$\cos x + \sin x \log(\sin x)$

Unit - III

Partial Differential equations

In general higher derivatives of the types

$\frac{\partial^2 u}{\partial x^2}$, $\frac{\partial^2 u}{\partial x \partial y}$, $\frac{\partial^3 u}{\partial x^3}$ may be of physical significance.

When the laws of physics are applied to a problem of this kind. we many sometimes, obtain a relation between the derivatives of the kind

$$\phi\left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial^2 u}{\partial x^2} \cdots\cdots \frac{\partial^2 u}{\partial x \partial y}\right) = 0$$

Such an equation relating partial derivatives is called a "Partial Differential equation."

A partial differential equation is an equation involving a function of two or more variables and some of its partial derivatives.

Therefore a partial differential equations contains

One dependent variable. Hence the main difference between the partial and ordinary differential equations is the number of independent variables involved in the equation.

Examples:

1. $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y}$ (u - dependent variable x, y independent variable)

2. $\left(\frac{\partial u}{\partial x}\right)^3 + \frac{\partial u}{\partial y} = 0$ (u - dependent variables x, y independent variable)

3. $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{\partial u}{\partial t} = 0$ (u - dependent variable x, y, t indep. var)

The order of a partial differential equation is the order of the highest partial derivative occuring in the equation.

In the above, e.g 1 is a second order equation in two variables, eg. 2 is a first order equation in two variables and eg. 3 is a first order equation in three variables.

Formation of partial differential equations:

In practice, there are two methods to form a partial differential equation

i) By elimination of arbitrary

ii) By elimination of arbitrary function.

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Formation of Partial Differential Equations by :

Elimination of Arbitrary constant :

$$\text{Let } f(x, y, z, a, b) = 0 \longrightarrow (1)$$

be an equation which contains two arbitrary contants a and b. we know that, to eliminate two constants we need atleast three equations.. Therefore partially differentiating equation (1) with respect to x and y we get two more equations. From there three equations we can eliminate the two constants 'a' and 'b' similary, for eliminating three constants we need four equations and so on.

where,

$$p = \frac{\partial z}{\partial x}, \; q = \frac{\partial z}{\partial y}, \; r = \frac{\partial^2 z}{\partial x^2}, \; s = \frac{\partial^2 z}{\partial x \partial y}, \; t = \frac{\partial^2 z}{\partial y^2}$$

Elimination of orbitary constant :

Form the partial differential equation by eliminatary arbitrary constant from $z = ax + by + a^2 + b^2$

Soln :

Given equation

$$z = ax + by + a^2 + b^2 \longrightarrow (1)$$

Differentiating we respect to x

$$\frac{\partial z}{\partial x} = a$$

$$p = a \longrightarrow (2)$$

Differentiating we respect to y

$$\frac{\partial z}{\partial y} = b$$

$q = b \rightarrow$ ②

Sub ② & ③ in ①

$$z = px + qy + p^2 + q^2$$

is the required partial differential equation after elimination of a and b

Form the partial differential equation by diminatary arbitary constant from $z = (x-a)^2 + (y-b)^2$

Soln:

Given equation

$$z = (x-a)^2 + (y-b)^2 + 1 \longrightarrow ①$$

Differentiating we respect to x

$$\frac{\partial z}{\partial x} = 2(x-a)$$

$$p = 2(x-a)$$

$$\frac{p}{2} = (x-a) \longrightarrow ②$$

D we respect to y

$$\frac{\partial z}{\partial y} = 2(y-b)$$

$$q = 2(y-b)$$

$$\frac{q}{2} = y-b \longrightarrow ③$$

Sub ② & ③ in ①

$$z = \left(\frac{p}{2}\right)^2 + \left(\frac{q}{2}\right)^2 + 1$$

$$z = \frac{p^2}{4} + \frac{q^2}{4} + 1$$

$$z = \frac{p^2+q^2+4}{4}$$

$$4z = p^2+q^2+4$$

Is the required partial differential equation after elimination a and b

Form the orbitary partial differential equation by eliminatary arbinatrary constant from $z=(x^2+a)(y^2+b)$

Soln :

Given equation

$$z = (x^2+a)(y^2+b) \longrightarrow ①$$

Differential we respecte to x

$$\frac{dz}{dx} = (y^2+b)\,2x$$

$$p = (y^2+b)\,2x$$

$$\frac{p}{2x} = y^2+b \longrightarrow ②$$

Differential we respect to y.

$$\frac{\partial z}{\partial y} = (x^2+a)\,2y$$

$$q = (x^2+2)\,2y$$

$$\frac{q}{2y} = (x^2+a) \longrightarrow ③$$

Sub ② & ③ in ①

$$z = \left(\frac{q}{2y}\right)\left(\frac{p}{2x}\right)$$

$$z = \frac{pq}{4xy}$$

$$pq = 4xyz$$

Is the required partial differential equation after elimination of a and b

Form the partial differential aquation by eliminatory arbinatray constant $\log(az-1) = x+ay+b$

Soln : Given equation

$$\log(az-1) = x+ay+b$$

Differential we respect to x

$$\frac{1}{az-1} \cdot \frac{a\,dz}{dx} = 1$$

$$\frac{ap}{az-1} = 1$$

$$ap = az - 1$$

$$1 = az - ap$$

$$1 = a(z-p)$$

$$a = \frac{1}{z-p} \longrightarrow ②$$

Differential we respect to y

$$\frac{1}{az-1} \cdot a\frac{dz}{dy} = a$$

$$\frac{aq}{az-1} = a$$

$$\frac{q}{az-1} = 1$$

$$q = az - 1 \longrightarrow ③$$

Sub ② in ③.

$$\therefore q = \frac{z}{z-p} - 1$$

$$q = \frac{z - z + p}{z-p}$$

$$qz - pq = p$$

$$qz = p + pq$$

$$qz = p(1+q)$$

Is the required partial differential equation after elimination a and b

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