

## Differential equations and Laplace Transforms

### Section – A

#### UNIT – I

1. Solve.  $\left(\frac{dy}{dx}\right)^2 - ax^2 = 0$
2. Solve.  $P = \log(px - y)$
3. Verify whether  $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$  is exact.
4. Solve.  $xy^2 + p(3x^2 - 2y^2) - 6xy = 0$
5. Solve.  $a(x dy + 2y dx) = xy dy$
6. Solve.  $y = (x-a)p - p^2$ .
7. Solve  $p^3 - 7p - 6 = 0$ .
8. Define periodic function .
9. Define particular Integrals
10. Define Lagrange's equation.
11. Define non-homogeneous.
12. Solvable for  $P^2 - 3P + 2 = 0$
13. Solvable for  $x = y + a \log P$

### SECTION – B

1. Solve.  $P^2 + \left(x + y - \frac{2y}{x}\right)P + xy + \frac{y^2}{x^2} - y - \frac{y^2}{x} = 0$
2. Solve.  $(a^2 - 2xy - y^2)dx - (x + y)^2 dy = 0$
3. Solve.  $y^2 \log y = xpy + p^2$
4. Solve.  $x = yp + ap^2$
5. Solve.  $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$
6. Solve.  $(D^2 - 4D + 3)y = \sin 3x \cos 2x$

7. 11. Solve  $x = \tan^{-1}(p) + \frac{p}{1+p^2}$  .
8. Solve  $\frac{dy}{dx} + \frac{y \cos x + \sin y + y}{\sin x + x \cos y + x} = 0$
9.  $(D^3 + 3D^2 + 3D + 1)y = e^{-x}$
10. Solve  $(D^2 - 4D + 4)y = e^{2x} + \cot 2x$  .

### SECTION -C

1. Solve  $(D^2 + 5D + 4)y = x^2 + 7x + 9$
2. Solve. (i)  $x - yp = ap_2$  (ii)  $y^2 = (1 + p^2)$
3. Solve. (i)  $xy(y - px) = x + yp$  (ii)  $x^2 p^2 - 2xyp + 2y^2 - x^2 = 0$
4. Solve.  $(y^2 + 2x^2 y)dx + (2x^2 - xy)dy = 0$  .
5. Find the general solution integral and singular solution of the equation
6.  $y = x \frac{dy}{dx} + \frac{a \frac{dy}{dx}}{\sqrt{1 + (\frac{dy}{dx})^2}}$
7. Solve  $x = \tan^{-1}(P) + \frac{P}{1 + P^2}$  .
8. Solve  $(D^2 + 9)y = (x^2 + 1)e^{3x}$

### UNIT - II

1. Solve.  $(D^4 - D^3 - 9D^2 - 11D - 4)y = 0$
2. Find the particular integral of  $(4D^2 + 12D + 9)y = 144e^{-3x}$
3. Solve.  $(D^4 - 4D^3 + 8D^2 - 8D + 4)y = 0$
4.  $(D^4 + 2D^2 n^2 - n^4)y = \cos mx$  Find the particular integrals.
5. Solve.  $\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + y = 0$

6. Find the particular integral of
7.  $(D^2 + 5D + 6)y = e^x$
8. Solve  $(D^2 - 6D + 5)y = e^{2x}$ .
9. Define Clairaut's equation.
10. Define Exact equation.

### SECTION- B

1. Solve  $(D^2 - 6D + 5)y = e^{2x}$ .
2. Solve.  $y = xp + x(1 + p^2)^{1/2}$
3. Solve.  $(D^3 - D^2 - D + 1)y = 1 + x^2$
4. Solve.  $(D^3 - D)y = 2 \cosh x$
5. Solve.  $(D^3 + 8)y = x^4 + 2x + 1$
6. Solve.  $(D^2 - 4D + 3)y = \sin 3x \cos 2x$
7. Solve.  $3x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = x$
8. Solve  $(D^2 - 2D + 1)y = x e^x \sin x$ .
9. Solve  $(D^2 + 6D + 8)y = e^{-2x} + \cos^2 x$ .
10. Solve  $p^2 + q^2 = 4$ .
11. Solve  $(D + D')z = e^{x-y}$
12. Solve  $(D^2 - DD')z = \sin x \sin 2y$ .

### SECTION-C

1. Solve the equation  $\frac{d^2y}{dx^2} + y = \sec x$  using method of variation of parameters.
2. Solve  $(D^2 - 2D + 2)y = e^x \tan x$  by the method variation of parameter.

3. Solve.  $(1-x)y^3 + (x^2-1)y^2 - x^2y + xy = 0$
4. Solve  $\frac{d^2y}{dx^2} + 4y = 4 \tan 2x$  using method of variation parameter
5. Solve  $(D^2 - D'^2)Z = XY$  Satisfying the continuous  $\frac{\partial Z}{\partial Y} = 0$   
where  $y = 0$ .
6. Solve  $\frac{dy}{dx} + \frac{y \cot x + \sin y + y}{\sin x + x \cos y + x} = 0$

### UNIT – III

1. Solve.  $\frac{d^2z}{dy^2} = \sin y$
2. Form the PDE by eliminating the arbitrary function from  
 $z = f(x^2 - y^2)$
3. Find the complete integral of  $z = px + qy + c\sqrt{1 + p^2 + q^2}$
4. Eliminate a and b from  $\frac{x^2 + y^2}{a^2} + \frac{z^2}{b^2} = 1$  and find partial differential equation.
5. Solve.  $PQ = K$ .
6. Define PDE.
7. Solve  $(D^3 - 4D^2D' + 4DD'^2)Z = 0$ .
8. Define particular Integrals

### UNIT – IV

1. Solve.  $(D^2 + DD' + 2D^{12})z = 0$
2. Find the complete integral of  $(D^2 - D^{12})Z = e^{x+y}$
3. Solve.  $(D^3 - 4D^2D' + 4DD'^2)z = 0$
4. Find the complete integral of  $z = px + qy + c\sqrt{1 + p^2 + q^2}$
5. Solve.  $\frac{d^2z}{dy^2} = \sin y$
6. Define Complete Integrals. .
7. Define particular Integrals
8.  $(D^2 + 5D + 6)y = e^x$

### SECTION-B

1. Solve  $(D^2 - 6D + 5)y = e^{2x}$ . Solve.  $\frac{\partial^2 z}{\partial x^2} = a^2 z$  given that when  $x = 0, \frac{\partial z}{\partial x} = a \sin y$  and  $\frac{\partial z}{\partial y} = 0$
2. Solve.  $p(1 + q^2) = q(z - 1)$  13. Form the PDE by eliminating arbitrary constant from  $\log (az - 1) = x + ay + b$ .
3. Solve  $x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = 2 \log x$ .
4. Solve.  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
5. Find the partial differential equation by Eliminating a, b, c from  $x(y^2 + z)p - y(x^2 + z)q = z(x^2 - y^2)$
6. Solve.  $(y + z)p + (z + x)q = x + y$
7. Solve.  $q = xp + p^2$ .

8. Form the PDE by eliminating arbitrary constant from
9.  $z = x f_1(x + t) + f_2(x + t)$ .
10. Solve  $3p^2 - 2q^2 = 4pq$
11. Solve  $p^2 + q^2 = 4$ .
12. Solve  $p^2 + q^2 = npq$ .
13. Solve  $p^2 + q^2 = x + y$

### SECTION -C

1. Solve  $px(y^2 + z) - qy(x^2 + z) = z(x^2 - y^2)$
2. Find the surface that contains the straight line  $x + y = 0, z = 1$
3. Find the complete Integral of  $pxy + pq + qy = yz$
4. Solve.  $x^2 \frac{\partial^2 z}{\partial x^2} + y^2 \frac{\partial^2 z}{\partial y^2} = (x + y)z$  Solve  $Z = Px + qy + \sqrt{1 + p^2 + q^2}$
5. Solve  $Z = Px + qy + \sqrt{1 + p^2 + q^2}$
6. Solve  $\frac{\partial^2 z}{\partial x^2} + 3 \frac{\partial^2 z}{\partial x \partial y} + 2 \frac{\partial^2 z}{\partial y^2} = x + y$ .

### UNIT - IV

1. Solve.  $(D^2 + DD' - 2D'^2)z = 0$
2. Write the particular integral of  $(D^2 - DD')z = \sin(x + y)$ .
3. Find the particular integral of  $(D^2 - 2DD' + D'^2)z = e^{2x+3y}$
4. Find the complete integral of  $(D^2 - D'^2)z = e^{x+y}$
5. Define laplace Transform.
6. Define periodic function Solve.  $(D^2 - DD')z = \sin x \sin 2y$
7. Solve.  $(D^2 - DD' + D')z = 2x + 3y$

8. Solve.  $\frac{\partial^2 x}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \cos mx \cos ny$
9. Solve.  $(2D^2 + 5DD' + 2D'^2)z = 24(y - x)$

### SECTION -B

1. Solve.  $\frac{\partial^2 z}{\partial x^2} + \frac{4\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = e^{2x-y}$
2. Solve.  $(D^2 - 4D'^2)z = \cos 4x \cot 3y$ . Form the PDE by eliminating arbitrary constant from

$$z = x f_1(x + t) + f_2(x + t).$$

3. Solve  $3p^2 - 2q^2 = 4pq$
4. Solve  $(D + D')^2 z = e^{x-y}$
5. Solve  $(D^2 - DD')z = \sin x \sin 2y$ .
6. Solve  $(D + D')^2 z = e^{x-y}$

### SECTION-C

1. Solve  $(D^2 - DD')z = \sin x \sin 2y$ .
2. Solve.  $(D^2 - DD' + D'^2)z = 2x + 3y$
3. Solve.  $(D^2 - DD' + D'^2)z = 2x + 3y$
4. Solve.  $(D^2 - 6DD' + 5D'^2)z = ex \sinh y + xy$
5. Solve  $p^2 - q^2 = n pq$ .
6. Solve  $(D^2 - D'^2)Z = XY$  Satisfying the continuous  $\frac{\partial z}{\partial y} = 0$  where  $y = 0$ .

**UNIT – V**

**SECTION -A**

1. Find  $[\sin^2 2t]$
2. Find  $L^{-1}\left[\frac{1}{S(S+a)}\right]$
3. Find  $L\left[\frac{\sin at}{t}\right]$
4. Find  $L^{-1}\left[\frac{s}{s^2+k^2}\right]$
5. Prove that  $L[f'(t)] = sL[f(t)] - f(0)$

**SECTION-B**

1. Find  $L^{-1}\left[\frac{1}{(s+a)^2}\right]$  Find  $L[f(t)] = \begin{cases} 1 & 0 < t < b \\ -1 & b < t < 2b \end{cases}$
2. Find  $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$
3. Find  $L[t^2 + \cos 2t + \sin 2t]$
4. Find  $L^{-1}\frac{s^2-s+2}{s(s-3)(s+2)}$
5. Find  $L^{-1}\left[\frac{1+2s}{(s+2)^2(s-1)^2}\right]$
6. Find  $L^{-1}\left[\frac{s+2}{(s^2+4s+5)^2}\right]$ .

**SECTION-C**

1. State and prove first shifting theorem .
2. Find  $L^{-1}\left[\log \frac{s-a}{s^2+a^2}\right]$



3. Solve  $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + dy = 4e^{-t}$  given that  $y = \frac{dy}{dt} = 0$ , when  $t=0$
4. Solve.  $t\frac{d^2y}{dt^2} - (2+t)\frac{dy}{dt} + 3y = t - 1$  when  $y(0) = 0$
5. Solve  $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 4e^{-t}$  given that  $y = \frac{dy}{dt} = 0$  when  $t=0$ .
6. State and prove convolution theorem.