# QUESTION BANK

## SUBJECT: ANALYTICAL GEOMETRY 3D

## CLASS: I B.Sc MATHS

## UNIT I

### Section A

1. What is meant by coplanar?
2. Write down the formula for the normal form of the equation of the plane.
3. (6,2,3) are direction ratio of a line. What are the direction cosines?
4. If the direction cosines of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$ then find the value of c
5. Find the distance between the points (4,3,-6) and (-2,1,-3)

### Section B

1. Find the equation of the plane through the point (1,1,1) and the intersection of the planes x + y + z = 6 and 2x + 3y + 4z + 5 = 0
2. The direction cosines l,m,n of two lines are connected by the relations l + m + n = 0, 2lm + 2ln - mn = 0
3. The direction cosines of two lines which are determined by the relations l + m - n = 0, mn + 6ln - 12lm = 0
4. Find the distances of the points (2,3,4) and (1,1,4) from the plane3x - 6y + 2z + 11 = 0

### Section C

1. A line makes angles $\alpha, \beta, \gamma, \delta$ with the four diagonals of a cube .Prove that $cos^2\alpha + cos^2\beta + cos^2\gamma + cos^2\ \delta$
2. Find the angle between lines whose direction cosines are $(l_{1,} m_{1,} n_1)$ and $l_{2,} m_{2,} n_2)$
3. Show that the straight lines whose direction cosines are given by the equations al + bm + cn = 0, $al^2 + vm^2 + wn^2$=0 are perpendicular or parallel according as $a^2(v + m) + b^2(w + u) + c^2(u + v) = 0$ or $\frac{a^2}{u} + \frac{b^2}{v} + \frac{c^2}{w} = 0$

## UNIT II

## Section A

1. Write down the condition for a line $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ in the plane ax + by + d =0
2. Define skew lines
3. Write condition for two lines are coplanar
4. Find the value of k , so that the lines $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-5}{1}=\frac{z-6}{-5}$ are perpendicular to each other
5. What is meant by unsymmetric form of the equation of a line?

## Section B

1. Show that the lines $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}, \frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ intersect and find the coordinates of the point of intersection
2. Find the equation of the plane passing through (-1,1,1), (1,-1,1) and perpendicular to the plane x + 2y + 2z = 5
3. Find the equation of the plane passing through the point (1,1,1) and the intersection of the plane x + y + z = 6 and 2x + 3y + 4z +5 = 0
4. A square ABCD of diagonal 2a is folded along the diagonal AC, so that the planes DAC, BAC are at right angles. Show that the shortest distance between DC and AB is then $2a/\sqrt{3}$

## Section C

1. Find the length and equation of the line of shortest distance between the lines $\frac{x+3}{-4}=\frac{y-6}{3}=\frac{z}{2}, \frac{x+2}{-4}=\frac{y}{1}=\frac{z-7}{1}$
2. Prove that the lines $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z-10}{8}, \frac{x-4}{1}=\frac{y+3}{-4}=\frac{z-1}{7}$ intersect. Find also their point of intersection and the plane through them
3. Find the magnitude and the equations of the line of shortest distance the lines $\frac{x-8}{3}=\frac{y+9}{-16}=\frac{z-10}{7}, \frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$

# UNIT III

## Section A

1. Define great circle in a sphere
2. Find the centre and radii of the sphere $2x^2 + 2y^2 + 2z^2 - 2x + 4y + 2z + 3 = 0$
3. Find the radius and centre of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z = 2$
4. When 2 spheres with radius $r_1$ and $r_2$ and centre $c_1$ and $c_2$ touch each other externally
5. Determine the equation of the sphere with centre (a, b, c) and radius r units

## Section B

1. Find the equation to the sphere through the four point (4, -1,2), (0, -2, 3), ( 1,-5,-1) and (2,0,1)
2. Show that the plane 2x +2y + z + 12 =0 touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$
3. If r be the radius of the circle $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0, lx + my + nz = 0.$ Prove that $(r^2 + d)(l^2 + m^2 + n^2) = (mw - nv)^2 + (nu - lw)^2 + (lv - mu)^2$
4. Find the centre and radius of the circle x + 2y + 2z = 15, $x^2 + y^2 + z^2 - 2y - 4z = 11$
5. Find the equations of the two tangent plane to the sphere , $x^2 + y^2 + z^2 = 9$ Which pass through the line x + y = 6,

## Section C

1. Derive the equation of the sphere drawn on the line joining $(x_{1,}y_{1,}z_1)$ and $(x_{2,}y_{2,}z_2)$ as diameter
2. Find the equation of the tangent planes to the sphere $x^2 + y^2 + z^2 + 2x - 4y + 6z - 7 = 0$ which intersects the line 6x – 3y – 2z = 0 =3z + 2
3. Show that the spheres $x^2 + y^2 + z^2 = 64$ and $x^2 + y^2 + z^2 - 12x + 4y - 6z + 48 = 0$ touch internally and find their point of contact

## UNIT IV

## Section A

1. Define Right circular cone
2. Write down the equation of the tangent plane $(x_{1,}y_{1,}z_1)$ to cone
3. Find the equation of the cone of the second degree which passes through the axes
4. Write down the condition for the second order homogeneous equation $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ represents (i) a cone (ii) a pair of planes

## Section B

1. Find the equation of the cone whose vertex is (1,2,3) and which passes through the circle $x^2 + y^2 + z^2 = 4$, x + y + z = 1
2. Show that $33x^2 + 13y^2 - 95z^2 - 144yz - 96zx - 48xy = 0$ represents a right circular cone whose axis is the line 3x = 2y = z. Find its vertical angle
3. Find the equations of the tangent planes to the cone $9x^2 - 4y^2 + 16z^2 = 0$ which contain the line $\frac{x}{32} = \frac{y}{72} = \frac{z}{27}$
4. Prove that the equation $2x^2 - 7y^2 + 2z^2 - 10yz - 8zx - 10xy + 6x + 12y - 6z - 3 = 0$ represents a cone whose vertex is $(\frac{1}{4}, \frac{1}{2}, \frac{1}{4})$
5. Find the equation to the right circular cone whose vertex is at the origin, whose has a vertical angles of $60^\circ$

## Section C

1. The axis of the right circular cone with the vertex at the right makes equal angles with the coordinates axes. If the equation of the con is $4(x^2 + y^2 + z^2) + 9(xy + yz + zx) = 0$. Prove that the semi vertical angle of the cone is $cos^{-1}[\frac{1}{3\sqrt{3}}]$
2. Find the condition for equation $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2fxy = 0$ to represent a right circular con

3. Show that the equation of a right cone which passes through (2,1,3) and has its vertex at the point (1,1,2) and axis the line $\frac{x-1}{2} = \frac{y-1}{-4} = \frac{z-2}{3}$ is $17x^2 - 7y^2 + 7z^2 + 24yz + 16xy - 12zx - 18x - 114y - 28z + 70 = 0$

# UNIT V

## Section A

1. Define central quadric
2. Write down the equation of the normal to the cone at $(x_{1,} y_{1,} z_1)$
3. Write the condition for the plane lx + my + nz = p to touch the conicoid $ax^2 + by^2 + cz^2 = 1$
4. Write down the condition that the cone has three mutually perpendicular generators
5. Write down the two tangent planes to a conicoid parallel to the plane lx + my + nz = 0

## Section B

1. Show that the cone whose vertex is at the origin and which passes through the circle of intersection of the sphere $x^2 + y^2 + z^2 = 3r^2$ and any plane at a distance r from the origin, has three mutually perpendicular generators
2. Find the intersection of a straight line and a quadric cone
3. Find the equations of the tangent plaes to the ellipsoid $\frac{x^2}{6} + \frac{y^2}{3} + \frac{z^2}{2} = 1$ which intersect on the line $\frac{x}{3} = \frac{y-3}{-3} = \frac{z}{1}$. Find also the coordinates of the point of contact.
4. Find the equations of the two tangent planes of the ellipsoid $2x^2 + 2y^2 + 2z^2 = 2$ which passes through the line z = 0, x + y = 10
5. Find the angle between the lines given by x + y + z = 0, $\frac{yz}{b-c} + \frac{zx}{c-a} + \frac{xy}{a-b} = 0$

## Section C

1. Show that the planes 3x + 2y + z = k touches the ellipsoid $3x^2 + 4y^2 + z^2 = 20$ if k = $\pm$10 and find the length of the chord of contact between the two tangent planes
2. Find the equations of the tangent planes to the cone $9x^2 - 4y^2 + 16z^2 = 0$ which contain the line $\frac{x}{32} = \frac{y}{72} = \frac{z}{72}$
3. Find the equation to the cone through the coordinate axes and the lines in which the plane lx + my + nz = 0 cuts the cone $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2fxy = 0$