

KRISHNA COLLEGE OF ARTS & SCIENCE

KOLLUTHANIPATTY - KARUR.

Major: 3

Title paper: Differential equations and
Laplace transform.

2-marks : questions :-

1. solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$.
2. solve the equation $\frac{d^4y}{dx^4} - y = 0$.
3. solve $(D^2 + 5D + 6)y = e^x$.
4. solve $(D^2 + D + 1)y = x^2$.
5. solve $(D^2 - 2mD + m^2)y = e^{mx}$.
6. Find $L(t^2 + 2t + 3)$
7. Find $L(\sin^2 2t)$
8. Find $L(\sin^3 2t)$
9. Find $L(t^3 - 3t^2 + 2)$
10. Find $L(\sinh 3t)$
11. Find $L(\sin^2 t)$
12. Find $L(\cos t \cos 2t)$
13. Find $L(t e^{-at})$
14. Find $L(t^2 e^{-3t})$
15. Find $L(t \sin at)$

16. Find $L (te^{-t} \sin t)$

17. Find $L (t^2 e^{3t})$

18. solve $\frac{dy}{dx} + y \cos x = \frac{1}{2} \sin 2x$.

19. solve $(1+x^2) \frac{dy}{dx} + y = e^{\tan^{-1} x}$

20. solve $\frac{dy}{dx} + \frac{3x^2 y}{1+x^3} = \frac{1+x^2}{1+x^3}$.

21. solve $x^2 p^2 + 3xyp + 2y^2 = 0$.

22. solve $xp^2 - 2yp + x = 0$.

23. solve $x = y^2 + \log p$.

24. Define Clairaut's form.

25. solve $y = (x-a)p - p^2$.

26. solve $y = 2px + y^2 p^2$.

27. solve $yx p^2 + p(3x^2 - 2y^2) - 6xy = 0$.

28. solve $(xp - y)^2 = a(1+p^2) \phi(x^2 + y^2)$

29. solve $x(1+p^2) = 1$

30. solve $x^2(y - px) = yp^3$ (put $x = x^2$, $y = y^2$)

31. Find $L^{-1} \left[\frac{1}{s(s+1)(s+2)} \right]$ ($y = y^2$)

32. Find $L^{-1} \left[\frac{1}{(s-3)s} \right]$.

33. Find $L^{-1} \left[\frac{1}{s(s-a)} \right]$

34. Find $L^{-1} \left(\frac{s}{(s+2)^2} \right)$

5 marks:

1. Solve $\frac{dy}{dx} = \frac{y+2}{x-1}$

2. Solve $e^x \tan y dx + (1-e^x) \sec^2 y dy = 0$

3. Solve $x\sqrt{1+y^2} + y\sqrt{1+x^2} \cdot \frac{dy}{dx} = 0$.

4. Solve $\frac{dy}{dx} + y \tan x = \cos^3 x$

5. Solve $\cos^2 x \frac{dy}{dx} + y = \tan x$

6. solve $\frac{dy}{dx} + \frac{3x^2 y}{1+x^3} = \frac{1+x^2}{1+x^3}$

7. solve $(x^2+1) \frac{dy}{dx} + 2xy = 4x^2$.

8. solve $\frac{dy}{dx} + y \cot x = 4x \operatorname{cosec} x$ given that $y=0$ when $x = \frac{\pi}{2}$

9. solve $p^3 - 4xy p + 8y^2 = 0$.

10. solve $xy p^2 + (y^2 - x^2) p - 2xy = 0$.

11. solve $xy(p^2+1) = (x^2+y^2)p$.

12. solve $(D^2 - 6D + 13)y = 5e^{2x}$.

13. solve $(D^2 - 3D + 2)y = e^{3x}$ which shall vanish for $x=0$ and for $x = \log 2$.

14. solve $(D^2 - 12D + 16)y = (e^x + e^{-2x})^2$.

15. solve $(D^2 - 3D + 2)y = \sin 3x$

16. Solve $(D^2 - 4D + 3)y = \sin 3x \cos 2x$

17. Solve $(D^2 + 16)y = 2e^{-3x} + \cos 4x$.

18. Solve $(D^2 - 8D + 9)y = 8 \cos 5x$

19. Solve $(D^2 - 3D + 2)y = \cos 3x \cos 2x$

20. Solve $(D^2 + 5D - 6)y = \sin 4x \sin x$.

21. Solve $(D^2 + 5D + 6)y = e^{-2x} + \sin 4x$

22. solve $(D^2 + 4)y = e^x + \sin 2x$.

23. solve $\frac{d^2y}{dx^2} + 4y = \cos 2x + e^{2x} + 4$.

24. $(D - 1)^2 y = x$.

25. solve $(D^2 + D + 1)y = x + \sin x$.

26. solve $(D^2 + 4)y = x^2 + \cos 2x$.

27. solve $(D^3 - D^2 - 6D)y = 1 + x^2$.

28. solve $(D^2 + 4)y = x e^{2x}$

29. solve $(D^2 + 4)y = (x^2 + 1)e^x$.

30. solve $(D^2 - 4D + 3)y = e^x \cos 2x$.

31. solve $(D^2 - 4)y = e^x \cos 2x$.

32. solve $x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2y = e^x$.

33. solve the equation

$$(x-1) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = (x-1)^2$$

34. solve the equation of Variation

$$\text{of parameters } \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} = e^x \sin x$$

35. Find $L\left(\frac{\cos 2t - \cos 3t}{t}\right)$

36. Find $L(t^2 \cosh at)$

37. Find $L(t \cos^2 t)$

38. Find $L(\sin at - at \cos at)$

39. Find $L\left(\frac{\sin^2 t}{t}\right)$

40. Find $L(t e^t \sin t \cos 2t)$.

41. Find $L^{-1}\left[\frac{s+2}{(s^2+4s+5)^2}\right]$

42. Find $L^{-1}\left[\frac{1}{(s+1)(s^2+2s+2)}\right]$.

43. Find $L^{-1}\left[\frac{1}{s^2(s+1)}\right]$

44. Find $L^{-1}\left[\frac{1}{(s+1)(s^2+1)}\right]$.

44. Find $L^{-1}\left[\frac{s}{(s^2+4)^2}\right]$

45. Find $L^{-1}\left[\frac{5s+3}{(s-1)(s^2+2s+5)}\right]$.

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10 Marks

1. Solve $\frac{dy}{dx} + y \cot x = 4x \operatorname{cosec} x$ given

that $y=0$ when $x=\frac{\pi}{2}$.

2. Show that the solution of the differential equation $\frac{d^2y}{dt^2} + 4y = A \sin pt$ which is such that

$y=0$ and $\frac{dy}{dt}=0$ when $t=0$

3. Solve the following equations:

(i) $(D^2 - 2D + 4)y = e^x \sin x$

(ii) $(D^2 - 2D + 4)y = e^x \cos x$

(iii) $(D^2 - 4)y = e^x \cos 2x$.

4. Find the Laplace transform of

(i) $f(t) = (t-1)^2$ when $t > 1$

$= 0$ when $t < 1$

(ii) $f(t) = e^{-t}$ when $0 < t < 4$

$= 0$ when $t > 4$

(iii) $f(t) = \cos t$ when $0 < t < \pi$

$= \sin t$ when $t > \pi$.

5. Find the inverse transforms of

(i) $\frac{s}{(s^2+4)^2}$

(ii) $\frac{5s+3}{(s-1)(s^2+2s+5)}$

6. Solve (i) $(D^4 - m^4)y = \sin mx$

(ii) $D^2(x^2+4)y = 96x^2$

(iii) $(D^2+2D+5)y = xe^x$

7. Solve the following equations by the method of variation of parameters:

(i) $\frac{d^2y}{dx^2} + n^2y = \operatorname{sech} x$

(ii) $\frac{d^2y}{dx^2} + 4y = \omega \operatorname{ec} 2x$

(iii) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x \sin x$

8. Eliminate the arbitrary function from the following functions:

(i) $ax+by+cz = f(x^2+y^2+z^2)$

(ii) $f(x^2+y^2, z-xy) = 0$

(iii) $\phi(x^2+y^2+z^2, x+y+z) = 0$

9. solve the following equations:

(i) $z = px + qy + pq$

(ii) $z = px + qy + 2\sqrt{pq}$

(iii) $z = px + qy + \frac{p}{q} - pq$

10. solve (i) $(y^2+z^2)p - xzq = -xz$

(ii) $(x^2-yz)p + (p^2-zx)q = z^2-xy$

(iii) $(y-z)p + (z-x)q = x-y$