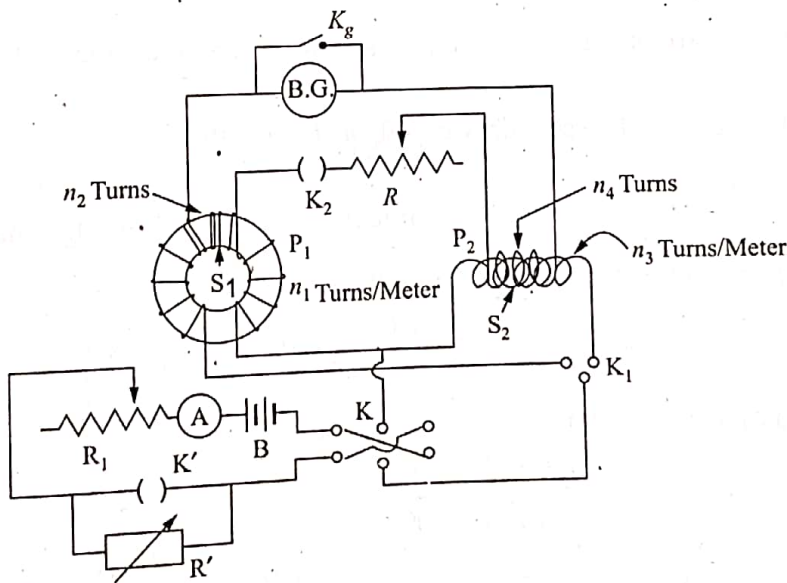


FIG. 14.30

A primary coil  $P_1$  is wound closely over the specimen ring. This winding is connected in series with a battery  $B$ , an ammeter  $A$ , a rheostat  $R_1$ , and a resistance  $R'$  through a reversing key  $K$  and a two-way key  $K_1$ . A tap key  $K'$  connected across  $R'$  facilitates either its inclusion or removal from the circuit. The secondary winding  $S_1$ , over the specimen, consists of a few turns of closely wound wire. This winding  $S_1$ , is connected in series with a rheostat  $R$ , a ballistic galvanometer and the secondary winding  $S_2$  of a standard solenoid through a key  $K_2$ .  $K_g$  is the damping key across the ballistic galvanometer.  $P_2$  is the primary winding of the standard solenoid. The two way key  $K_1$  connects either  $P_1$  or  $P_2$  to the battery circuit.