The standard Error some of ... are give below Statistics for large samples

Statistics	Standard Error
1. Sample Mean : $\bar{x}$	$\sigma/\sqrt{n}$
2. sample standard divation : s	$\sqrt{\sigma^2/2n}$
3. Sample Variance : S^2	$\sigma^2\sqrt{2/n}$.
4. Difference of two sample Means : $\bar{x}_1 - \bar{x}_2$	$\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$.
5. Different of two Sample standard divation is : $S_1 - S_2$	$\sqrt{\frac{\sigma_1^2}{2n_1}+\frac{\sigma_2^2}{2n_2}}$.
6. Absenved sample proposition : P	$\sqrt{PQ/n}$.
7. Different of two Sample proportion : $P_1 - P_2$	$\sqrt{\frac{P_1 q_1}{n_1}+\frac{P_2 q_2}{n_2}}$

Testing of hypothesis :- As sample investigation Produce results and with these results discision Made on the population with an assemsion is called statistical hypothesis

Hypothesis :- Hypothesis is an assumption which May or may not be true about the population Parameter

Types of hypothesis :- i) Null Hypothesis
ii) Alternative Hypothesis..

Null hyp[illegible] the hypothesis which set up on null and nutrial basis then that is called as null hypothesis and it is denoted by H_0

Alternative hypothesis:- Rejection H_0 least to the acceptance of the alternative hypothesis which is denoted by H_1

Example:

$H_0 : \mu = 50$ Vs H_1:

When we are Conducting a test we make, Come across four types, of situation

i) We May reject H_0, when H_0 is true (Type 1 error)

ii) We May accept H_0, when H_0 is false (Type II error)

H_0 true Truly iii) Accept H_0, when H_0 is true (correct decision)

iv) Reject H_0, when H_0 is false

H_0 true Truly not guilty

Therefore,

α = probability of Type I error

(ie) = p(Reject H_0/ H_0 is true)

β = probability of Type II error

= p(accept H_0/H_0 is false)

Type I and Type II Error:-

Rejecting a null hypothesis when it is true is called type I error in other words is Called as producer risk.

Accepting a null hypothesis when it is false is called type II error is other words it is called as Consumer

Level of significance:-

The Maximum probability of committing Type I error which we specified in a test that is the probability that a random value (α) the statistics lies in the critical region is called the level of significance.

And is usually expressed as a percentage in other words the total area of the critical region expressed as α% (percentage in the LoS

from the study of Normal distibution it is known that probability of

$$P\{E(t) - 1.96\, S.E(t) < t < E(t) + 1.96\, S.E(t)\} = 0.95$$

(ie) $P\left\{\left|\frac{t - E(t)}{S.E(t)}\right| < 1.96\right\} = 0.95$

$$P\{|Z| > 1.96\} = 0.05 \text{ (or) } 5\%$$

Thus went t lies in either of two region constuting the critical region given above the LoS is 5%.

Note:

The specification of critical region and the choice of LoS will depends upon the nature of the problem and is a matter of judgement for those who carried out the investication.

$\alpha = 5\%$, 1% & 2%

Critical Region:-

The range of variance has two regions.

i) Acceptance region

ii) Rejection (or) Critical region

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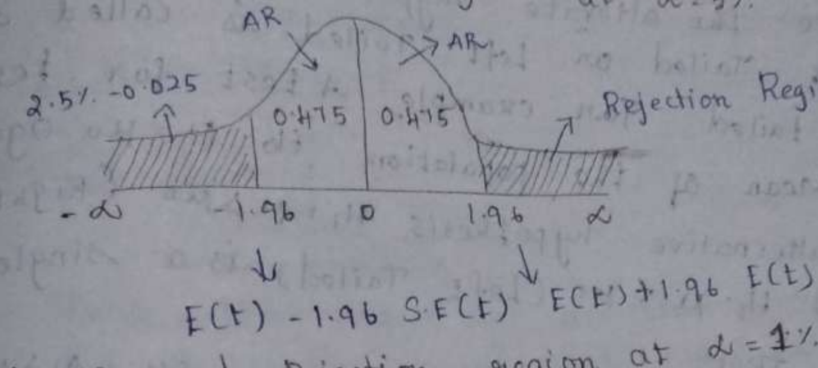
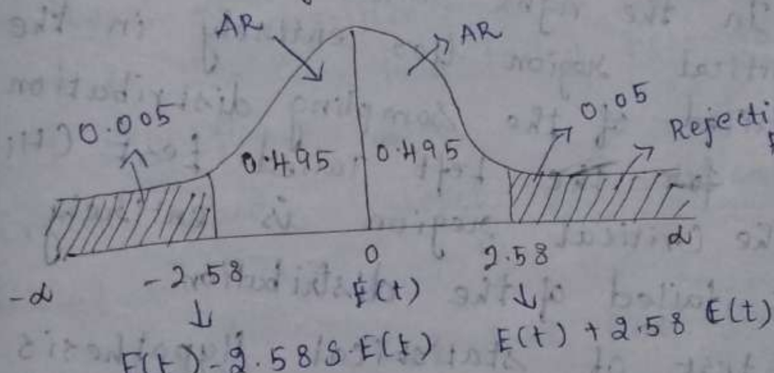
If a sample statistics falls in a critical region we have two reject H_0, and it is false in acceptance region we accept H_0.

Acceptance and Rejection region at $\alpha = 5\%$.



Acceptance and Rejection region at $\alpha = 1\%$.



Test statistics:

If t is a statistics in a large sample then t follows a Normal distribution with mean $E(t)$, which is the corresponding population parameter, and standard deviation $= S.E(t)$

Hence $z = \dfrac{t - E(t)}{SE(t)}$ is a standard Normal variate that z is called test statistics. ~~Statx~~ Statistics follows a Normal distribution with mean zero (0) and standard deviation is unity, Symbologically noted as $z = \frac{t - E(t)}{SE(t)} \sim N(0,1)$

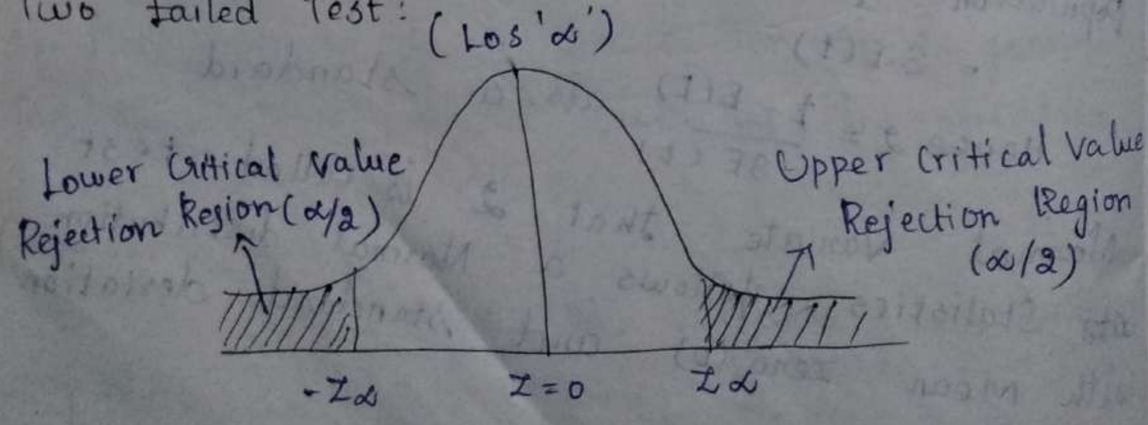
One tailed and Two tailed Test.-

A test of any statistical hypothesis where the alterate hypothesis is one tailed right Tailed or Left Tailed is called a one tailed. for example, A test for testing the Mean of the population $H_0: \mu = \mu_0$ against the alternative hypothesis $H_1: \mu > \mu_0$ (Right Tailed) $H_1: \mu < \mu_0$ (Left Tailed) is a single Tailed Test.

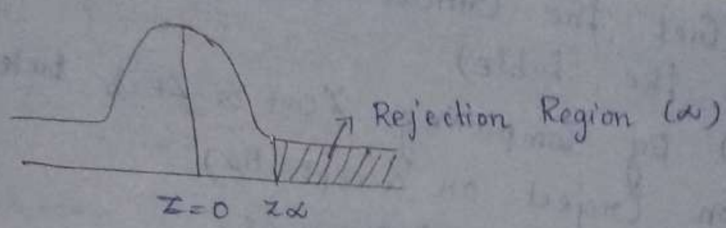
In the right Tailed Test ($H_1: \mu > \mu_0$) the critical region lies entirely in the right tailed of the sampling distribution of $\bar{x}$, while for the Left Tailed test ($H_1: \mu < \mu_0$) the critical region is entirely in the left tailed of the distribution.

A test of statistical Hypothesis where the alterative Hypothesis is two tailed such as $H_0: \mu = \mu_0$, against the alternative Hypothesis $H_1: \mu \neq \mu_0$ ($\mu > \mu_0$ & $\mu < \mu_0$) is known as two tailed Test.

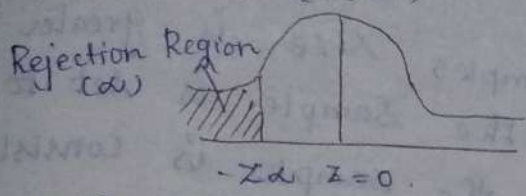
Two tailed Test: (Los 'α')



Right



Left Tailed Test (Los'α')



Thus the significant are critical value of Z for a single Tailed That (right or left) at the Level of significant α is same as critical value of Z for a two tailed test at the level of significant '2α'

Critical value (Z_α)	Level of significance (α) 1%	5%	10%
Two-tailed test	$\lvert Z_\alpha \rvert = 2.58$	$\lvert Z_\alpha \rvert = 1.96$	$\lvert Z_\alpha \rvert = 1.645$
Right tailed test	$Z_\alpha = 2.33$	$Z_\alpha = 1.645$	$Z_\alpha = 1.28$
Left tailed test	$Z_\alpha = -2.33$	$Z_\alpha = -1.645$	$Z_\alpha = -1.28$

Steps in testing of hypothesis:-

The following steps to be followed in every testing of hypothesis problem.

i) Formulate the null Hypothesis for a given problem (H_0)

ii) Find a suitable alternative Hypothesis

iii) Deside the Level of significance (α)

iv) Find appropriate test statistics

$Z = \frac{t - E(t)}{}$, under H_0 (Z_{cal}) 2020-5-17 19:12

v) Get the critical value (z_{tab}) (normally from the table)

vi) By comparing z_{cal} & z_{tab} take proper decision (reject or accept H_0)

vii) Draw the inferences.

Large Sample Test:-

When the samples size is greater than or equal to 30 ($n \geq 30$), the sample is set to be large sample as the sample is consisted as large we can use the standard normal distribution function as our test statistics we will see the following test

i) Test for single Mean.
ii) Test for differents of two Means
iii) Test for single propotion.
iv) Test for two sample propotion.

Z test for single Mean:-

To test $H_0: \mu = \mu_0$, the test statistics given by $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$, where $\bar{x}$ is the given sample Mean,

μ_0 is the specified population Mean

σ is the standard deviation.

n is the sample size.

Decision:-

If $z_{cal} < z_{tab}$ (critical value) accept the null hypothesis, otherwise reject it.

test statistical is given by,

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Decision:

If $Z_{cal} < Z_{tab}$ we accept the Null hypothesis.

v) Test of significance of the difference between sample SD and population SD is given by:

$$Z = \frac{S - \sigma}{\sigma/\sqrt{2n}}.$$

vi) Test of significance of the difference between standard deviation (SD) of 2 Large Sample is given by,

$$Z = \frac{S_1 - S_2}{\sqrt{\frac{S_1^2}{2n_2} + \frac{S_2^2}{2n_1}}}$$

S_1 - Small deviation sample 1

S2 - Small deviation sample 2.

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Unit - 5.

5M (or) 10(M)

Queueing Theory:-

Def:-

Queueing system:-

A Queueing system can be completely described by.

i) Arrival pattern.

ii) service pattern.

iii) Queue discipline.

iv) Customer Behaviour.

Kendal's Notation:-

Generally, Queueing Model May be Completely specified in the following symbol from,

(a|b|c): (d|e) where α / A = probability Law for the arrival.

B = Probability Law accroding to which Customer are served.

C = Number of Channels.

D = Capacity of the system.

E = Queue Discipline.

Model - I:- $\boxed{(M|M|1): (\infty|FCFS)}$ → First come First service

probability distribution of Queue Length is given by $\underline{p^n(1-\rho)}$ where

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$$\rho = \frac{\lambda \rightarrow \text{Arrival pattern}}{\mu \rightarrow \text{service pattern}} < 1 \ \& \ n \geq 0$$

Formula's:-

i) To find the average (Excepted) Number of Units in the system

$$L_s = \frac{\lambda/\mu}{1-\lambda/\mu}$$

$$\boxed{L_s = \frac{\rho}{1-\rho}} \quad \rho = \frac{\lambda}{\mu}$$

ii) To find the average length of Queue.

1. $L_s = \dfrac{\rho}{1-\rho}$

2. $L_q = L_s - \dfrac{\lambda}{\mu}$.

$$L_q = \frac{\rho^2}{1-\rho}$$

iii) Excepted waiting time in the system.

3. $W_s = \dfrac{L_s}{\lambda}$ (or)

$$W_s = \frac{1}{\mu - \lambda}$$

iv) ~~Expepted~~ Expected waiting time in the Queue.

4. $W_q = \dfrac{L_q}{\lambda}$ (or)

$\dfrac{\lambda}{\ }$

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v) Expected waiting time of a customer how has to wait.

5. $(W/W>0)$

$$W_n = \frac{1}{\mu - \lambda}.$$

vi) Expected length of the Non-empty Queue

6. $(L/L>0)$

$$L_n = \frac{\mu}{\mu - \lambda}$$

vii) probability of Queue size Exceed

$$N = \rho^N.$$

Viii) Traffic Intensity :- (or) Utilization factor.

$$\rho = \frac{\lambda}{\mu}$$

1) In a railway Marshalling yard goods trains arrive at a rate of 30 trains per day assuming that inter arrival time and service time distribution follows an exponential distribution with an average of 30 Minutes calculate the following

i) The Mean queue size

ii) The probability that queue size exceeds 10

iii) If the input of the train increases to and average of 33 perday what will be the changes in, i, & ii

Soln:

Given $\lambda = 30$ trains per day $= \frac{30}{24 \times 60} = 1/48$ Mins.

$\boxed{\mu = \frac{1}{30} \text{ per mins.}}$

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48

To find, i) Mean queue size

$$Ls = \frac{\rho}{1-\rho} \quad \rho = \frac{\lambda}{\mu} = \frac{1/48}{1/30} = 1/48 \times 30$$

$$\rho = 5/8$$

$$Ls = \frac{5/8}{1-5/8} = \frac{5/8}{\frac{8-5}{8}} = \frac{5/8}{3/8}$$

$$= 5/8 \cdot 8/3$$

$$\boxed{Ls = 5/3 = 1.67 \simeq 2 \text{ trains}}$$

ii) To find probability that the queue size exceed 10

$$= \rho^N \quad N=10 \quad \rho = 5/8$$

$$= (5/8)^{10}$$

$$= 0.0091$$

iii) To find input the train average 33 per day

$$\lambda = 33 \text{ per day} = \frac{3\cancel{8}}{24\times 60} = \cancel{48}\ \cancel{43.6364} \quad 0.0229$$

$$\mu = 1/30 \text{ per min} = 0.0333$$

$$Ls = \frac{\rho}{1-\rho} \quad \rho = \frac{\lambda}{\mu} = \frac{43.6364}{\cancel{0.0333}} = 1310.4024$$

$$\cancel{Ls} = \frac{1310.4024}{1-\cancel{1310.4024}} = \frac{1310.4024}{-1309.4024} = \cancel{-1.0008}$$

$$P\ \cancel{Ls} = \frac{0.0229}{0.0339} = 0.6877$$

$$Ls = \frac{0.6877}{1-0.6877} = \cancel{0.5209}\ 2.2020 \simeq$$

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ii) ρ^N, $N=10$.

$\rho=(0.6877)^{10}=0.0237$.

H.W (X).

IN a superMarket that average arrival rate of customers is 10 every 30 miuntes following Poisson process. The average time taken by a cashier to List and calculate the customers purchase is two and a half Minutes following Expondial distribution what is the probability that the queue Length execeed 6 ?

What is the expected time spend by a customer in the system?

$$\lambda=\frac{10}{30} \text{ per minute}$$

$$\mu = 2.5 \text{ Minute} = \frac{1}{2.5} \text{ per mins.}$$

i) To find the probability that the queue Size exceeds 6.

$$N=\rho^N, \quad \rho=\frac{\lambda}{\mu} \quad N=6.$$

$$\rho^N=\left(\frac{0.333}{0.4}\right)^6=(0.8325)^6$$

$$=0.3329.$$

ii) To find the Expected waiting Time in the System.

$$W_s=\frac{1}{\mu-\lambda}$$

$$=\frac{1}{0.4-0.333}$$

$$=\frac{1}{0.0670}=14.925=15$$

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3) In a public telephone booth the arrivals are on the average 15 per hours, a call on the average takes 3 mins If the always there is Just one phone find.

i) Expected number of callers in the both at any time

ii) the proposition of the time the booth is expected to be idle

Soln:- $\lambda = 15$ per hours $\mu = 3$ per Minutes

$= 1/3$ per Minutes

$= 1/3 \times 60$

$\mu = 20$ per hours.

i) To find the Expected length of the Non-empty queue.

$$L_n = \frac{\mu}{\mu - \lambda} = \frac{20}{20-15} = 20/5 = 4.$$

ii) Probability of the time the booth is expected to be idle.

$$P_0 = 1 - \rho$$

$$= 1 - \lambda/\mu = 1 - 15/20 = \frac{20-15}{20} = 5/20 = 1/4.$$

$= 0.25$ hours.

corr

4) A TV repairman find that the time spend on his Job has an exponstial distribution with Mean 30 mins. Is he repairs self in the order in which they came in

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and if the arrival of sets is poisson with an average rate of 10 per 8 hour day what is the expected idle time day? How many Jobs are ahead of the average set just brought in.

Soln:-

$\mu = 1/30$ per mins $\quad \lambda = 10/8$ per hours

$= \frac{1}{30} \times 60$ $\quad \lambda = 1.25$ per hours

$\mu = 2$ hours

i) To find the Expected idle time day.

$$\rho = \frac{\lambda}{\mu} = \frac{1.25}{2}$$

$$\rho = 0.6250.$$

8 hours day repairmen busy time

$= 8(0.6250) = 5$

The idle time day $= 8 - 5 = 3$ hours.

ii) To find how many Jobs are ahead of the average set just brought in.

$$L_s = \frac{\rho}{1-\rho} = \frac{0.6250}{1-0.6250}$$

$$= \frac{0.6250}{0.3750} = 1.667$$

$= \simeq 2$ Set of Tv.

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5) Cars arrive at a petrol pump having one petrol unit in poison fashion with an average of 10 cars per hour. The service time is distributed Expenantially with a Mean of 3 Mins find

i) Average Number of cars in the system

ii) Average waiting time in the queue.

iii) Average queue Length

iv) The probability that the Number of cars in the system is 2 (ii)

Soln:-

$\lambda = 10$ ~~cars~~ per hour $= 10$ per hrs.

$\mu = 3$ Mins. $= \frac{1}{3}$ per Mins $= \frac{1}{3} \times 60 = 20$ per hrs

To find, average number of cars in the system

$$Ls = \frac{\lambda/\mu}{1-\lambda/\mu} \text{ (or) } \frac{\rho}{1-\rho}$$

$$\rho = \frac{\lambda}{\mu} = \frac{10}{20} = 0.5$$

$$Ls = \frac{\rho}{1-\rho} = \frac{0.5}{1-0.5} = 1.$$

$$\boxed{Ls = 1 \text{ car}}$$

To find, Average waiting time in the queue

$$Wq = \frac{Lq}{\lambda}$$

$$Lq = \frac{\rho^2}{1-\rho^2} = \frac{(0.5)^2}{1-(0.5)} = 0.05 \qquad Wq = \frac{\lambda}{\mu(\mu-\lambda)}$$

$$= 0.05$$

To find, Average queue Length.

$$Lq = \frac{\rho^2}{1-\rho^2} = \frac{(0.5)^2}{1-(0.5)^2} = 0.5$$

$$\boxed{Lq = 0.5}$$

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iv) To find, Probability that the Number of cars in the system 2

$n = 2$

$$P_n = p^n (1-p)$$

$$P_2 = p^2 (1-p)$$

$$= (0.5)^2 (1-0.5)$$

$$= 0.25 \times 0.5$$

$$\boxed{P_2 = 0.1250} \quad P_2 = 1/8$$

6) Customers arrive at a one window drive in bank according to poison distribution with Mean 10 per hrs service time per customer is expantitional with Mean 5 Mins. The space infront of the window including that one for the serviced car can accommodate a Maximum of 3 Cars. other can wait outside this Space

i) what is the Probability that an arriving Customer can drive directly to the space in front of the window.

ii) What is the probability that an arriving customer will have to wait out side the idegated space.

iii) How long the arriving Customer is expacted to wait before starting service.

Soln:- Given,

$\lambda = 10$ per hurs

$\mu = 5$ Mins $= 1/5$ per $\quad 1/5 \times 60 = 12$ per

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Soln:-

$$\boxed{p^n = \rho^2(1-\rho)}$$

Probability that N units in the system

i) $P_0+P_1+P_2 = \rho^0(1-\rho)+\rho^1(1-\rho)+\rho^2(1-\rho)$

$= (1-0.8333)+0.8333(1-0.8333)+$

$P^0 (0.8333)^2(1-0.8333)$

$=(1-0.8333)\left[1+0.8333+(0.8333)^2\right]$

$= 0.1667\left[1+0.8333+0.6944\right]$

$= 0.1667\left[2.5277\right]$

$= 0.4214$

ii) Probability that an arriving customer will have to space waiting out side the inticated space

$1-(P_0+P_1+P_2)$

$= 1-(0.4214)$

$= 0.5786$

iii) Waiting before starting service

$$W_q = \frac{\lambda}{\mu-(\mu-\lambda)}$$

$$= \frac{10}{12\cdot(12-10)}$$

$$= \frac{10}{12\times 2}$$

$$= \frac{10}{24} = 0.4167$$

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7) people Arrive at a theatre ticket Booth in poisson distributed arrival rate of 25 per hour. service time is constant at 2 Mins. Calculate

i) The Mean Number in the waiting Line

ii) The Mean waiting time.

iii) The utilization factors.

$$\lambda = 25 \text{ per hour}$$

$$\mu = 2 \text{ Mins} = \frac{1}{2} \times 60 = 30 \text{ per hours.}$$

i) To find the Mean Number in the waiting line.

$$W_q = \frac{\lambda}{\mu \times (\mu - \lambda)}$$

$$= \frac{25}{30 \times (30-25)} = \frac{25}{30 \times 5} = \frac{25}{150} = 1/6.$$

$$= 0.1667.$$

ii) To find the Mean waiting time

$$W_s = \frac{1}{\mu - \lambda}$$

$$= \frac{1}{30-25} = \frac{1}{5} = 0.2$$

iii) To find the utilization factor.

$$\rho = \frac{\lambda}{\mu} = \frac{25}{30} = 0.8333.$$

8) At a public Telephone Booth in a post office arrivals are considered to be poisson with an average inter arrival time of 12 Mins. The Length of the phone call May be assumed to be distributed exponentially with an

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average of 4 mins Calculate the following

i) what is the Probability that a fresh arrival will not have to wait for the phone.

ii) what is the average length of Queues formed from time to time

$\lambda = 12$ mins $= 1/12$ per mins.

$\mu = 4$ mins $= 1/4$ per mins

i) $\rho = \frac{1/12}{1/4} = 1/4 . = 0.3333$

$P_0 = 1 - \rho$

$= 1 - 0.3333$

$= 0.6667$

ii) $L_n = \frac{\mu}{\mu - \lambda}$.

$= 1/4 \big/ 1/4 - 1/12 = \frac{0.25}{0.1667} = 1.4997 = 1.5$

9) Arrival at a telephone Booth are considered to be poisson with an average time of 10 Mins between one Arrival at the Next the length of a phone call is assumed to be distributed expontionally with Mean 3 Mins.

i) what is the probability that a person arriving at the booth will have to wait?

ii) what is the Average length of the Queue that forms from time to time?

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second booth when convinced that an arrival would have to wait atleast 3 Mins for the phone by how much must the flow of arrivals be increased in order to Justify a second booth.

$$\lambda = 10 \text{ Mins} = \tfrac{1}{10} \text{ per mins}$$

$$\mu = 3 \text{ Mins} = \tfrac{1}{3} \text{ per Mins.}$$

i) To find the probability a person arriving

$$W_s = \frac{1}{\mu - \lambda}$$

$$= \frac{1}{1/3 - 1/10} = \frac{1}{1/3 - 1/10}$$

$$= \frac{1}{0.3333 - 0.1000} = \frac{1}{0.2333}$$

$$= 4.2863.$$

ii) Average Length of the Queue.

$$L_s = \frac{\mu}{\mu - \lambda} = \frac{1/3}{1/3 - 1/10}$$

$$= \frac{0.3333}{0.3333 - 0.1000}$$

$$= \frac{0.3333}{0.2333} = 1.4286.$$

iii) $$W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{1/10}{1/3\,(1/3 - 1/10)}$$

$$= \frac{0.1000}{0.3333\,(0.3333 - 0.1000)} = \frac{0.1000}{0.0778}$$

$$= 1.2860$$

$$3 = \frac{\lambda}{\frac{1}{3}\left(\frac{1}{3}-\lambda\right)} \qquad \frac{1}{\lambda} = \frac{1}{10}$$

$$\frac{1}{\lambda} = \frac{1}{\frac{1}{3}\left(\frac{1}{3}-\lambda\right)\frac{1}{3}}$$

$$\frac{1}{\lambda} = \frac{1}{\frac{1}{9}\left(\frac{1}{3}-\lambda\right)}$$

$$\frac{1}{9}\left(\frac{1}{3}-\lambda\right) = \lambda$$

$$\frac{1}{27} - \frac{1}{3}\lambda = \lambda$$

$$\frac{1}{27} = \frac{1}{3}\lambda^2$$

$$\boxed{\lambda = \frac{1}{3}}$$

$$3 = \frac{3\lambda}{\frac{1}{3}-\lambda} \Rightarrow \qquad u = \frac{1}{3}$$

$$1 = \frac{\lambda}{\frac{1}{3}-\lambda}$$

$$\frac{1}{3} - \lambda = \lambda$$

$$\frac{1-3\lambda}{3} = \lambda$$

$$1 - 3\lambda = 3\lambda$$

$$= \frac{8}{24} = \lambda^2 - 3\lambda - 3\lambda + 1 = 0$$

$$6\lambda + 1 = 0$$

$$+6\lambda = +1$$

$$\boxed{\lambda = \frac{1}{6}}$$

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