Unit - I.

Binominal Exponential & logarithmic series.

Partial Fraction :

Defination :

If $f(x)$ and $g(x)$ are two fractional ~~Integral~~ algebraic fraction of x, and the fraction $\frac{f(x)}{g(x)}$ can be expressed as a sum of simple fraction, then $\frac{f(x)}{g(x)}$ is called Partial fraction.

Rule : I.

1) Resolve into partial fraction $\frac{2x-7}{(3x-5)(x-2)(x+4)}$.

Given $\frac{2x-7}{(3x-5)(x-2)(x+4)}$

solu:

$$\frac{2x-7}{(3x-5)(x-2)(x+4)} = \frac{A}{(3x-5)} + \frac{B}{(x-2)} + \frac{C}{(x+4)}$$

$$\frac{2x-7}{\cancel{(3x-5)(x-2)(x+4)}} = \frac{A(x-2)(x+4)+B(3x-5)(x+4)+C(3x-5)(x-2)}{\cancel{(3x-5)(x-2)(x+4)}}$$

$$2x-7 = A(x-2)(x+4)+B(3x-5)(x+4)+C(3x-5)(x-2)$$

put $x=2$.

$2(2)-7 = A(0) + B(6-5)(2+4) + C(0)$

$4-7 = B(1)(6)$

$-3 = 6B$

$\boxed{B = -\frac{1}{2}}$

put $x = -4$

$2(-4)-7 = A(0)+B(0)+C(3(-4)-5)(-4-2)$

$-8-7 = C(-12-5)(-6)$

$-15 = C(-17)(-6)$

$\frac{-\cancel{15}^{\,5}}{-17\times-\cancel{6}_{\,2}} = C$

$\boxed{C = -\frac{5}{34}}$

put $x = 5/3$.

$2\left(\frac{5}{3}\right)-7 = A\left(5/3-2\right)\left(5/3+4\right)+B(0)+C(0)$

$\frac{10}{3}-7 = A\left(\frac{5-6}{3}\right)\left(\frac{5+12}{3}\right)$

$\frac{10-21}{\cancel{3}} = A\left(-1/\cancel{3}\right)\left(17/3\right)$

Sub A, B & C in equ ①.

$\frac{2x-7}{(3x-5)(x-2)(x+4)} = \frac{33}{17}\left(\frac{1}{(3x-5)}\right) - \frac{1}{2}\left(\frac{1}{(x-2)}\right) - \frac{5}{34}\left(\frac{1}{x+4}\right)$

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Rule : II ~~Resolve~~

a) $$\frac{f(x)}{g(x)} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3}.$$

1) Resolve into partial fraction $\dfrac{9}{(x-1)(x+2)^2}$.

Given $\dfrac{9}{(x-1)(x+2)^2}$.

Soln: $$\frac{9}{(x+1)(x+2)^2} = \frac{A}{(x-1)} + \frac{B}{(x+2)} + \frac{C}{(x+2)^2} \rightarrow (1)$$

$$\frac{9}{\cancel{(x-1)(x+2)^2}} = \frac{A(x+2)^2 + B(x-1)(x+2) + C(x-1)}{\cancel{(x-1)(x+2)^2}}$$

$$9 = A(x+2)^2 + B(x-1)(x+2) + C(x-1) \rightarrow (2)$$

put $x = -2$,

$$9 = A(0) + B(0) + C(-2-1)$$

$$9 = C(-3)$$

$$C = 9/-3$$

$$\boxed{C = -3}$$

put $x = 1$

$$9 = A(1+2)^2 + B(0) + C(0)$$

$$9 = A(3)^2$$

$$9 = A(9)$$

$$\boxed{A = 1}$$

$② \Rightarrow 9 = A(x^2+4+4x) + B(x^2+2x-\cancel{x}-2) + C(x-1).$

Equating the co-efficient of x^2

$0 = A+B$

$0 = 1+B$

$\boxed{B=-1}$

$① \Rightarrow \frac{9}{(x-1)(x+2)^2} = \frac{1}{(x-1)} - \frac{1}{(x+2)} - \frac{3}{(x+2)^2}.$

Rule II:

If the quartatic factor ax^2+bx+c. repeated, then the corresponding partial fractio is $\frac{Ax+B}{ax^2+bx+c} + \frac{Cx+D}{(ax^2+bx+c)^2}$

1) $\frac{2x+1}{(x-1)(x^2+1)^2}$.

Solu:

Given $\frac{2x+1}{(x-1)(x^2+1)}$

$\frac{2x+1}{(x-1)(x^2+1)} = \frac{A}{(x-1)} + \frac{Bx+C}{(x^2+1)} \rightarrow ①.$

$\frac{2x+1}{(x-1)\cancel{(x^2+1)}} = \frac{A(x^2+1)+(Bx+C)(x-1)}{\cancel{(x-1)(x^2+1)}}$

$2x+1 = A(x^2+1)+(Bx+C)(x-1)$

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put $x = 1$.

$2(1) + 1 = A(1+1) + (B(1) + C)(0)$

$3 = A(2)$

$3/2 = A$.

$\therefore \boxed{A = 3/2}$

Equating the co-efficients of x^2

$0 = A + B$

$0 = 3/2 + B$

$-3/2 = B$.

$\therefore \boxed{B = -3/2}$

Equating constants

$1 = A - C$

$1 = 3/2 - C$

$C = 3/2 - 1$

$C = \frac{3-2}{2} = 1/2$.

$\therefore \boxed{C = 1/2}$

$$① \Rightarrow \frac{2x+1}{(x-1)(x^2+1)} = 3/2\left(\frac{1}{(x-1)}\right) + \frac{(-3/2)x + (1/2)}{(x^2+1)}$$

$$= 3/2 \frac{1}{x-1} + \frac{\frac{-3x+1}{2}}{x^2+1}$$

$$\frac{2x+1}{(x-1)(x^2+1)} = 3/2 \frac{1}{x-1} + 1/2 \frac{(-3x+1)}{x^2+1}$$