Algebra, Analytical Geomentry (3D) & Trigonomentry.

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Unit - II

Matrices

Definition 1:

A rectangular array of mn numbers consisting of m row and n columns,

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{m1} & a_{m2} & & a_{mn} \end{pmatrix}$$

is termed a <u>matrix.</u> A matrix A is also denoted briefly as.

$$A = (a_{ij}) \qquad \left(\begin{array}{l} i = 1.2 \ldots m; \\ j = 1.2 \ldots n \end{array} \right)$$

Definition 2:

In any matrix if the number of row is equal to the number of column m=n then the matrix is called <u>square matrix.</u>

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

Definition 3:

A determinant consisting of the element of a square matrix is called the determinant of the matrix.

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$$\det(A) = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{vmatrix}$$

~~Deter~~

Note:

A non square matrix ~~does~~ not have a determinant.

Definition 4:

A matrix A^T is called the <u>transpose</u> of ~~the~~ A. if the columns of a A are the row of

Note:

Transpose of a matrix A is also denoted by A':

Eg:

Let

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}$$ then the matrix

$$A^T = \begin{pmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{pmatrix}$$ is the transpose of A.

Note:

The transpose of the transpose of any matrix is itself.

$$(A^T)^T = A \text{ (or) } (A')' = A.$$

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Definition 5 :

A square matrix A is said to be Symmetric about the principal diagonal if $a_{ij} = a_{ji}$. It is a clear from this that a Symmetric matrix coincides with its transpose.

Eg :

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 3 & 1 \\ -1 & 1 & 4 \end{pmatrix}$$ is a symmetric matrix,

$$A^T = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 3 & 1 \\ -1 & 1 & 4 \end{pmatrix}$$

Note :

$A^T = A$ where A is a symmetric matrix.

Definition 6 :

Skew - Symmetric matrices :

A square $\cancel{\text{matrices}}$ matrix of $A = (a_{ij})$ is said to be skew symmetric if

$$a_{ij} = -a_{ji}$$

$$i, j = 1, 2 \ldots n$$

Note :

If A is a skew-symmetric matrix, then

$A^T = -A$.

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Definition 7:

A square matrix whose element outside the principal diagonal are all zero is called diagonal matrix.

Eg:

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$ is a diagonal matrix.

Definition 8:

If the elements of diagonal matrix on the principal diagonal are all unity, then the matrix is termed a unit matrix (identity matrix) and is denoted by I.

$$I = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \cdots & & & & \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}$$

Definition 9:

A zero or null matrix is composed of entirely zeros. The matrix is denoted by 0.

Eg:

$$0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Note:

There is no real danger of confusing a zero matrix and the number zero.

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Conjucate of Matrix :

A matrix obtained from the given matrix A by replacing its elements by the corresponding conjucate complex numbers is called the conjucate of A and it is denoted by $\bar{A}$.

Eg :

Let

$$A = \begin{bmatrix} 1+i & 1 \\ 2 & 3+i \end{bmatrix}$$

Then $\begin{bmatrix} 1-i & 1 \\ 2 & 3-i \end{bmatrix}$

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Eg :

$$\begin{bmatrix} a & b+ic \\ b-ic & d \end{bmatrix}$$

Skew Hermitan matrix :

In a square matrix $A = [a_{ij}]$ if

$a_{ij} = -\overline{a_{ji}}$ [the (ij) th element of A is equal to the negative of the conjucate of (ji) th element] then the matrix A is called Skew Hermitan. matrix.

Eg :

$$\begin{bmatrix} 0 & 1+i \\ -1+i & 0 \end{bmatrix}.$$

Unit - II.

Problems :

1) If $A = \begin{bmatrix} 3 & 1 & -1 \\ 0 & 1 & 2 \end{bmatrix}$, find AA' & $A'A$.

Sol :

Given

$$A = \begin{pmatrix} 3 & 1 & -1 \\ 0 & 1 & 2 \end{pmatrix}$$

$$A' = \begin{pmatrix} 3 & 0 \\ 1 & 1 \\ -1 & 2 \end{pmatrix}$$

$$AA' = \begin{pmatrix} 3 & 1 & -1 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 1 & 1 \\ -1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 9+1+1 & 0+1-2 \\ 0+1-2 & 0+1+4 \end{pmatrix}$$

$$= \begin{pmatrix} 11 & -1 \\ -1 & 5 \end{pmatrix}$$

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$$A'A = \begin{pmatrix} 3 & 0 \\ 1 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 3 & 1 & -1 \\ 0 & 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 9+0 & 3+0 & -3+0 \\ 3+0 & 1+1 & -1+2 \\ -3+0 & -1+2 & 1+4 \end{pmatrix}$$

$$= \begin{pmatrix} 9 & 3 & -3 \\ 3 & 2 & 1 \\ -3 & 1 & 5 \end{pmatrix}$$

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Orthogonal Matrices:

Definition:

A Square Matrix A (with real element) is said to be orthogonal. If $AA' = A'A = I$.

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1) Show that the Matrix $\frac{1}{3}\begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$

Sol:

Given $A = \frac{1}{3}\begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$

$$A = \begin{bmatrix} 2/3 & 2/3 & 1/3 \\ -2/3 & 1/3 & 2/3 \\ 1/3 & -2/3 & 2/3 \end{bmatrix}$$

$$A' = \begin{bmatrix} 2/3 & -2/3 & 1/3 \\ 2/3 & 1/3 & -2/3 \\ 1/3 & 2/3 & 2/3 \end{bmatrix}$$

$$AA' = \begin{bmatrix} 2/3 & 2/3 & 1/3 \\ -2/3 & 1/3 & 2/3 \\ 1/3 & -2/3 & 2/3 \end{bmatrix}\begin{bmatrix} 2/3 & -2/3 & 1/3 \\ 2/3 & 1/3 & -2/3 \\ 1/3 & 2/3 & 2/3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{9}+\frac{4}{9}+\frac{1}{9} & \frac{-4}{9}+\frac{2}{9}+\frac{2}{9} & \frac{2}{9}-\frac{4}{9}+\frac{2}{9} \\ \frac{-4}{9}+\frac{2}{9}+\frac{2}{9} & 4/9+1/9+4/9 & -2/9-2/9+4/9 \\ \frac{2}{9}-4/9+2/9 & -2/9-2/9+4/9 & 1/9+4/9+4/9 \end{bmatrix}$$

$$AA' = \begin{bmatrix} 9/9 & 0 & 0 \\ 0 & 9/9 & 0 \\ 0 & 0 & 9/9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I.$$

$\therefore AA' = I. \quad \therefore A$ is orthogonal.

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Unitary Matrices :

Define :

A Square Matrix A is said to Unitary. If $(\bar{A})'A = A(\bar{A})' = I$, where $\bar{A}$ conjucate matrix of A.

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1) Show that the matrix $A = \begin{bmatrix} 1/\sqrt{2} & i/\sqrt{2} \\ -i/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$ is unitary.

SOL:

Given $A = \begin{bmatrix} 1/\sqrt{2} & i/\sqrt{2} \\ -i/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$

$$\bar{A} = \begin{bmatrix} 1/\sqrt{2} & -i/\sqrt{2} \\ i/\sqrt{2} & +1/\sqrt{2} \end{bmatrix}$$

$$(\bar{A})' = \begin{bmatrix} 1/\sqrt{2} & i/\sqrt{2} \\ -i/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$

$$(\bar{A})'A = \begin{bmatrix} 1/\sqrt{2} & i/\sqrt{2} \\ -i/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & i/\sqrt{2} \\ -i/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} \quad [i^2 = -1]$$

$$= \begin{bmatrix} 1/2 + 1/2 & +i/2 \mp i/2 \\ -i/2 + i/2 & 1/2 + 1/2 \end{bmatrix} = \begin{bmatrix} 2/2 & 0 \\ 0 & 2/2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I.$$

$(\bar{A})'A = I.$

$\therefore$ A is unitary matrix.

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