Newton's - Raphson Method

Let x_0 be an approximate root of the equation $f(x)=0$ let $x_1 = x_0 + h$ be the exact root where h - is very small, positive or negative.

$$\therefore f(x_1) = 0 \quad \text{——— (1)}$$

By taylor's series expansion, we have

$$f(x_1) = f(x_0+h) = f(x_0) + \frac{h}{1!} f'(x_0) + \frac{h^2}{2!} f''(x_0) + \ldots\ldots$$

since $f(x_1)=0$ and h is very small h^2 and higher powers of h can be neglected

Hence, $f(x_0) + hf'(x_0) = 0$

$$h = \frac{f(x_0)}{f'(x_0)} \text{ if } f'(x_0) \neq 0$$

Hence $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ is a first approximation to the root

similiarly starting with x_1 we get the next approximation to the root given by

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

In general,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2 \ldots\ldots$$

and it is known as Newton - Rephson's iteration formula or simply Newton - Rephson's formula.

1. Find the real roots of the equation $x^3 - 2x - 5 = 0$ correct 4 decimal places using Newton Raphson Method.

Solution :-

Given that, $f(x) = x^3 - 2x - 5$

put $x = 0 \Rightarrow f(0) = -ve$

$x = 1 \Rightarrow f(1) = -ve$

$x = 2 \Rightarrow f(2) = -ve$

$x = 3 \Rightarrow f(3) = +ve$

The roots lies between 2 & 3

$$x_{n+1} = x_n - \left(\frac{f(x_n)}{f'(x_n)}\right) \quad \text{—— (1)}$$

W.K.T

$$f(x) = x^3 - 2x - 5$$

$$f'(x) = 3x^2 - 2$$

put, $n=0$ in equation (1) $\Rightarrow$

$$x_1 = x_0 - \left(\frac{f(x_0)}{f'(x_0)}\right)$$

Let $x_0 = \frac{a+b}{2} = \frac{2+3}{2} = 2.5$

$x_0 = 2.5$

$$x_1 = 2.5 - \left(\frac{f(2.5)}{f'(2.5)}\right)$$

$$f(2.5) = (2.5)^3 - 2(2.5) - 5$$
$$= 5.625$$

$$f'(2.5) = 3(2.5)^2 - 2$$
$$= 16.75$$

$$x_1 = 2.5 - \left(\frac{5.625}{16.75}\right)$$

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$x_1 = 2.5 - 0.3858$

$x_1 = 2.1642$

put $n = 1$ in equation ① $\Rightarrow$

$$x_2 = x_1 - \left(\frac{f(x_1)}{f'(x_1)}\right)$$

$$x_2 = 2.1642 - \left(\frac{f(2.1642)}{f'(2.1642)}\right)$$

$$f(2.1642) = (2.1642)^3 - 2(2.1642) - 5$$
$$= 0.8081$$

$$f'(2.1642) = 3(2.1642)^2 - 2$$
$$= 12.051$$

$$x_2 = 2.1642 - \left(\frac{0.8081}{12.051}\right)$$

$$x_2 = 2.1642 - 0.0670$$

$$x_2 = 2.0971$$

put, $n = 2$ in equation ① $\Rightarrow$

$$x_3 = x_2 - \left(\frac{f(x_2)}{f'(x_2)}\right)$$

$$x_3 = 2.0971 - \left(\frac{f(2.0971)}{f'(2.0971)}\right)$$

$$x_3 = 2.0971 - \left(\frac{0.0284}{11.193}\right)$$

$x_3 = 2.0971 - 0.0025$

$x_3 = 2.0946.$

put, $n = 3$ in equation ① =>

$$x_4 = x_3 - \left(\frac{f(x_3)}{f'(x_3)}\right)$$

$$x_4 = 2.0946 - \left(\frac{f(2.0946)}{f'(2.0946)}\right)$$

$$x_4 = 2.0946 - \left(\frac{0.00051}{11.662}\right)$$

$$x_4 = 2.0946 - 0.000048$$

$$x_4 = 2.0946$$

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2. Find the real roots of the equation $x \sin x + \cos x = 0$. correct 3 decimal places using N.R.M :-

Solution :-

Given that $f(x) = x\sin x + \cos x$

$f(0) = +ve$

$f(1) = +ve$

$f(2) = +ve$

$f(3) = -ve$

$\therefore$ the roots lies between 2 & 3.

$$x_{n+1} = x_n - \left(\frac{f(x_n)}{f'(x_n)}\right) \quad \text{——— ①}$$

$$x_1 = x_0 - \left(\frac{f(x_0)}{f'(x_0)}\right)$$

$f(x) = x \sin x + \cos x$

$f'(x) = x \cos x + \sin x - \sin x$

$f'(x) = x \cos x$

put ; $n = 0$ in equation ① =>

$$x_1 = x_0 - \left(\frac{f(x_0)}{f'(x_0)}\right)$$

Let , $x_0 = \frac{a+b}{2} = \frac{2+3}{2} = 2.5$

$\boxed{x_0 = 2.5}$

$$x_1 = 2.5 - \left(\frac{f(2.5)}{f'(2.5)}\right)$$

$$x_1 = 2.5 - \left(\frac{0.6950}{-2.0028}\right)$$

$x_1 = 2.8470$

put , $n=1$

$$x_2 = x_1 - \left(\frac{f(x_1)}{f'(x_1)}\right)$$

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$$x_2 = 2.8470 - \left(\frac{-0.1302}{-2.7243}\right)$$

$$= 2.8470 - 0.0477.$$

$$x_2 = 2.7993$$

put; $n = 2$,

$$x_3 = x_2 - \left(\frac{f(x_2)}{f'(x_2)}\right)$$

$$= 2.7993 - \left(\frac{-0.0024}{-2.6369}\right)$$

$$= 2.7993 - 0.0009$$

$$\therefore x_3 = 2.7984$$

put $n = 3$

$$x_4 = x_3 - \left(\frac{f(x_3)}{f'(x_3)}\right)$$

$$= 2.7984 - \left(\frac{-0.0000}{-2.6352}\right)$$

$$\therefore x_4 = 2.7984 - 0$$

$$x_4 = 2.7984 //.$$

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3. Find the real roots of the equation $xe^x - 2 = 0$ correct to 3 decimal play using Newton Raphson Method.

Solution :-

Given that

$xe^{x} - 2 = 0$

put $x = 0 \Rightarrow f(0) = -ve$

$f(1) = +ve$.

The roots lies between 0 & 1

$f(x) = xe^{x} - 2$

$f'(x) = xe^{x} + e^{x}.1$

$f'(x) = e^{x}(x+1)$

$x_0 = \frac{a+b}{2} = \frac{0+1}{2} = 0.5$

Newton Raphson formula :-

$$x_{n+1} = x_0 - \left(\frac{f(x_n)}{f'(x_n)}\right)$$

put, $n=0 \Rightarrow$

$$x_1 = x_0 - \left(\frac{f(x_0)}{f'(x_0)}\right)$$

$$x_1 = 0.5 - \left(\frac{-1.1756}{2.473}\right)$$

$= 0.5 + 0.4753$

$x_1 = 0.9753$

put $n=1$ equation ① $\Rightarrow$

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$$x_2 = x_1 - \left(\frac{f(x_1)}{f'(x_1)}\right)$$

$$x_2 = 0.9753 - \left(\frac{0.5864}{5.2384}\right)$$

$$= 0.8634$$

put $n=2$ in equation (F) ⇒

$$x_3 = x_2 - \left(\frac{f(x_2)}{f'(x_2)}\right)$$

$$= 0.8634 - \left(\frac{f(0.8634)}{f'(0.8634)}\right)$$

$$= 0.8634 - \left(\frac{0.0473}{4.4185}\right)$$

$$\therefore x_3 = 0.8526$$

put $n=3$

$$x_4 = x_3 - \left(\frac{f(x_3)}{f'(x_3)}\right)$$

$$x_4 = 0.8526 - \left(\frac{0.001}{4.348}\right)$$

$$x_4 = 0.8526$$

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Lagrange's Interpolation formula.

formula :

$$y(x) = \frac{(x-x_1)(x-x_2)(x-x_2)\cdots\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots\cdots\cdots(x_0-x_n)} \times y_0 +$$

$$\frac{(x-x_2)(x-x_3)(x-x_4)\cdots\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots\cdots\cdots(x_1-x_n)} \times y_2 + \cdots$$

$$+ \frac{(x-x_0)(x-x_1)\cdots\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots\cdots(x_n-x_{n-1})} \times y_n$$

1. use lagrange's interpolation formula to find the value of y at $x=6$ from the following data

	x_0	x_1	x_2	x_3
x	3	7	9	10
y	y_0 168	y_1 120	y_2 72	y_3 63.

Solution :

given that

x	x_0 3	x_1 7	x_2 9	x_3 10
y	168 y_0	120 y_1	72 y_2	63 y_3.

formula :-

$$y(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \times y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$$

$$\times y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \times y_2 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \times y_3 .$$

Here $x=6$; $x_0 = 3$; $x_1 = 7$; $x_2 = 9$; $x_3 = 10$

$y_0 = 168$; $y_1 = 120$; $y_2 = 72$; $y_3 = 63$

$$y(6) = \frac{(6-7)(6-9)(6-10)}{(3-7)(3-9)(3-10)} \times 168 + \frac{(6-3)(6-9)(6-10)}{(7-3)(7-9)(7-10)}$$

$$\times 120 + \frac{(6-3)(6-7)(6-10)}{(6-3)(6-7)(6-10)} \times 72 + \frac{(6-3)(6-7)(6-9)}{(10-3)(10-7)(10-9)} \times 63$$

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$$y(6) = \frac{(-12)}{(-168)} \times 168 + \frac{36}{24} \times 120 + \left(\frac{12}{-12}\right) \times 72 + \left(\frac{9}{21}\right) \times 63$$

$$y(6) = (0.071 \times 168) + (1.5 \times 120) + (-1 \times 72) + (0.428 \times 63)$$

$$y(6) = 11.928 + 180 - 72 + 26.964$$

$$y(6) = 146.892$$

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