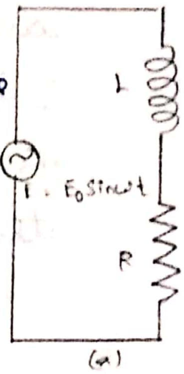
Unit - IV AC circuits

Alternating Emf applied to series circuit containing LC, LR and CR:

AC circuit containing Inductance (L) and Resistance (R) in series: Let an alternating emf $E = E_0 e^{i\omega t}$ be applied to a circuit having an inductance L and a non-inductance resistance R in series. The potential drop across the inductance is,



(a)

$$V_L = j\omega L I$$

The potential drop across resistance is

$$V_R = RI$$

Here, I is the current at any instant t.

$$E = j\omega L I + RI$$

current in the circuit,

$$I = \frac{E}{R + j\omega L} \quad \text{——— (1)}$$

$$\text{But } I = \frac{E}{Z} \quad \text{——— (2)}$$

Impedence of the R-L circuit.

$$Z = R + j\omega L \quad \text{——— (3)}$$

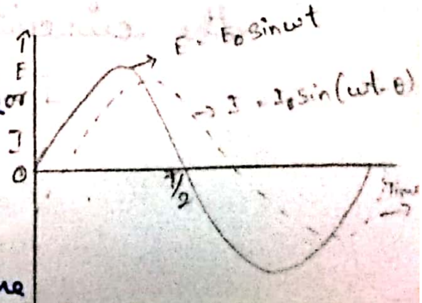
$$I = \frac{E_0 e^{j\omega L}}{\sqrt{R^2+\omega^2L^2)}\, e^{j\theta}} \quad \left(\text{where } \tan\theta = \frac{\omega L}{R}\right).$$

$$I = \frac{E_0}{\sqrt{R^2+\omega^2L^2)}}\, e^{j(\omega t - \theta)} \quad \text{——— (4)}$$

$$= I_0\, e^{j(\omega t - \theta)} \quad \text{——— (5)}$$

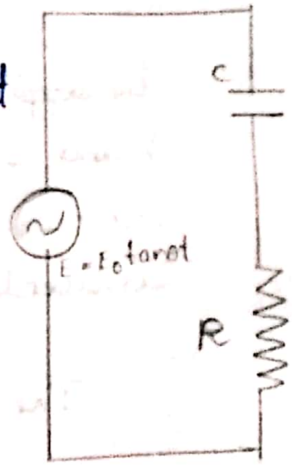
$$\text{Here } I_0 = \frac{E_0}{\sqrt{R^2+\omega^2L^2}} \quad \text{——— (6)}$$

* It represents the peak value of the current through the circuit.
* The impedence Z of the circuit is given by the term $\sqrt{R^2+\omega^2L^2}$.
* The current inphase behind the emf of an angle $\theta = \tan^{-1}\frac{\omega L}{R}$
* The variation of instantaneous values of emf and current with time are Represented graphically in fig (b)



(R) in series:

Let an alternating voltage $E = E_0 e^{i\omega t}$ be applied to a circuit containing resistance R and a capacitor C in series. Let I be the current through the circuit and q, the charge on the plates of the capacitor at any instant.



Then, $I\left(R + \frac{1}{i\omega C}\right) = E = E_0 e^{j\omega t}$ —— (1)

$$x = \frac{E}{R + \frac{1}{i\omega C}} \quad \text{—— (2)}$$

But $I = \frac{E}{Z}$ —— (3)

Impedance of series R-C circuits,

$$Z = R + \frac{1}{j\omega C} = R - \frac{j}{\omega C}$$

$$I = \frac{E}{R - j/\omega C} = \frac{E_0 e^{j\omega t}}{\sqrt{\left[\left(R^2 + \frac{1}{\omega^2 C^2}\right)\right]} e^{-j\theta}} \quad \text{—— (4)}$$

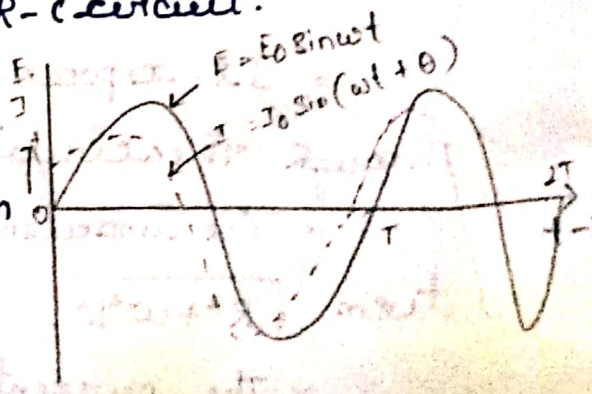
Here, $\theta = \tan^{-1}\left(\frac{1/\omega C}{R}\right)$ —— (5)

$$I = \frac{e^{j(\omega t + \theta)}}{\sqrt{\{R^2 + (1/\omega C)^2\}}} \quad \text{—— (6)}$$

The current in this circuit thus leads the applied voltage by an angle θ.

The impedance of series R-C circuit.

$$Z = \sqrt{\left[R^2 + (1/\omega C)^2\right]}$$



The actual current in the circuit is

$$I = I_0 \sin(\omega t + \theta).$$

Parallel Resonance circuit:

A parallel resonance circuit consisting of an inductance L and a capacitance C connected in parallel to the alternating emf is shown in fig.

Let the applied emf be

$$E = E_0 \sin \omega t .$$

current through the inductance lags behind the applied emf $\pi/2$ and is given by

$$I_L = \frac{E_0}{\omega L} \sin(\omega t - \pi/2)$$

Hence ωL is the reactance of the inductance. The current through the capacitor leads the applied emf by $\pi/2$ and is given by.

$$I_C = \frac{E_0}{(1/\omega C)} \sin(\omega t + \pi/2).$$

Here $(1/\omega C)$ is the reactance of the capacitance. The total current through the circuit is given by.

$$I = I_L + I_C$$

$$I = E_0/\omega L \sin(\omega t - \pi/2) + \frac{E_0}{(1/\omega C)} \sin(\omega + \pi/2)$$

$$I = -\frac{E_0}{\omega L} \cos \omega t + \frac{E_0}{(1/\omega C)} \cos \omega t.$$

$$I = E_0\left(\omega C - \frac{1}{\omega L}\right) \cos \omega t \quad \text{——} (1).$$

If for a particular frequency ν_0, $\omega C = \frac{1}{\omega L}$, then from eqn (1), the current becomes zero. The circuit now offers an infinite impedance.

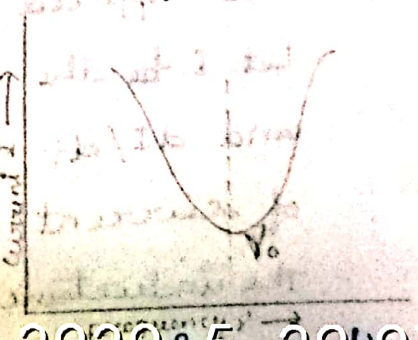
The circuit now offers an infinite impedance. Such a circuit which offers an infinite impedance to the A.C is called a parallel resonant circuit. ν_0 frequency is called the resonant frequency.

Thus at ν_0, we have,

$$\omega C = \frac{1}{\omega L}, \quad \omega^2 = \frac{1}{LC} \text{ (or)}$$

$$\omega = \frac{1}{\sqrt{LC}}, \quad 2\pi\nu_0 = \frac{1}{\sqrt{LC}}$$

$$\nu_0 = \frac{1}{2\pi\sqrt{LC}} \quad \text{——} (2)$$



The variation of current with frequency

Alternating EMF applied to circuits containing L, C and R.

(i) A.C circuit containing Resistance only:

Suppose an alternating sinusoidal voltage

$$E = E_0 e^{j\omega t}$$

is applied across a pure resistance R. The current through the resistance at any instant t is,

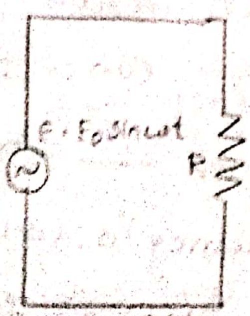
$$I = \frac{E}{R} = \frac{E_0 e^{j\omega t}}{R} = I_0 e^{j\omega t}$$

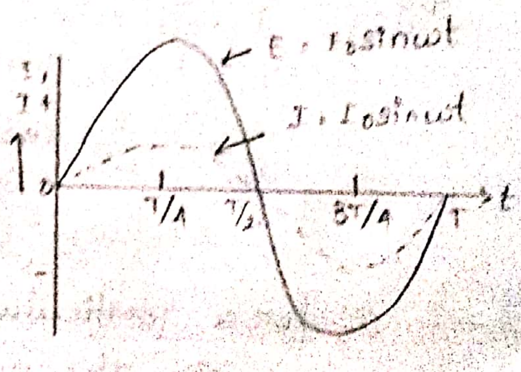
$$I_0 = \frac{E_0}{R}$$

Actual value of current = Imaginary part of $I_0 e^{j\omega t} = I_0 \sin\omega t$.

$$I = I_0 \sin\omega t.$$

Hence the current is in phase with the driving voltage.



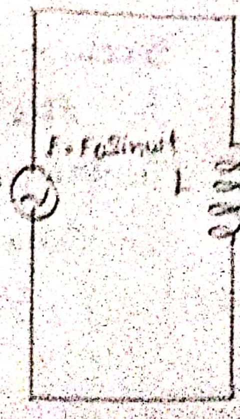


(ii) A.C circuits containing Inductance only,

Let an alternating voltage

$$E = E_0 e^{j\omega t}$$

be applied across an inductance L. Let I be the instantaneous current and dI/dt be the rate of change of current with time through the inductance.



The voltage induced across the inductance $= -L \frac{dI}{dt}$

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$$L \frac{dI}{dt} = E_0 e^{j\omega t}$$

$$dI = \frac{E_0}{L} e^{j\omega t} dt$$

Integrating

$$I = \frac{E_0}{j\omega L} e^{j\omega t} = \frac{E}{j\omega L} \quad \text{—(1)}$$

$$I = \frac{E_0}{\omega L} e^{j(\omega t - \pi/2)}$$

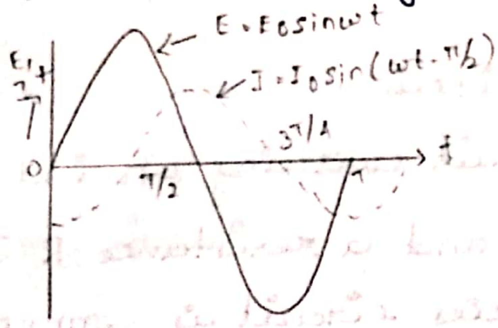
Let $I_0 = \frac{E_0}{\omega L}$ be the maximum current, Then

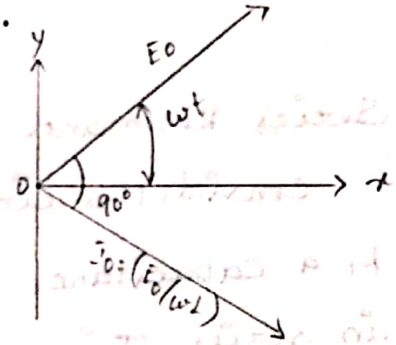
$$I = I_0 e^{j(\omega t - \pi/2)}$$

The actual current in the circuit is

$$I = I_0 \sin(\omega t - \pi/2)$$

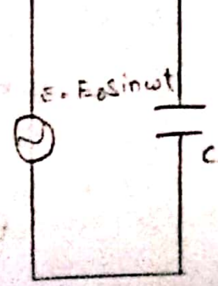
In a pure Inductor, The current lags behind the emf by $\pi/2$.





(iii) A.c circuit containg capacitance only:

Let an alternating voltage $E = E_0 e^{j\omega t}$ be applied across the plates of a capacitance C. Let q be the charge at any instant across the plates of the capacitor. The P.d across the plates at any instant must be equal to the applied emf at that instant. Thus



$$q/C = E_0 e^{j\omega t}$$

The current through the circuit is given by,

$$I = dq/dt = \frac{d}{dt}(CE_0 e^{j\omega t}) = j\omega C E_0 e^{j\omega t}$$

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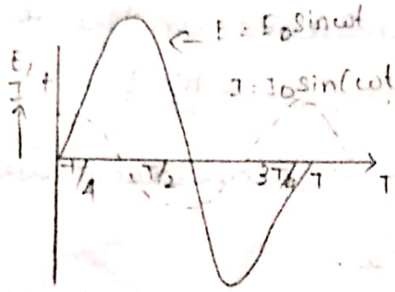
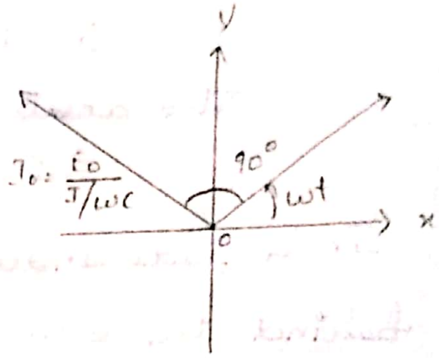
$$I = \frac{E_0}{1/\omega C} e^{j(\omega t + \pi/2)} = I_0 e^{j(\omega t + \pi/2)}$$

$$I_0 = \frac{E_0}{1/\omega C}$$ is the peak value of alternating current. The actual current in the circuit is $I = I_0 \sin(\omega t + \pi/2)$

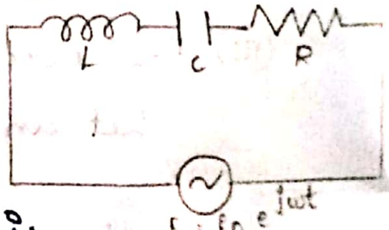
In a pure capacitor, the current leads the emf by $\pi/2$.





Series Resonance circuit:

Consider a circuit containg an inductance L, a capacitance C and a resistance R Joined in series. This series circuit is connected to an AC supply given by



$$E = E_0 e^{j\omega t} \quad \text{—— (1)}$$

The total complex impedance is

$$Z = Z_R + Z_L + Z_C$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$= \sqrt{R^2 + (\omega L - 1/\omega C)^2}\, e^{j\phi}$$

where $\tan\phi = \dfrac{(\omega L - 1/\omega C)}{R}$ —— (2)

using ohm's law in complex form, the 'complex' current in the circuit is.

$$I = \frac{E}{Z} = \frac{E_0 e^{j\omega t}}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}\, e^{j\phi}}$$

$$I = \frac{E_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\, e^{j(\omega t - \phi)} \quad \text{———} \; (3)$$

$$I_0 = \frac{E_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

$$I = I_0 e^{j(\omega t - \phi)}$$

The actual emf is the imaginary part of the equivalent complex emf. Hence the actual current in the circuit is obtained by taking the imaginary part of the above complex current.

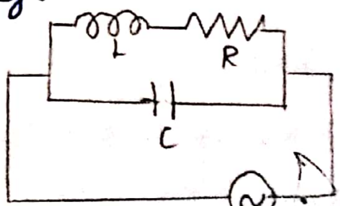
$$i = Im_j (I) = \frac{E_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \sin(\omega t - \phi) \quad \text{———} \; (5)$$

The equivalent impedance of the series LCR circuit is $\sqrt{R^2 + (\omega L - 1/\omega C)^2}$

The current 'lags' behind the voltage by an angle $\phi = \tan^{-1} \frac{(\omega L - 1/\omega C)}{R}$.

Parallel resonant circuit.

Here, capacitor C is connected in parallel to the series combination of resistance R and Inductance L. The combination is connected across the AC source. The applied voltage is sinusoidal represented by.



$E = F_0 e^{j\omega t}$

$$E = E_0 e^{j\omega t}$$

complex impedance of L-branch

$$Z_1 = R + jL\omega$$

complex impedance of C-branch

$$Z_2 = 1/jC\omega$$

Z_1 and Z_2 are in parallel.

$$\frac{1}{Z} = \frac{1}{R+j\omega L} + \frac{1}{1/j\omega C} = \frac{1}{R+j\omega L} + j\omega C$$

$$= \frac{R - j\omega L}{(R+j\omega L)\times(R-j\omega L)} + j\omega C$$

$$= \frac{R}{R^2+(L\omega)^2} + j\left[C\omega - \frac{L\omega}{R^2+(L\omega)^2}\right]$$

The current $I = E/Z = E \times 1/Z$

$$I = E\left[\frac{R}{R^2+(L\omega)^2} + j\left(C\omega - \frac{L\omega}{R^2+(L\omega)^2}\right)\right]$$

Let $A\cos\phi = \frac{R}{R^2+(L\omega)^2}$; $A\sin\phi = C\omega - \frac{L\omega}{R^2+(L\omega)^2}$

$$I = E(A\cos\phi + jA\sin\phi)$$

$$= EAe^{j\phi} = E_0 Ae^{j(\omega t+\phi)}$$

$$\phi = \tan^{-1}\frac{C\omega - \left(\frac{L\omega}{R^2+(L\omega)^2}\right)}{\frac{R}{R^2+(L\omega)^2}}$$

$$A^2 = \frac{R^2}{(R^2+\omega^2L^2)^2} + \left(C\omega - \frac{L\omega}{R^2+\omega^2L^2}\right)^2$$

The magnitude of the admittance.

$$Y = \frac{1}{Z} = \frac{\sqrt{\left[R^2 + (\omega CR^2+\omega^3L^2C-\omega L)^2\right]}}{R^2+\omega^2L^2}$$

The admittance will be minimum, when.

$$\omega CR^2 + \omega^3L^2C - \omega L = 0$$

$$\omega = \omega_0 = \sqrt{\left[\frac{1}{LC} - \frac{R^2}{L^2}\right]}$$

$$\nu_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}.$$

This is the resonant frequency of the circuit.

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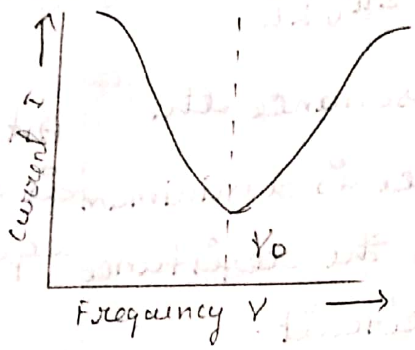
If R is very small so that $\frac{R^2}{L^2}$ is negligible compared to $\frac{1}{LC}$.

$$\nu_0 = \frac{1}{2\pi\sqrt{(LC)}}$$

At such a minimum admittance .ie maximum impedance, the circuit current is minimum. The graph between current and frequency is shown.



Impedance at resonance

At resonance, $Z = \frac{R^2 + (L\omega)^2}{R}$

$R^2 + (L\omega)^2 = L/C$ at resonace.

$$Z = \frac{L}{RC}.$$

The smaller the resistance R, larger is the impedance. If R is negligible, the mpedance is infinite at resonance.

Rejector circuit:

The parallel resonant circuit does not allow the ~~the~~ current of the same frequency as the natural frequency of the circuit. Thus it can be used to suppress the current of this particular frequency out of currents of many other frequencies. Hence the circuit is known as a 'rejector' or 'filter' circuit.

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Comparison between series and parallel.

Series resonant circuit	parallel resonant circuit.
1. An acceptor circuit	A rejector circuit.
2. Resonant frequency $\nu_r = \frac{1}{2\pi\sqrt{LC}}$	Resonant frequency $\nu_r = \frac{1}{2\pi\sqrt{LC}}$.
3. At resonance the impedance is a minimum equal to the resistance in the circuit.	At resonance the impedance is maximum nearly equal to infinity.
4. Selective	Selective.
5 used in the tuning circuit to separate the wanted frequency from the incoming frequencies by offering low impedance at that frequency.	used to present a maximum impedance to the wanted frequency, usually in the plate circuit of values.

Sharpness of resonance.

The sharpness of resonance is defined as the rapidity with which the current falls from its resonant value (E_0/R) with change in applied frequency.

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wattless current:

The average power dissipated during a complete cycle is $E_v I_v \cdot \cos\phi$. The current in A.C circuit is said to be wattless when the average power consumed in the circuit is zero.

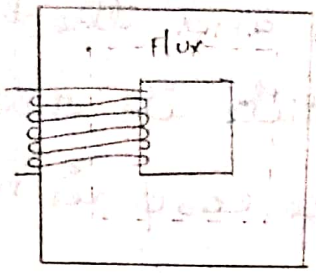
If an ac circuit is purely inductive or purely capacitive with no ohmic resistance, phase angle $\phi = \pi/2$ so that $\cos\phi = 0$ or the power consumed is zero. The current in such a circuit does not perform any useful work and is rightly called the wattless or idle current. In this situation, The circuit does not consume any power, through it offers a resistance to the flow of alternating current in it. It is the principle of coke coil.

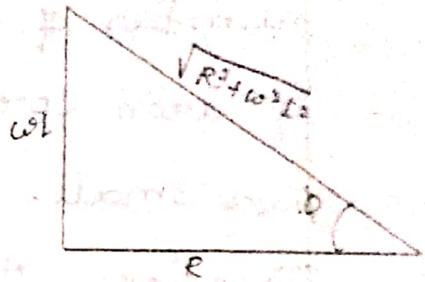
Unit – IV

choke coil:

A choke coil is an inductance coil which is used to control the current in an ac circuit.

construction:





A choke consists of a coil of several ~~times~~ turns of insulated thick copper wire of low resitance but large inductance. wound over a laminated core. The core is layered and is made up of thin sheets of stalloy to reduce hysteresis losses. The laminations are coated with shellac to insulate and bound together firmly so as to minimise loss of energy due to eddy currents.

principle:

The average power dissipated in the choke coil is given by

$$P = \frac{1}{2} E_0 I_0 \cos\phi .$$

If the resistance of the choke coil is R and the inductance of the coke coil is L, Then the power factor $\cos\phi$ is given by

$$\cos\phi = \frac{R}{\sqrt{(R^2 + \omega^2 L^2)}}$$

The inductance L of the coke coil is quite large on account of its large number of turns and the high permeability of iron core, while its resistance R is very small. Hence $\cos\phi$ is nearly zero. Therefore, the power absorbed by the coil is Extremely Small. Thus the choke coil reduces the strength of the current without appreciable wastage of energy. The only waste of energy is due to the hysteresis loss in the iron core. The loss due to eddy currents is minimised by making the core laminated.

Preferance of choke coil over an ohmic resistance for diminishing the current.

The current in an A.C. circuit can also be diminished by using an ordinary ohmic resistance (rheostat) in the circuit. But such a method of Controlling A.C is

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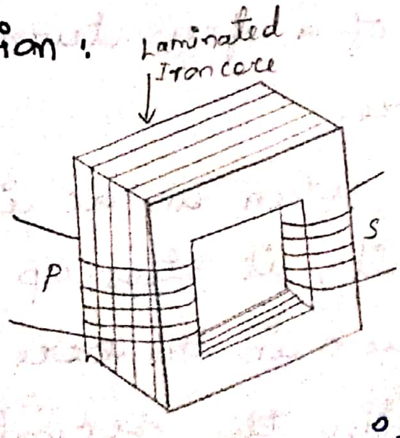
not economical as much of the electrical energy (I^2Rt) supplied by the source is wasted as heat. Hence the choke coil is to be preferred over the ohmic resistance. The energy used in establishing the magnetic field in the choke coil is restored when the magnetic field collapses. Hence to regulate ac, it is more economical to use a choke than a resistance.

choking coils are very much used in electronic circuits, mercury lamps and sodium vapour lamps.

The Transformer:

It is a device for converting a low alternating voltage at high current into a high alternating voltage at low current and vice-versa. It is an electrical device based on the principle of mutual induction between the coils.

construction:



A transformer consists of two coils, called the primary P and secondary S, which are insulated from each other and wound on a common soft-iron laminated core.

The alternating voltage to be tansformed is connected to the primary while the load is connected to the Secondary. Transformers which convert low voltages into higher voltages are called step-up transformers. Transformers which convert high voltages into lower voltages are called step-down transformers.

In a step-up transformer, the primary coil consists of a few turns of thick insulated copper wire of large number of turns of thin copper wire. In a step-down transformer, The primary consists of a large number of turns of thin copper wire and the Secondary of a few turns of thick copper wire.

Now when an ac is applied to the primary coil, it steps up an alternating magnetic flux in the core which also gets linked with the secondary. This change in flux linked with the secondary coil

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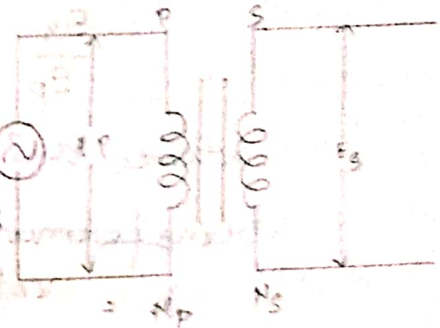
induces an alternating emf in the secondary coil. Thus the energy supplied to the primary is transferred to the secondary through the changing magnetic flux in the core.

Theory:

(i) Transformer on no load

Let N_p and N_s be the number of turns in the primary and secondary respectively of the transformer. When an alternating emf is applied across the primary, a current flows in its winding. This develops a magnetic flux in the core. Here it is assumed That there are no losses. and no leakage of flux.



Let ϕ = Flux linked with each turn of either coil. This magnetic flux is linked up with both the primary and the secondary.

By Faraday's law of electromagnetic induction, the emf induced in the primary is given by

$$e_p = -\frac{d(N_p\phi)}{dt} = -N_p\frac{d\phi}{dt}.$$

and the emf induced in the secondary

$$e_s = -\frac{d(N_s\phi)}{dt} = -N_s\frac{d\phi}{dt}$$

$$\frac{\mathcal{E}_s}{\mathcal{E}_p} = \frac{N_s}{N_p}.$$

In an ideal transformer, the resistance of the primary circuit is negligible and there are no energy losses. So the induced emf $\mathcal{E}_p$ in the primary is numerically equal to the applied voltage E_p across the primary. If the secondary is on open circuit its resistance is infinite. So the voltage E_s across the terminals of the secondary is equal to the induced emf $\mathcal{E}_s$.

$$\frac{E_s}{E_p} = \frac{\mathcal{E}_s}{\mathcal{E}_p} = \frac{N_s}{N_p} = k,$$

where k is called the turns ratio or transformation ratio of the transformer.

$$k = \frac{\text{Voltage obtained across secondary}}{\text{Voltage applied across primary}}$$

$$= \frac{\text{No. of turns in secondary}}{\text{no. of turns in primary}}.$$

In a step-up transformer $\mathcal{E}_s > \mathcal{E}_p$ and hence $N_s > N_p$.

In a step-down transformer $\mathcal{E}_s < \mathcal{E}_p$ and hence $N_s < N_p$.

Let I_p and I_s be the currents in primary and secondary at any instant.

In this case power output is equal to power input.

power in the secondary = power in the primary

$$E_s \times I_s = E_p \times I_p$$

$$\frac{I_p}{I_s} = \frac{E_s}{E_p} = \frac{N_s}{N_p} = k$$

Thus, when the voltage is stepped up the current is correspondingly reduced in the same ratio, and vice-versa.

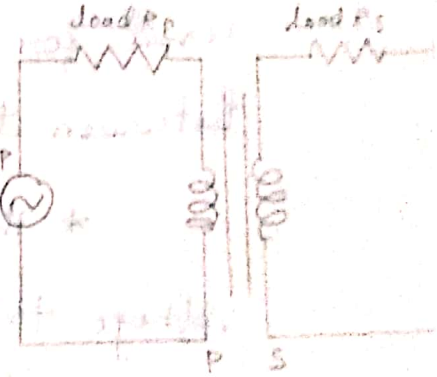
(ii) Transformer on load :

If the primary circuit has an appreciable resistance R_p, the difference between the applied voltage E_p and the back emf ε_p must be equal to the potential drop $I_p \times R_p$ in primary coil.

$$E_p - \varepsilon_p = I_p \times R_p$$

(or) $\varepsilon_p = E_p - I_p \times R_p$.

Again, if the secondary circuit is closed having finite resistance (load) R_s, a part of the induced emf ε_s in the secondary overcomes the potential drop $I_s \times R_s$. Hence the available P.D across the secondary is given by,



$$E_s = \varepsilon_s - I_s \times R_s$$

$$\varepsilon_s = E_s + I_s \times R_s$$

$$\frac{\varepsilon_s}{\varepsilon_p} = \frac{E_s + I_s R_s}{E_p - I_p R_p} = K$$

$$E_s + I_s R_s = K(E_p - I_p R_p) \qquad (I_p = K I_s)$$

$$E_s = K E_p - I R_s - K I_p R_p$$

$$= K E_p - I_s R_s - K^2 I_s R_p$$

$$= K E_p - I_s (R_s + K^2 R_p)$$

In this case E_s / E_p is not a constant but decreases as more current is drawn from the secondary circuit.

Uses of Transformers.

* The step-up and step-down transformers are used in a.c electrical power distribution for the domestic and industrial purposes.

* The audio-frequency transformers are used in radio receivers, radio-telephony, radio-telegraphy and in televisions.

* The radio frequency transformers are used in radio-communications at frequencies of the order of mega-cycles.

* The impedance transformers are used for matching the impedance between two circuits in radio communicati

* The constant current and constant voltage transformers are designed to give constant output current and voltage respectively even when the input voltage varies considerably.

Skin Effect:

The current density (J) remains constant over any given section when a steady current passes through a uniform wire. But, when an alternating current of high frequency flows through a wire, the current density is not uniform at all points across a section. There is a greater current density at the surface layers than at the interior layers of the conductor. When the frequency is very large, the current is almost entirely confined to the surface layer. This phenomenon is called skin effect.