Ordinary Differential Equation, Partial Differential Equation, Lap Lace Transforms And Vector Analysis.

Section – A

UNIT – I

1. $x^2 p_2 + 3xyp + 2y^2 = 0$
2. Find the particular integrands of $\left(D^2 + 16\right) y = \cos 4x$
3. Define : Linear differential equation.
4. Eliminate the arbitrary function. $z = f\left(\frac{xy}{z}\right)$
5. Find the particular integrals of $o + \left(D^2 + 16\right) y = e^{-3x}$
6. Define Clairaut’s equation.
7. Find the PDE by eliminating the ordinary constant form $z = px + by + {}^{a}\!/_{b}$.
8. Solve $xp + yq = x$.

Section – B

1. Solve. $x - yp = ap^2$
2. Solve. $\left(D^3 - D^2 - D + 1\right) y = 1 + x^2$
3. Solve. $\left(D^3 - 8D + 9\right) y = 8\sin 5x$
4. Solve. $\left(D^2 + 3D^2 - 4\right) y = e^x + \cos 3x$
5. Solve. $y = xp + x\left(1 + p^2\right)^{1/2}$
6. Solve $x = P^2 + y$
7. Solve $x = \tan^{-1}(P) + \frac{P}{1 + P^2}$
8. $(D_2 - 6D + 5)\ y = e^{2x}$
9. Form the P.D.E by eliminating arbitrary constant of equt $z = ax + by + a^2\ b^2$
10. Solve $P + \left(1 + q\right) = qz$.
11. Solve $P^2 + q^2 = x + y$

Section – C

1. Solve. $\left(D^2 - 2D + 4\right) y = e^x \cos x$
2. Solve. $\left(D^3 - 2D + 4\right) y = e^x \cos x$
3. Solve. $\left(D^2 + 1\right) y = x^2 e^{2x} + x \cos$
4. Solve $P^2 + 2yp\, Cotx = y^2$.
5. Solve the equation $\left(D^2 + 5D + 4\right) y = x^2 + 7x + 9$.

UNIT –II

Section – A

1. Eliminate the arbitrary function from $z = f\left(x^2 + y^2\right)$ and find the partial differential equation.
2. Solve. $q = y^2 q^2$
3. Solve. $\left(d^3 - 3d^2 + 4\right) y = 0$
4. Write down claimant's form.
5. Solve. $z = px + qy + pq$
6. Define Lagrange's linear equation.
7. $P\,(1+q) = qz$

Section – B

1. Solve. $\left(3z - 4y\right) p + \left(4x - 2z\right) q = 2y - 3x$
2. Solve. $q\left(p - \sin x\right) = \cos y$
3. Solve. $p\left(1 - q^2\right) = q\left(z - 1\right)$
4. Solve. $z = px + qy + \sqrt{1 + p^2 + q^2}$
5. Solve. $pxy + pq + qy = y^2$
6. $\sqrt{p} + \sqrt{q} = 1$
7. Solve $z = px + qy + 2\left(\sqrt{pq}\right)$.

Section – C

1. Solve. (i) $z = p^2 + q^2$ (ii) $z = px + qy - 2\sqrt{pq}$

2. Determine the surface which satisfies the differential equation $\left(x2 - a2p\right) + \left(xy - az\tan\alpha\right) \quad q = xz - ay\cot\alpha$ and passes through the curve $x^2 + y^2 = a^2$, z=0

3. Obtain the complete solution of $xp^2 - ypq + y^3q - y^2z = 0$

4. Solve $z = px + qy + \sqrt{1 + p^2 + q^2}$.

UNIT – III

Section – A

1. Find $L\left(\frac{1-e^t}{t}\right)$

2. Find $L^{-1}\left(\frac{1}{s\left(s+a\right)}\right)$

3. Find $L\left(t^2 + 2t + 3\right)$

4. State final value theorem in Laplace transform.

5. Find $L\left(\cosh at\right)$

Section – B

1. Find $L^{-1}\left[\frac{s+2}{\left(s^2+4s+5\right)^2}\right]$

2. Find $L^{-1}\left[\frac{s}{s^2a^2+b^2}\right]$

3. Find $L\left(\frac{\sin^2 t}{t}\right)$

4. Evaluate $\int_0^\infty te^{-3t}\cos t dt$

5. Find $L\left[te^{-t}\sin t\right]$

Section – C

1. Using Leplace transform solve

$$\frac{d^2y}{dt^2}+2\frac{dy}{dt}-3y=\sin t,\ \mathrm{y}(0)=0 \text{ and } \mathrm{y}'(0)=0$$

2. Evaluate the following integrads $(a)\int_0^\infty \frac{e^{-3t}-e^{bt}}{t}dt \quad (b)\int_0^\infty \frac{e^{-t}\sin^{2t}}{t}dt$

3. Using Leplace transform solve $y''-2y'+y=(t+1)2,\ \mathrm{y}(0)=4$ and $\mathrm{y}'(0)=-2$

UNIT-IV

Section – A

1. Prove that $div\overline{\gamma}=3$

2. If $\overline{f}$ and $\overline{\mathrm{g}}$ are irrotational, show that $\overline{f}\times\overline{g}$ is solenoidal.

3. Find $L^{-1}\left(\frac{1}{(s+2)^2+16}\right)$

4. Find $L^{-1}\left(\frac{s}{(s+2)^2}\right)$

5. Find $\nabla\phi$ if $\phi=xy^2+yz^3$

Section – B

1. Find the directional derivative of $\phi=xy+yz+zx$ at (1, 2, 0) in the direction of the vector $\overline{i}+2\overline{j}+2\overline{k}$

2. Prove that $div\left(\overline{u}\times\overline{v}\right)=\overline{v}.cur/\overline{u}-\overline{u}.cur/\overline{v}$

3. Find $L^{-1}\left[\frac{1}{(s+1)(s^2+2s+2)}\right]$

4. Find $L^{-1}\left[\frac{2(s+1)}{(s^2+2s+2)^2}\right]$

5. Prove that $div\left(\overline{r}+\overline{a}\right)=o$ where $\overline{a}$ is a constant vector.

Section – C

1. Prove that $(i)\, div$ grad $r^n = n(n+1)r^{n-2}$ $(ii)\, curl$ grad $r^n = 0$
2. Solve the differential equation $\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + 24y = 24x$ given that $y = \frac{dy}{dx} = 0$ when x = 0
3. Prove that $div(r^n \vec{r}) = (n+3)r^n$. Deduce that $r^n \vec{r}$ is solenoidal if and only if n = -3

UNIT – V

Section – A

1. State stoke's Theorem.
2. Define line Integral.
3. Find the unit normal to the surface $x^3 + xyz + z^3 = 1$ at (1,-1,1)
4. If $\phi = x^2 z + e^{y/x}$ and $\psi = 2z^2 y - xy^2$ find $\nabla(\phi + \psi)$
5. Evaluate $\int_0^1 \int_0^1 (x^2 + y^2)\, dxdy$

Section – B

1. $\overline{f} = (3x^2 + 6y)\bar{i} - 14yzj + 20xz^2k,$ Evaluate along the path from (0, 0, 0) to (1, 1, 1) $\int_c \overline{f}.d\overline{r}$ along the path $x = t,$ y=t^2, z=t^3
2. Evaluate $\int_c \overline{f}.\overline{n}$ where $\overline{F} = 18z\bar{i} - 12\overline{j} + 3y\overline{k}$ and S is the surface of the plane. 2x+3y+6z=12 in the first octant.
3. Find div f and Curl f for $f = (y^2 - z^2 + 3yz - 2x)i + (3xz + 2xy)j + (3xy - 2xz + 2z)k$
4. If $\phi = \log r,$ where $\overline{r} = x\bar{i} + y\bar{i} + z\overline{k},$ Prove that $\nabla^2(\log r) = \frac{1}{r^2}$
5. (i) Define surface integral.

   (ii) If $f = x^2\vec{i} - xy\vec{j}$ and C is the straight line joining the points (0, 0) and (1, 1) find $\int_c \vec{f}.d\vec{r}$

Section-C

1. Verify Gauss divergence theorem for $\overline{F} = 4x\bar{i} - 2y^2\overline{j} + z^2\overline{k}$ taken over the region bounded by $x^2 + y^2 = 4$, z=0 and z=3.

2. Prove that $(a)\left(r^n\overline{r}\right) = (n+3)rn$ $(b) Curl\left(r^n\overline{r}\right) = 0$

3. Verify stake's theorem for $\vec{f} = y^2z\vec{i} + z^2x\vec{y} + x^2y\vec{k}$ where S is the open surface of the cube forward by the planes $x = \pm a$, y= ± a and z= ± a, in which the plane Z=-a is cut.