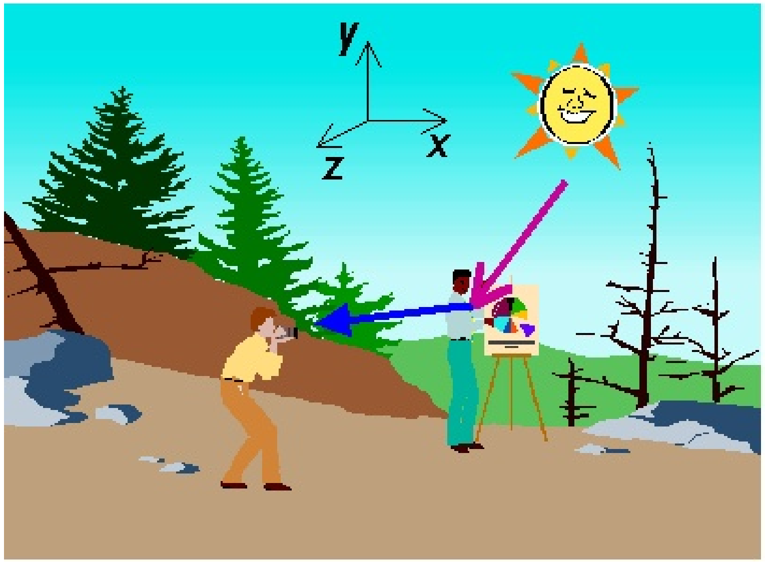
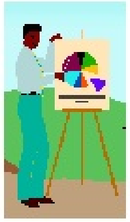
Image Formation

- For natural images we need a light source *(λ: wavelength of the source)*
  - $E(x, y, z, \lambda)$: incident light on a point *(x, y, z world coordinates of the point)*
- Each point in the scene has a reflectivity function.
  - $r(x, y, z, \lambda)$: reflectivity function
- Light reflects from a point and the reflected light is captured by an imaging device.
  - $c(x, y, z, \lambda) = E(x, y, z, \lambda) \times r(x, y, z, \lambda)$: reflected light.

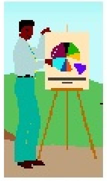


→ $E(x, y, z, \lambda)$

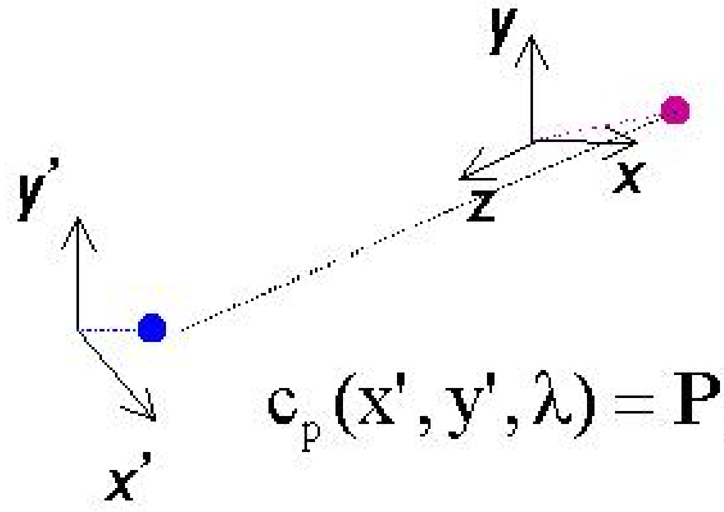
→ $c(x, y, z, \lambda) = E(x, y, z, \lambda) \cdot r(x, y, z, \lambda)$

Camera($c(x, y, z, \lambda)$) = 

Inside the Camera - Projection

Camera$(\mathrm{c}(\mathrm{x}, \mathrm{y}, \mathrm{z}, \lambda))$ **=** 

- Projection $(\mathcal{P})$ from world coordinates (x, y, z) to camera or image coordinates (x', y') $\left[c_p(x', y', \lambda) = \mathcal{P}(c(x, y, z, \lambda))\right]$.



$$\mathrm{c}_{\mathrm{p}}(\mathrm{x}', \mathrm{y}', \lambda) = \mathbf{P}(\mathrm{c}(\mathrm{x}, \mathrm{y}, \mathrm{z}, \lambda))$$

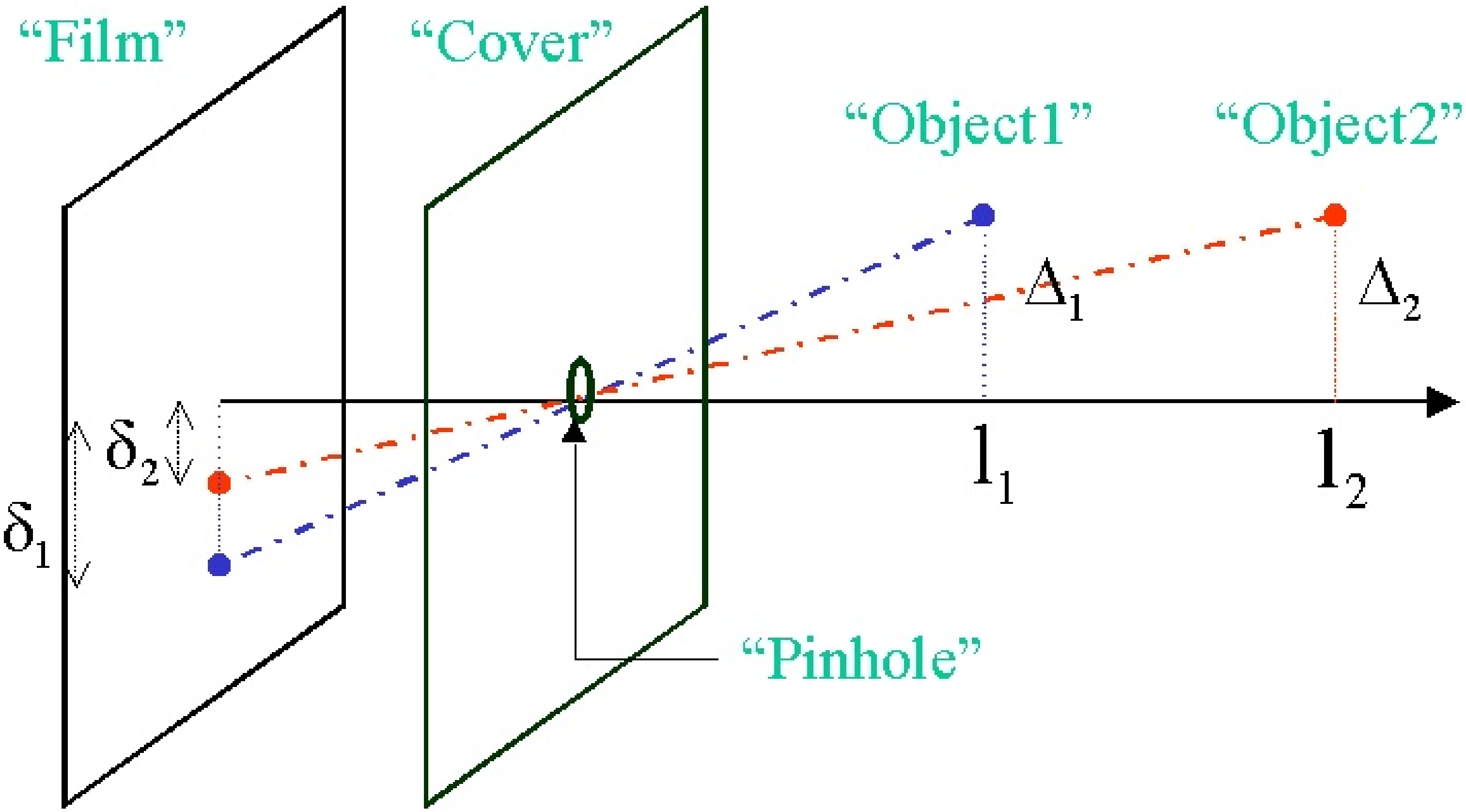
Projections

- There are two types of projections ($\mathcal{P}$) of interest to us:

   1. Perspective Projection
      - Objects closer to the capture device appear bigger. Most image formation situations can be considered to be under this category, including images taken by camera and the *human eye*.
   2. Ortographic Projection
      - This is "unnatural". Objects appear the same size regardless of their distance to the "capture device".

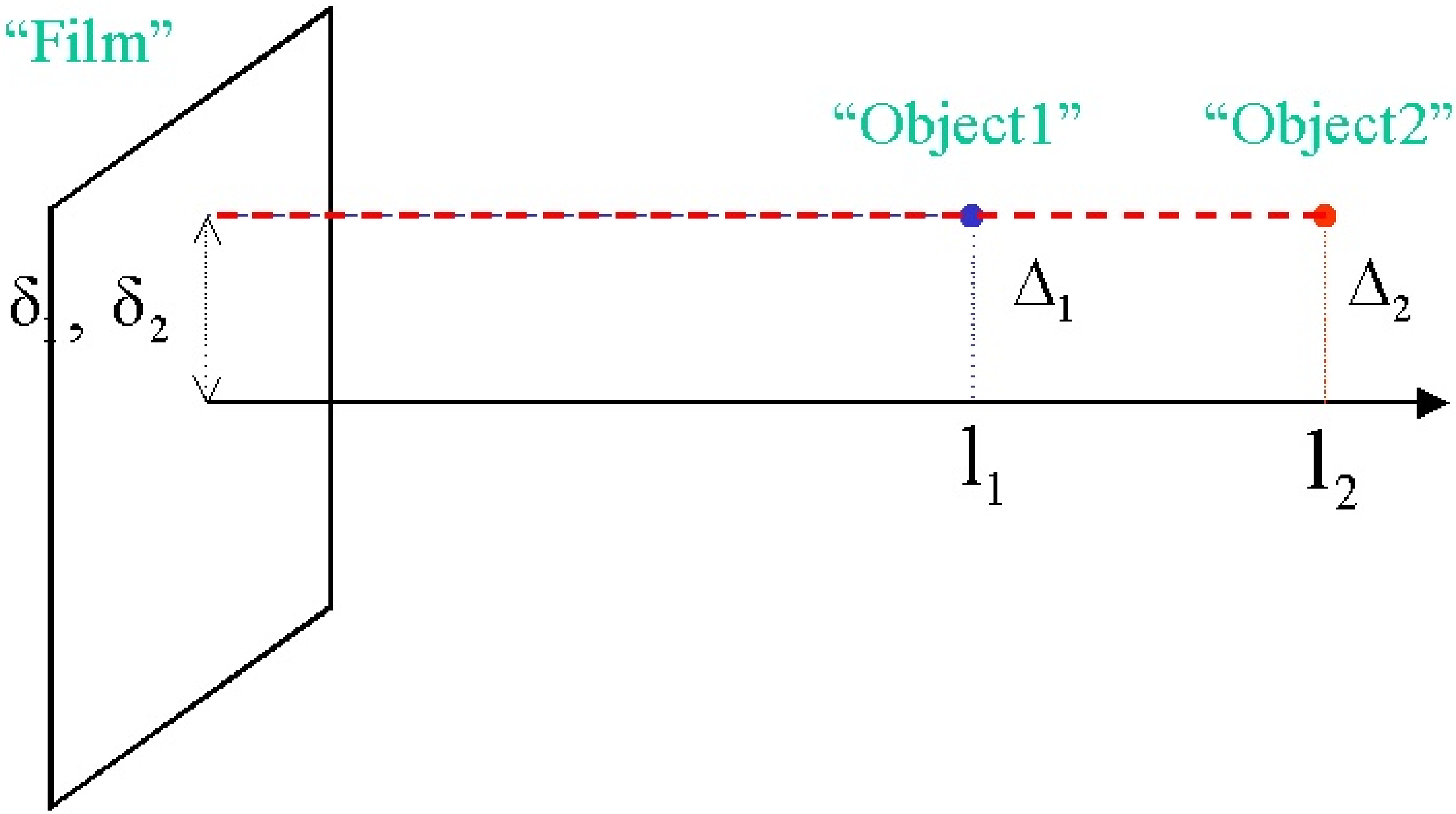
- Both types of projections can be represented via mathematical formulas. Ortographic projection is *easier* and is sometimes used as a mathematical convenience. For more details see [1].

Example - Perspective



- Perspective Projection: $\Delta_1 = \Delta_2$, $l_1 < l_2 \rightarrow \delta_2 < \delta_1$.

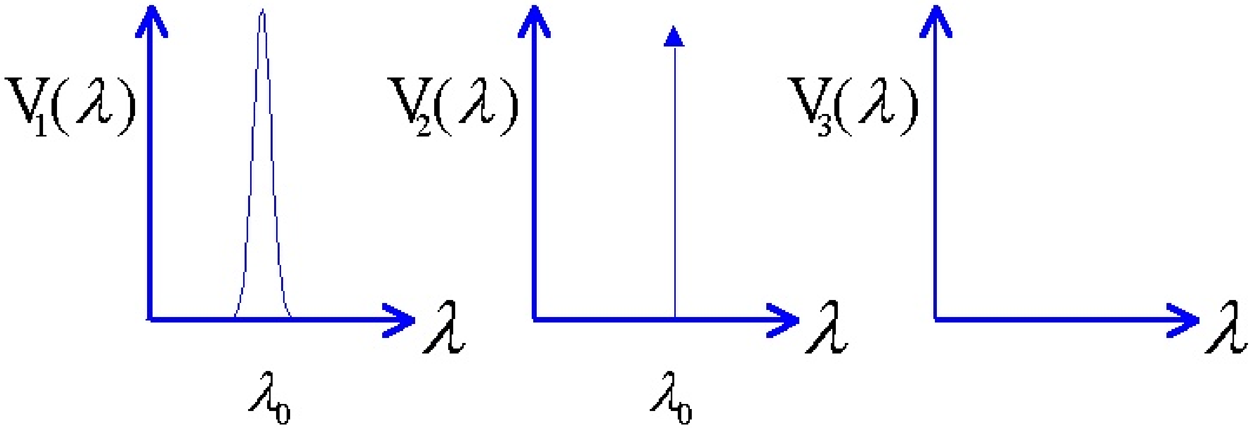
Example - Ortographic



- Ortographic Projection: $\Delta_1 = \Delta_2$, $l_1 < l_2 \rightarrow \delta_2 = \delta_1$.

Inside the Camera - Sensitivity

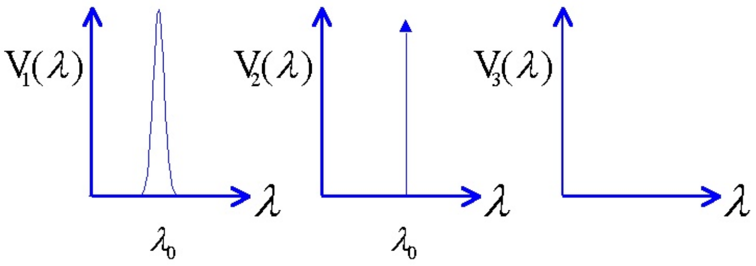
- Once we have $c_p(x', y', \lambda)$ the characteristics of the capture device take over.
- $V(\lambda)$ is the *sensitivity function* of a capture device. Each capture device has such a function which determines how sensitive it is in capturing the range of *wavelengths* (λ) present in $c_p(x', y', \lambda)$.



- The result is an "image function" which determines the amount of reflected light that is captured at the camera coordinates (x', y').

$$f(x', y') = \int c_p(x', y', \lambda) V(\lambda) d\lambda \tag{1}$$

Example



Let us determine the image functions for the above sensitivity functions imaging the same scene:

1. This is the most realistic of the three. Sensitivity is concentrated in a band around λ_0.

$$f_1(x', y') = \int c_p(x', y', \lambda) V_1(\lambda) d\lambda$$

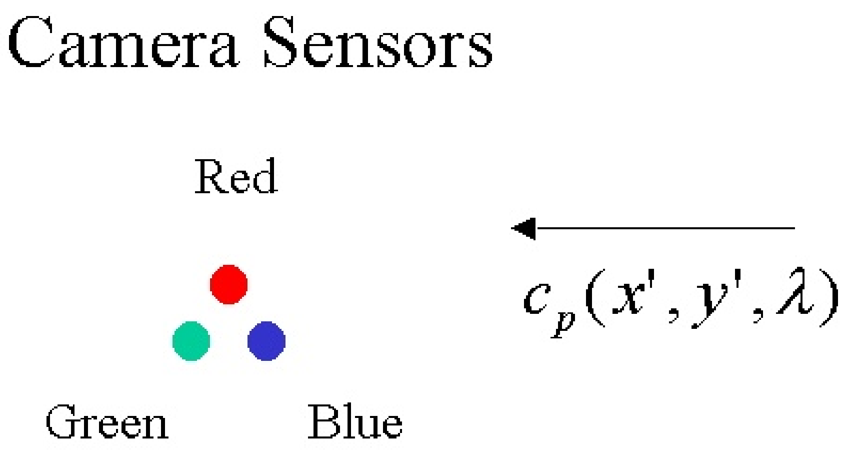
2. This is an unrealistic capture device which has sensitivity only to a single wavelength λ_0 as determined by the delta function. However there are devices that get close to such "selective" behavior.

$$\begin{aligned} f_2(x', y') &= \int c_p(x', y', \lambda) V_2(\lambda) d\lambda = \int c_p(x', y', \lambda) \delta(\lambda - \lambda_0) d\lambda \\ &= c_p(x', y', \lambda_0) \end{aligned}$$

3. This is what happens if you take a picture without taking the cap off the lens of your camera.

$$\begin{aligned} f_3(x', y') &= \int c_p(x', y', \lambda) V_3(\lambda) d\lambda = \int c_p(x', y', \lambda)\ 0\ d\lambda \\ &= 0 \end{aligned}$$

Sensitivity and Color



- For a camera that captures color images, imagine that it has *three sensors* at each $(x',\ y')$ with sensitivity functions tuned to the colors or wavelengths red, green and blue, outputting *three* image functions:

$$\begin{aligned} f_{\mathrm{R}}(x',y') &= \int c_p(x',y',\lambda)V_{\mathrm{R}}(\lambda)d\lambda \\ f_{\mathrm{G}}(x',y') &= \int c_p(x',y',\lambda)V_{\mathrm{G}}(\lambda)d\lambda \\ f_{\mathrm{B}}(x',y') &= \int c_p(x',y',\lambda)V_{\mathrm{B}}(\lambda)d\lambda \end{aligned}$$

- These three image functions can be used by display devices (such as your monitor or your eye) to show a “color” image.

Summary

- The image function $f_{\mathrm{C}}(x', y')$ ($C = R,\ G,\ B$) is formed as:

$$f_{\mathrm{C}}(x', y') \;=\; \int c_p(x', y', \lambda) V_{\mathrm{C}}(\lambda) d\lambda \tag{2}$$

- It is the result of:

1. Incident light $E(x, y, z, \lambda)$ at the point (x, y, z) in the scene,
2. The reflectivity function $r(x, y, z, \lambda)$ of this point,
3. The formation of the reflected light $c(x, y, z, \lambda) = E(x, y, z, \lambda) \times r(x, y, z, \lambda)$,
4. The projection of the reflected light $c(x, y, z, \lambda)$ from the *three* dimensional world coordinates to *two* dimensional camera coordinates which forms $c_p(x', y', \lambda)$,
5. The sensitivity function(s) of the camera $V(\lambda)$.

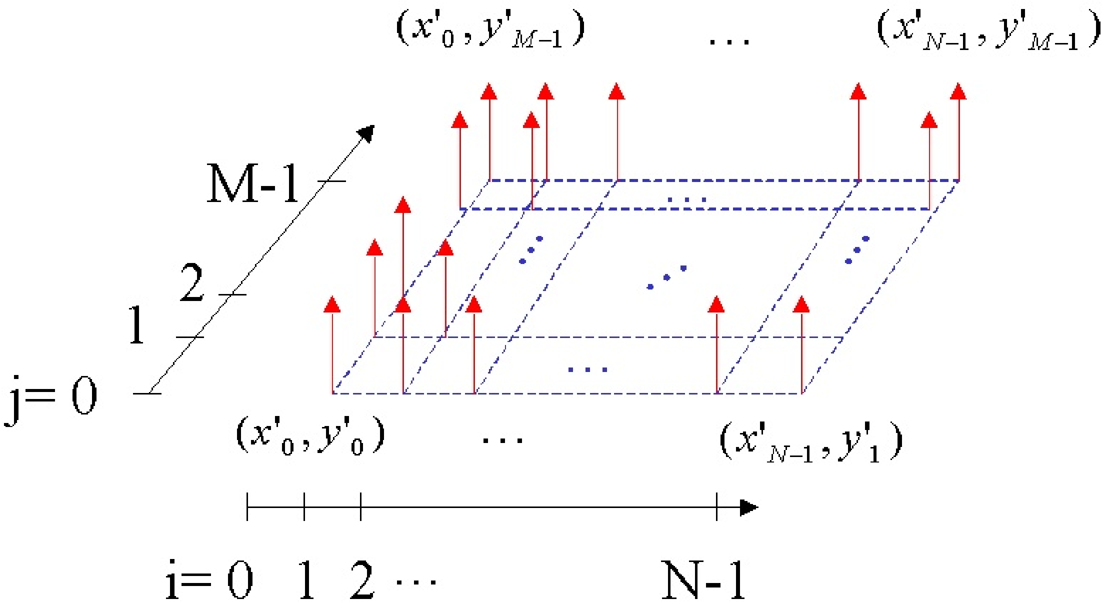
Digital Image Formation

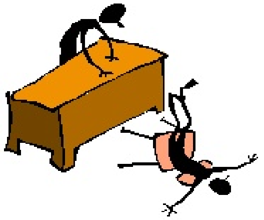
- The image function $f_{\mathrm{C}}(x', y')$ is still a function of $x' \in [x'_{min}, x'_{max}]$ and $y' \in [y'_{min}, y'_{max}]$ which vary in a continuum given by the respective intervals.
- The values taken by the image function are real numbers which again vary in a continuum or interval $f_{\mathrm{C}}(x', y') \in [f_{min}, f_{max}]$.
- Digital computers cannot process parameters/functions that vary in a continuum.
- We have to *discretize*:

  1. $x', \;\; y' \Rightarrow x'_i, \;\; y'_j \quad (i = 0, \ldots, N-1, \;\; j = 0, \ldots, M-1)$: Sampling
  2. $f_{\mathrm{C}}(x'_i, y'_j) \Rightarrow \hat{f}_{\mathrm{C}}(x'_i, y'_j)$: Quantization.

Sampling

comb(x',y')

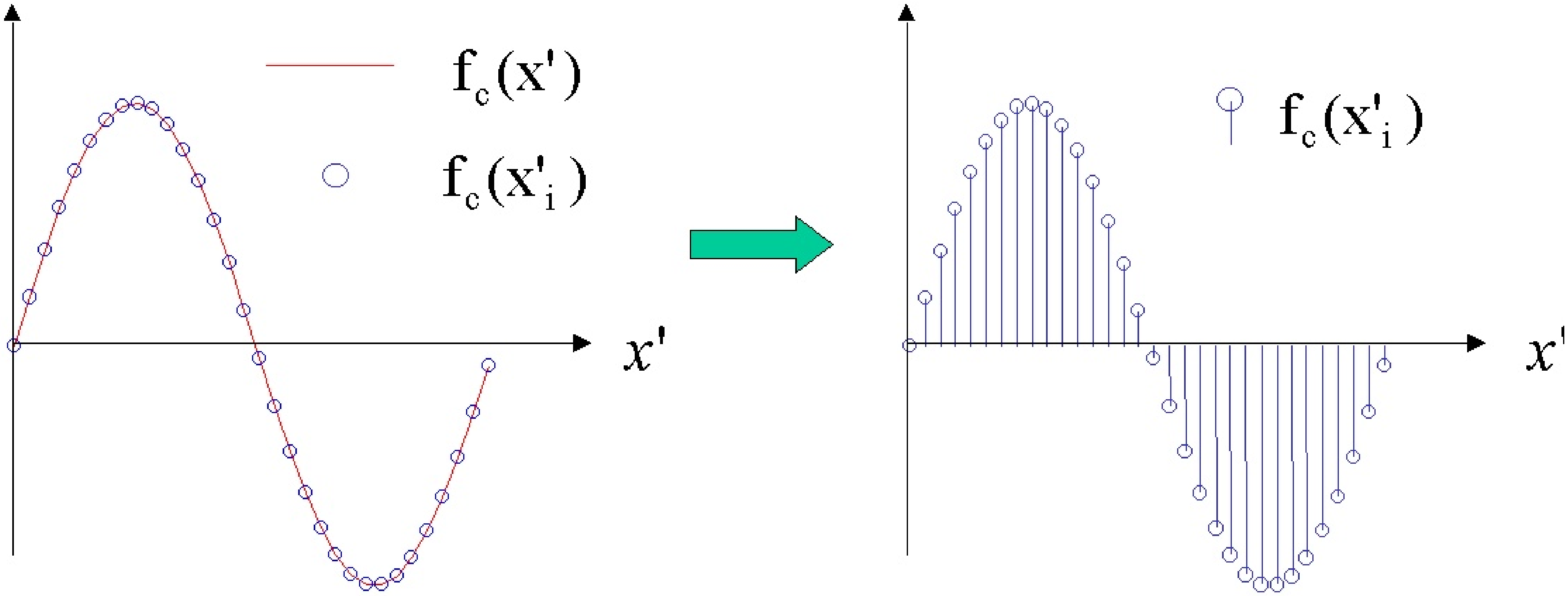




$$comb(x', y') = \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} \delta(x' - i\Delta_x, y' - j\Delta_y) \tag{3}$$

- Obtain sampling by utilizing $f_C(x', y') \times comb(x', y')$.

Example



Idealized Sampling

- We will later see that by utilizing $f_{\mathrm{C}}(x', y') \times comb(x', y')$, it is possible to discretize (x', y') and obtain a new "image function" that is defined on the discrete grid $(x'_i, y'_j) \quad (i = 0, \ldots, N-1, \; j = 0, \ldots, M-1)$.

- For now assume that we somehow obtain the sampled image function $f_{\mathrm{C}}(x'_i, y'_j)$.
- To denote this discretization refer to $f_{\mathrm{C}}(x'_i, y'_j)$ as $f_{\mathrm{C}}(i, j)$ from now on.

Quantization

- $f_{\mathrm{C}}\,(i,j) \quad (i=0,\ldots,N-1,\;\; j=0,\ldots,M-1)$. We have the second step of discretization left.
- $f_{\mathrm{C}}\,(i,j) \in [f_{min}, f_{max}],\;\; \forall(i,j)$.
- Discretize the values $f_{\mathrm{C}}\,(i,j)$ to P *levels* as follows:

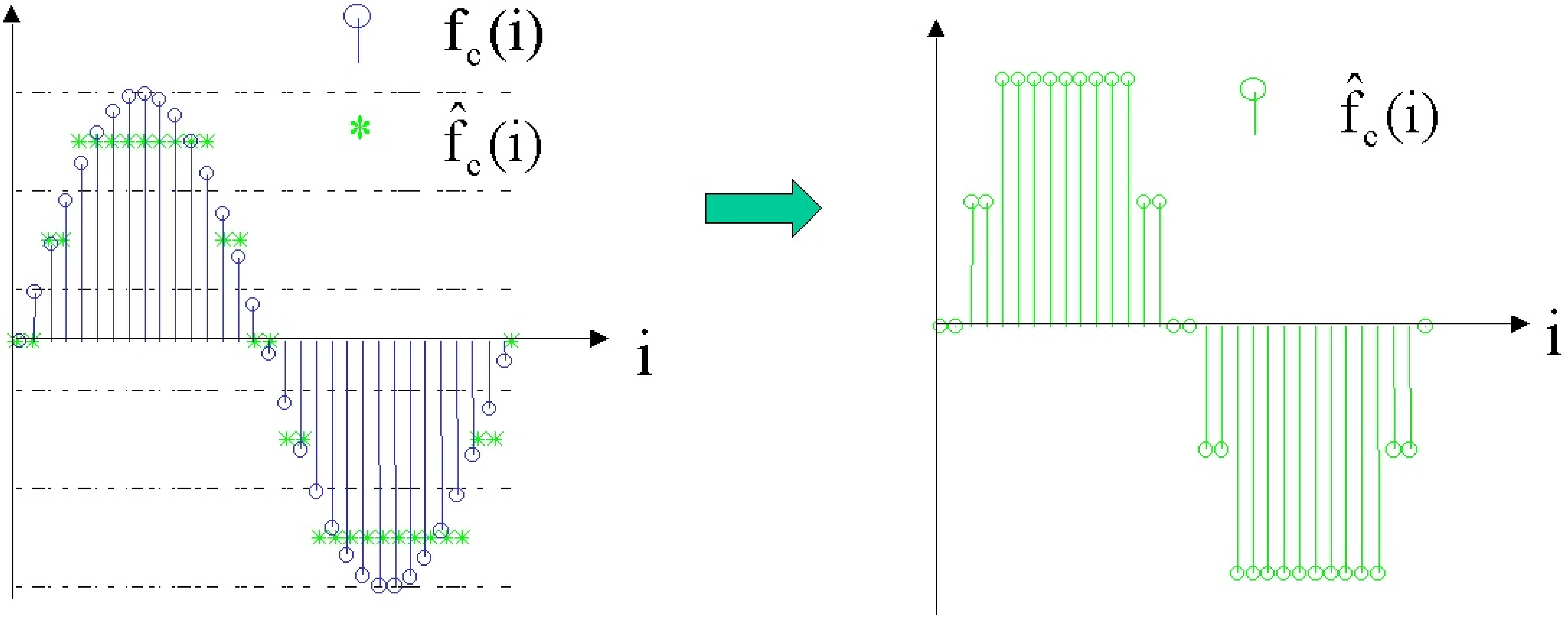
  Let $\Delta_Q = \frac{f_{max}-f_{min}}{P}$.

$$\hat{f}_{\mathrm{C}}\,(i,j) = Q(f_{\mathrm{C}}\,(i,j)) \tag{4}$$

where

$$\begin{aligned} Q(f_{\mathrm{C}}(i,j)) \;\; &= \;\; (k+1/2)\Delta_Q + f_{min} \\ & \;\;\;\;\;\; \textit{if and only if } f_{\mathrm{C}}(i,j) \in [f_{min}+k\Delta_Q, f_{min}+(k+1)\Delta_Q) \\ & \;\;\;\;\;\; \textit{if and only if } f_{min}+k\Delta_Q \leq f_{\mathrm{C}}(i,j) < f_{min}+(k+1)\Delta_Q \\ & \;\;\;\;\;\; \text{for } k=0,\ldots,P-1 \end{aligned}$$

Example



Quantization to P levels

- Typically $P = 2^8 = 256$ and we have $log_2(P) = log_2(2^8) = 8$ bit quantization.
- We have thus achieved the second step of discretization.
- From now on omit references to $f_{min},\ f_{max}$ and unless otherwise stated assume that the original digital images are quantized to 8 bits or 256 levels.
- To denote this refer to $\hat{f}_{\mathrm{C}}\,(i,j)$ as taking integer values k where $0 \leq k \leq 255$, i.e., let us say that

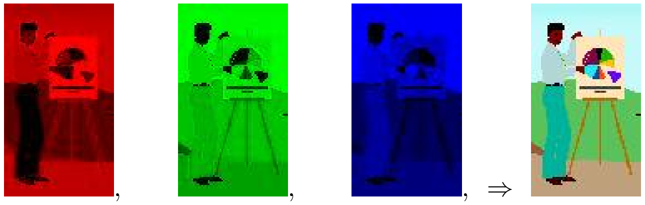
$$\hat{f}_{\mathrm{C}}(i,j) \in \{0,\ldots,255\} \tag{5}$$

Summary

- “Let there be light” → incident light → reflectivity → reflected light → projection → sensitivity → $f_{\mathrm{C}}\,(x', y')$.
- Sampling: $f_{\mathrm{C}}\,(x', y') \rightarrow f_{\mathrm{C}}\,(i, j)$.
- Quantization: $f_{\mathrm{C}}(i, j) \rightarrow \hat{f}_{\mathrm{C}}(i, j) \in \{0, \ldots, 255\}$.

(R,G,B) Parameterization of Full Color Images

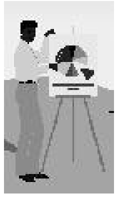
$$\hat{f}_{\mathrm{R}}(i,j),\ \hat{f}_{\mathrm{G}}(i,j),\ \hat{f}_{\mathrm{B}}(i,j)\ \rightarrow \text{full color image}$$



- $\hat{f}_{\mathrm{R}}(i,j)$, $\hat{f}_{\mathrm{G}}(i,j)$ and $\hat{f}_{\mathrm{B}}(i,j)$ are called the (R, G, B) parameterization of the "color space" of the full color image.
- There are other parameterizations, each with its own advantages and disadvantages (see chapter 3 of the textbook [2]).

Grayscale Images

"Grayscale" image $\hat{f}_{\text{gray}}(i,j)$



- A grayscale or luminance image can be considered to be *one* of the components of a different parameterization.
- Advantage: It captures most of the "image information".
- Our emphasis in this class will be on general processing. Hence we will mainly work with grayscale images in order to avoid the various nuances involved with different parameterizations.

Images as Matrices

- Recalling the image formation operations we have discussed, note that the image $\hat{f}_{\text{gray}}(i,j)$ is an $N \times M$ *matrix* with integer entries in the range $0,\ldots,255$.
- From now on suppress $\hat{(\ )}_{\text{gray}}$ and denote an image as a matrix "$\mathbf{A}$" (or $\mathbf{B}$,..., etc.) with elements $A(i,j) \in \{0,\ldots,255\}$ for $i = 0,\ldots,N-1$, $j = 0,\ldots,M-1$.
- So we will be processing matrices!
- Warning: Some processing we will do will take an image $\mathbf{A}$ with $A(i,j) \in \{0,\ldots,255\}$ into a new matrix $\mathbf{B}$ which may *not* have integer entries!

  In these cases we must suitably *scale* and *round* the elements of $\mathbf{B}$ in order to display it as an image.

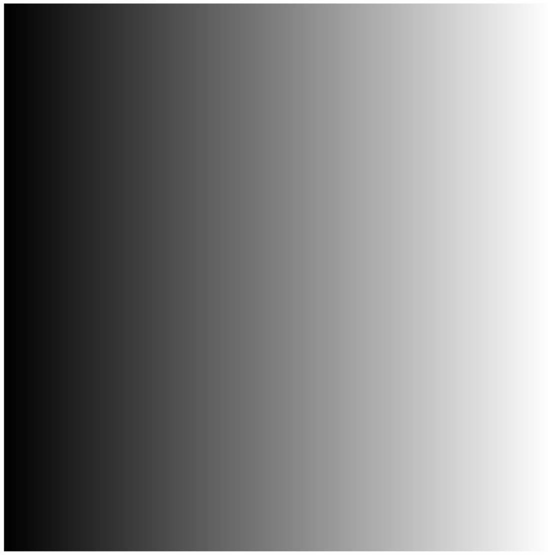
Matrices and Matlab

- The software package Matlab is a very easy to use tool that specializes in matrices.
- We will be utilizing Matlab for all processing, examples, homeworks, etc.
- If you do not have access to Matlab, a copy licensed for use in EL 512 will be provided to you. See the end of this lecture for instructions and details.

Example - I

- The image of a ramp (256×256):

$$\mathbf{A} = \left.\begin{bmatrix} 0 & 1 & 2 & \ldots & 255 \\ 0 & 1 & 2 & \ldots & 255 \\ \vdots & & & & \\ 0 & 1 & 2 & \ldots & 255 \end{bmatrix}\quad\right\} 256 \text{ rows}$$

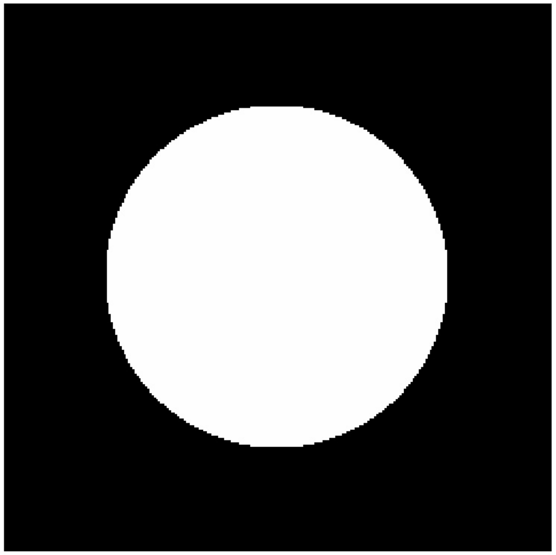


```
>>  for i = 1 : 256
      for j = 1 : 256
        A(i, j) = j - 1;
        end
      end
>>  image(A);
>>  colormap(gray(256));
>>  axis('image');
```

Example - II

- The image of a circle (256×256) of radius 80 pixels centered at $(128, 128)$:

$$B(i,j) = \begin{cases} 255 & \text{if } \sqrt{(i-128)^2 + (j-128)^2} < 80 \\ 0 & \text{otherwise} \end{cases}$$

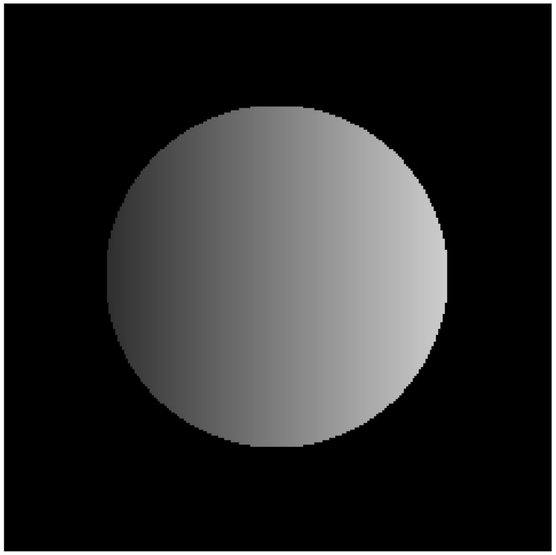


```
>>  for i = 1 : 256
      for j = 1 : 256
        dist = ((i − 128)^2 + (j − 128)^2)^(.5);
        if (dist < 80)
          B(i, j) = 255;
        else
          B(i, j) = 0;
          end
        end
      end
>>  image(B);
>>  colormap(gray(256));
>>  axis('image');
```

Example - III

- The image of a "graded" circle (256×256):

$$C(i,j) = A(i,j) \times B(i,j)/255$$



```
>>  for $i = 1 : 256$
      for $j = 1 : 256$
        $C(i,j) = A(i,j) * B(i,j)/255;$
        end
      end
>>  image($C$);
>>  colormap(gray(256));
>>  axis('image');
```

Homework I

1. If necessary, get a copy of matlab including a handout that gives an introduction to matlab. You may do so at the Multimedia Lab. (LC 008) in the Brooklyn campus. The preferred time is Wednesday afternoon.
2. Get a picture of *yourself* taken. Make sure it is 8 bit grayscale and *learn* how to read your picture into matlab as a matrix. Again, you may do so at the Multimedia Lab. (LC 008) in the Brooklyn campus.
3. Display your image and obtain a printout. Try ">> help print" for instructions within matlab.

If you are at a different campus and have problems with the above instructions please send me email. Please note that this is a one time event, i.e., do your best.

References

[1] B. K. P. Horn, *Robot Vision.* Cambridge, MA: MIT Press, 1986.

[2] A. K. Jain, *Fundamentals of Digital Image Processing.* Englewood Cliffs, NJ: Prentice Hall, 1989.