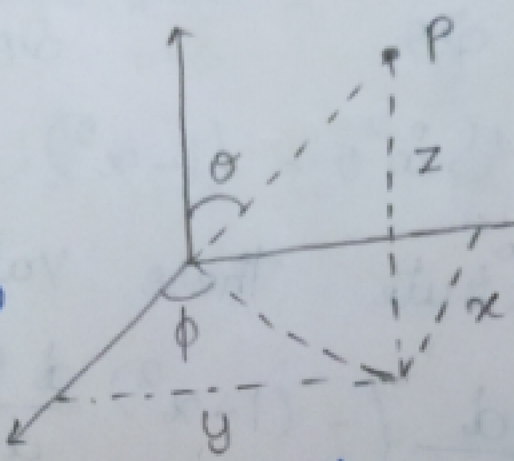
3/5/16 Hydrogen atom:

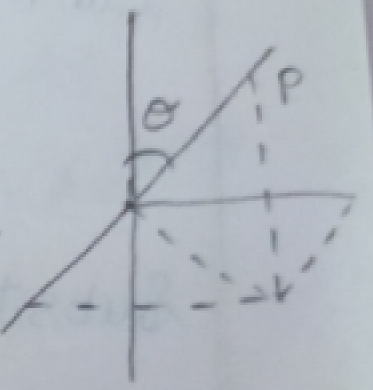
The Proton and electron rotate with the Centre Point.

$x = r \sin\theta \cos\phi$

$y = r \sin\theta \sin\phi$

$z = r \cos\theta.$





from spherical polar co-ordinates :-

$$\frac{1}{r^2} \cdot \frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi}{\partial r}\right) + \frac{1}{r^2} \cdot \frac{1}{\sin\theta} \frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial \psi}{\partial \theta}\right)$$

$$+ \frac{1}{r^2 \sin^2\theta} \cdot \frac{\partial^2 \psi}{\partial \phi^2} + \frac{2\mu}{\hbar^2}\left(E - V(r)\right)\psi = 0. \quad \to (1)$$

here. $\mu \to$ reduced mass because proton mass is very small.

$10^{-15} \to$ fermi (atom

$10^{-10} \to$ N

Seperation of Variable :-

$$\psi(r, \theta, \phi) = R(r)\, \Theta\theta\, \Phi\phi$$

$$\frac{1}{R} \cdot \frac{1}{r^2} \cdot \frac{\partial}{\partial r}\left(r^2 \frac{\partial R}{\partial r}\right) + \frac{1}{\Theta} \frac{1}{r^2} \frac{1}{\sin\theta} \frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial \Theta}{\partial \theta}\right)$$

$$+ \frac{1}{r^2 \sin^2\theta} \cdot \frac{1}{\Phi} \cdot \frac{\partial^2 \Phi}{\partial \phi^2} + \frac{2\mu}{\hbar^2}\left(E - V(r)\right)\psi = 0$$

Multiply $(r^2 \sin^2\theta)$ on both sides

$$\frac{\sin^2\theta}{R} \cdot \frac{\partial}{\partial r}\left(r^2 \frac{\partial R}{\partial r}\right) + \frac{1}{\Theta} \sin\theta \frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial \Theta}{\partial \theta}\right)$$

$$+ \frac{1}{\Phi} \frac{\partial^2 \Phi}{\partial \phi^2} + \frac{2\mu}{\hbar^2}(r^2 \sin^2\theta)\left(E - V(r)\right)\psi = 0 \quad \to (2)$$

here: $\frac{1}{\Phi} \cdot \frac{\partial^2 \Phi}{\partial \phi^2} = -m^2 \to$ mass of electron, $-m$ proton, $+m$

$\pm m^2 \to +$ can be neglected so become $\boxed{-m^2}$

$$\frac{1}{\Phi} \cdot \frac{\partial^2 \Phi}{\partial \phi^2} + m^2 = 0.$$

$\frac{\partial^2 \Phi}{\partial \phi^2} = -m^2 \Phi$

$\frac{\partial^2 \Phi}{\partial \phi^2} + m^2 \Phi = 0.$

The soln of Φ is :-

$$\Phi = A e^{\pm im\phi}.$$

Normalisation Condition:-

$\Phi = A e^{im\phi}$

$\Phi^* = A e^{-im\phi}$

A^2

$$\int \Phi \Phi^* \, d\tau = 1$$

$$A^2 \int e^{im\phi} e^{-im\phi} \, d\tau = 1$$

$$A^2 (2\pi) = 1$$

$$A^2 = \frac{1}{2\pi}$$

$$A = \frac{1}{\sqrt{2\pi}}$$

$$A^2 = \frac{1}{2\pi}$$

$$A = \frac{1}{\sqrt{2\pi}}$$

$$\therefore \Phi = \frac{1}{\sqrt{2\pi}} e^{\pm i m\phi} \rightarrow (A)$$ here: $m = 0, \pm 1, \pm 2, \dots$

Soln of Θ:

$$\frac{\sin^2\theta}{R} \cdot \frac{\partial}{\partial r}\left(r^2 \frac{\partial R}{\partial r}\right) + \frac{1}{\Theta} \sin\theta \frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial\Theta}{\partial\theta}\right) - m^2 + \frac{r^2 \sin^2\theta \, 2\mu}{\hbar^2}(E - V(r))\psi = 0$$

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Divide $(\sin^2\theta)$ on both sides:

$$\frac{1}{R} \cdot \frac{\partial}{\partial r}\left(r^2 \frac{\partial R}{\partial r}\right) + \frac{1}{\Theta \sin\theta} \cdot \frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial\Theta}{\partial\theta}\right) - \frac{m^2}{\sin^2\theta} + \frac{r^2 2\mu}{\hbar^2}(E - V(r))\psi = 0$$

$$\frac{-1}{\Theta \sin\theta} \cdot \frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial\Theta}{\partial\theta}\right) + \frac{m^2}{\sin^2\theta} = \frac{1}{R}\frac{\partial}{\partial r}\left(r^2 \frac{\partial R}{\partial r}\right) + \frac{2\mu r^2}{\hbar^2}(E - V(r))$$

$$\frac{-1}{\Theta \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial\Theta}{\partial\theta}\right) + \frac{m^2}{\sin^2\theta} = \lambda$$

$$\frac{-1}{\Theta \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial\Theta}{\partial\theta}\right) = \left[\lambda - \frac{m^2}{\sin^2\theta}\right]$$

$$\frac{-1}{\sin\theta} \cdot \frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial\Theta}{\partial\theta}\right) = \left[\lambda - \frac{m^2}{\sin^2\theta}\right]\Theta$$

$$\frac{1}{\sin\theta} \cdot \frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial\Theta}{\partial\theta}\right) + \left[\lambda - \frac{m^2}{\sin^2\theta}\right]\Theta = 0 \rightarrow (B)$$

Continuous in 2 pages after.

$$\frac{1}{r} \cdot \frac{\partial}{\partial r}\left(r^2 \frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2} + \frac{2\mu}{\hbar^2}(E - V)\psi) = 0$$

Non-de

The small force can be fr... created a large ... is called perturbati...

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Th

(Continuous in H_2 atom)

$$x = \cos\theta \;;\; dx = -\sin\theta \, d\theta \Rightarrow \frac{dx}{d\theta} = -\sin\theta.$$

$$\frac{\partial\Theta}{\partial\theta} = \frac{\partial\Theta}{\partial x} \cdot \frac{\partial x}{\partial\theta}$$

$$\frac{\partial\Theta}{\partial\theta} = -\sin\theta \frac{\partial\Theta}{\partial x}$$

$$\frac{\partial}{\partial\theta} = -\sin\theta \frac{\partial}{\partial x}$$

$$\frac{\partial}{\partial x} = \frac{-1}{\sin\theta} \cdot \frac{\partial}{\partial\theta}$$

$$\frac{\partial\Theta}{\partial\theta} \times \sin\theta = \frac{\partial\Theta}{\partial\theta} = -\sin^2\theta \frac{\partial\Theta}{\partial x}.$$

$$= -(1 - \cos^2\theta)\frac{\partial\Theta}{\cdots}$$

$$\frac{\partial \Theta}{\partial \theta} \times \sin\theta = \frac{\partial \Theta}{\partial \theta} = -\sin^2\theta \frac{\partial \Theta}{\partial x}.$$

$$= -(1-\cos^2\theta)\frac{\partial \Theta}{\partial x}.$$

$$\sin\theta \frac{\partial \Theta}{\partial x} = -(1-x^2)\frac{\partial \Theta}{\partial x}.$$

equ (3) becomes:

$$(3) \Rightarrow \frac{1}{\sin\theta}\cdot\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right) + \left[\lambda - \frac{m^2}{\sin^2\theta}\right]\Theta = 0$$

$$\left(\frac{\cancel{-}\partial}{\partial x}\right)\left(\cancel{-}(1-x^2)\frac{\partial\Theta}{\partial x}\right) + \left[\lambda - \frac{m^2}{1-x^2}\right]\Theta = 0$$

$$\frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial\Theta}{\partial x}\right] + \left[\lambda - \frac{m^2}{1-x^2}\right]\Theta = 0.$$

Then soln is:

$$\Theta = BP_l^m(x) \Rightarrow \Theta = BP_l^m(\cos\theta).$$

here,

$B \rightarrow$ normalisation constants.

$$B = \left\{\frac{(2l+1)}{2}\cdot\frac{l-|m|!}{l+|m|!}\right\}^{1/2}$$

$$\Theta = \sqrt{\frac{(2l+1)}{2}\cdot\frac{l-|m|!}{l+|m|!}}\cdot P_l^m \cos\theta \rightarrow (8)$$

Soln for R:

$$\frac{1}{R}\cdot\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) + \frac{2\mu r^2}{\hbar^2}[E - V(r)] = \lambda$$

$$\frac{1}{R}\cdot\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) + \frac{2\mu r^2}{\hbar^2}[E - V(r)] - \lambda = 0$$

Multiply R/r^2

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) + \frac{2\mu R}{\hbar^2}[E - V(r)] - \lambda = 0$$

$\hookrightarrow (4)$

here:

$$\lambda = \frac{\hbar^2 l(l+1)}{2\mu r^2} R$$

$$V(r) = -\frac{Ze^2}{r}$$

$$V(r) = \frac{Ze^2\alpha}{e}$$

$$\frac{1}{r^2}\cdot\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) + \frac{2\mu}{\hbar^2}\left[E - V(r) - \frac{\hbar^2 l(l+1)}{2\mu r^2}\right]R = 0.$$

Dimensionless independent variable:-

$$\rho = \alpha r$$

$$r = \rho/\alpha \Rightarrow \frac{1}{r} = \frac{\alpha}{\rho} \Rightarrow \frac{1}{r^2} = \frac{\alpha^2}{\rho^2}$$

$$\frac{\alpha^2}{\rho^2}\cdot\frac{\partial}{\partial\rho}\left[\frac{\rho^2}{\alpha^2}\cdot\frac{\partial R}{\partial\rho}\right] + \frac{2\mu}{\hbar^2}\left[E + \frac{Ze^2\alpha}{\rho} - \frac{\hbar^2 l(l+1)\alpha^2}{2\mu\rho^2}\right]R = 0$$

$\hookrightarrow (5)$

Hence the particle in ground state :-

$E<0$; $E=-|E|$ (ie) $|E|=-E$.

There, E is a negative.

$$\frac{\alpha^2}{\rho^2}\cdot\frac{\partial}{\partial\rho}\left[\frac{\rho^2}{\alpha^2}\cdot\frac{\partial R}{\partial\rho}\right]+\frac{2\mu}{\hbar^2}\left[-|E|+\frac{ze^2\alpha}{\rho}-\frac{\hbar^2 l(l+1)}{2\mu\rho^2}\alpha^2\right]R=0$$

$$\frac{\alpha^2}{\rho^2}\cdot\frac{\partial}{\partial\rho}\left[\frac{\rho^2}{\alpha^2}\cdot\frac{\partial R}{\partial\rho}\right]+\left[\frac{-2\mu|E|}{\hbar^2}+\frac{2\mu ze^2\alpha}{\hbar^2\rho}-\frac{2\mu\hbar^2 l(l+1)}{\hbar^2 2\mu\rho^2}\alpha^2\right]R=0$$

$\div\alpha^2$ on both sides.

$$\frac{1}{\rho^2}\cdot\frac{\partial}{\partial\rho}\left[\rho^2\frac{\partial R}{\partial\rho}\right]+\left[\frac{-2\mu|E|}{\hbar^2\alpha^2}+\frac{2\mu ze^2\alpha}{\hbar^2\rho\alpha^2}-\frac{l(l+1)\alpha^2}{\rho^2\alpha^2}\right]R=0$$

$$\frac{1}{\rho^2}\cdot\frac{\partial}{\partial\rho}\left[\rho^2\frac{\partial R}{\partial\rho}\right]+\left[\frac{-2\mu|E|}{\hbar^2\alpha^2}+\frac{2\mu ze^2}{\hbar^2\rho\alpha}-\frac{l(l+1)}{\rho^2}\right]R=0. \rightarrow (6)$$

here:-

$$\frac{2\mu|E|}{\hbar^2\alpha^2}=\frac{1}{4}\ ;\ \alpha^2=\left(\frac{8\mu|E|}{\hbar^2}\right)\quad \alpha=\left(\frac{8\mu|E|}{\hbar^2}\right)^{1/2}$$

$$\frac{2\mu ze^2}{\hbar^2\alpha}=\lambda'=\frac{2ze^2\mu}{\hbar^2}\left(\frac{\hbar^2}{8\mu|E|}\right)^{1/2}\quad \mu=\mu^{1/2}\mu^{1/2}\quad \mu=(\mu^2)^{1/2}$$

$$=\frac{\cancel{2}ze^2}{\hbar^{\cancel{2}}}\left(\frac{\mu^2}{2\mu|E|}\right)^{1/2}\cancel{\hbar}\times\frac{1}{\cancel{2}}\quad 8=(4\times2)^{1/2}=2(2)^{1/2}\quad (\hbar^2)^{1/2}=\hbar$$

$$\lambda'=\frac{ze^2}{\hbar}\left(\frac{\mu}{2|E|}\right)^{1/2}\rightarrow (6a).$$

Then, equ (6) becomes :

$$\frac{1}{\rho^2}\cdot\frac{\partial}{\partial\rho}\left[\rho^2\frac{\partial R}{\partial\rho}\right]+\left[\frac{-1}{4}+\frac{\lambda'}{\rho}-\frac{l(l+1)}{\rho^2}\right]R=0.$$

ρ is larger values, so equ redues to.

$$\frac{1}{\rho^2}\cdot\frac{\partial}{\partial\rho}\left[\rho^2\frac{\partial R}{\partial\rho}\right]-\frac{1}{4}R=0\rightarrow (7)$$

Soln for this equ :-

$$R(\rho)=e^{\rho/2}\ (or)\ R(\rho)=e^{-\rho/2}.$$

The general soln is :-

$$R(\rho)=F(\rho)e^{-\rho/2}\rightarrow (8)$$

$F(\rho)$ the radial equation is converted into a Laugre equ. The soln of Laugre equ is :-

$$R(r)=Ce^{-\rho/2}\rho^{l}L_{2n+1}^{2l+1}(\rho).$$

$n\rightarrow$ radial Quantum number

$C\rightarrow$ Constant.

The value of C is obtained by normalisation method :-

$\iint\psi\psi^{*}d\tau$

The general soln is :-

$$R(\rho) = F(\rho)\, e^{-\rho/2} \rightarrow (8)$$

$F(\rho)$ the radial equation is Converted into a Lauggre equ. The soln of Lauggre equ is :-

$$R(r) = C e^{-\rho/2} \rho^{l} L_{2n+1}^{2l+1} (\rho).$$

$n \Rightarrow$ radial Quantum number

$C \rightarrow$ Constant.

The value of C is obtained by normalisation method :-

$$\iint R R^{*} \, d\tau = 1.$$

then,
$$C = \left[\left(\frac{\rho}{r}\right)^{3} \frac{(n-l-1)!}{2n\{(n+l)!\}^{3}} \right]^{1/2}$$

$$\therefore R(r) = \sqrt{\left(\frac{\rho}{r}\right)^{3} \frac{(n-l-1)!}{(2n)\{(n+l)!\}^{3}}}\; e^{-\rho/2} \rho^{l} L_{2n+1}^{2l+1}(\rho)$$
$\rightarrow (9)$.

here:
$$\rho = \frac{2Zr}{na_0}$$

$$\therefore R(r) = \sqrt{\left(\frac{2Z}{na_0}\right)^{3} \frac{(n-l-1)!}{2n\{(n+l)!\}^{3}}}\; \exp\left[\frac{-Zr}{na_0}\right] \left[\frac{2Zr}{na_0}\right]^{l} L_{2n+1}^{2l+1} \left[\frac{2Zr}{na_0}\right].$$
$\rightarrow (10)$

here:
$a_0 \rightarrow$ atomic radius.

$$\frac{2\mu Ze^{2}}{\hbar^{2}\alpha} = \lambda' \quad \& \quad \frac{2\mu|E|}{\hbar^{2}\alpha^{2}} = \frac{1}{4}$$

$$\alpha^{2} = \frac{8\mu|E|}{\hbar^{2}} \qquad \alpha = \left(\frac{8\mu|E|}{\hbar^{2}}\right)^{1/2}$$

$$\lambda' = \frac{2\mu Ze^{2}}{\hbar^{2}} \left(\frac{\hbar^{2}}{8\mu|E|}\right)^{1/2}$$

$$= \frac{Ze^{2}}{\hbar} \left(\frac{\mu^{2}}{2\mu|E|}\right)^{1/2}$$

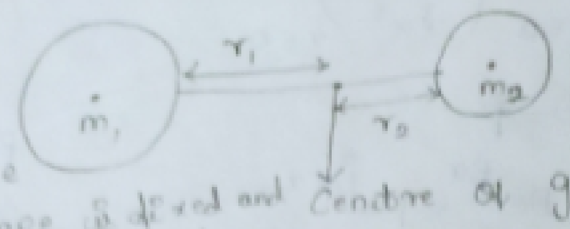
$$\lambda' = \frac{Ze^{2}}{\hbar} \left(\frac{\mu}{2|E|}\right)^{1/2}$$

$$\frac{1}{r^{2}} \cdot \frac{\partial}{\partial r}\left(r^{2} \frac{\partial \psi}{\partial r}\right) + \frac{1}{r^{2}} \cdot \frac{1}{\sin\theta} \frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^{2}\sin^{2}\theta} \cdot \frac{\partial^{2}\psi}{\partial\phi^{2}}$$

Rigid Rotator:

One part of the object is joined to another part of the object is freely.

Rigid body: An idealized extended whole size and shape is fixed and remains unaltered, when force are applied.



(finite distance).

both are rotate in opposite direction.

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(i) when, Energy of a rigid rotator

$$E = 0$$

Centre of Mass → A point in the system at which whole mass of the body is supposed to concentrated.

(ii) The total radial distance

$$r = r_1 + r_2$$

(iii) The total mass of a rigid body

(reduced mass).
$$\frac{1}{m} = \frac{1}{m_1} + \frac{1}{m_2} = \frac{m_1 m_2}{m_1 + m_2} = \mu$$

The mass does not vary during rotation but, assumed that mass can be varied.

(iv) Moment of inertia

$$I = mr^2 \quad (\because r = 1)$$

because we assumed rigid rotator, so r is unity (fixed distance is finite.)

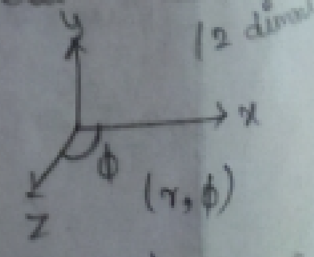
$$\boxed{I = m}$$

(v) $\frac{\partial \psi}{\partial r} = 0$ (During rotation of rigid body they can be produce a wave function w.r to distance r) is constant.

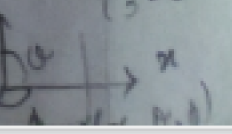
$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi}{\partial r}\right) + \frac{1}{r^2}\frac{1}{\sin\theta}$$

$$\frac{1}{r^2 \sin^2\theta}$$

Spherical Coordinates (2 dimension)



Polar coordinates (3-dimension)



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Schroedinger equation:

$$\nabla^2 \psi = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}$$

r θ φ

$$\nabla^2 \psi + \frac{2m}{\hbar^2}(E - V)\psi = 0.$$

from spherical polar coordinates:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi}{\partial r}\right) + \frac{1}{r^2}\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial \psi}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\cdot\frac{\partial^2 \psi}{\partial \phi^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0.$$

Conditions:

$r^2 = 1$, $\frac{\partial \psi}{\partial r} = 0$, $V = 0$, $m = I$ apply in above equation:

$$\frac{1}{\sin\theta}\cdot\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial \psi}{\partial \theta}\right) + \frac{1}{\sin^2\theta}\cdot\frac{\partial^2 \psi}{\partial \phi^2} + \frac{2EI}{\hbar^2}\psi = 0$$

Seperation of Variable:

$$\psi(\theta, \phi) = \Theta(\theta)\,\Phi(\phi)$$

here: $\psi = \Theta\Phi$

multiply $\times \frac{\sin^2\theta}{\Theta\Phi}$

Apply seperation of variable in above equation.

$$\frac{\Phi}{\sin\theta}\cdot\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial \Theta}{\partial \theta}\right) + \frac{\Theta}{\sin^2\theta}\cdot\frac{\partial^2 \Phi}{\partial \phi^2} + \frac{2EI}{\hbar^2}\Theta\Phi = 0$$

Multiply $\left(\frac{\sin^2\theta}{\Theta\Phi}\right)$ on both sides.

$\sin\theta$

Multiply $\left(\frac{\sin^2\theta}{\Theta\,\Phi}\right)$ on both sides.

$$\frac{\sin^2\theta}{\Theta\,\Phi}\times\frac{\cancel{\Phi}}{\sin\theta}\cdot\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right)+\frac{\sin^2\theta}{\cancel{\Theta}\,\Phi}\times\frac{\cancel{\Theta}}{\sin^2\theta}\cdot\frac{\partial^2\Phi}{\partial\phi^2}$$

$$+\frac{\sin^2\theta}{\Theta\,\Phi}\times\frac{2IE}{\hbar^2}\,\Theta\,\Phi=0$$

$$\frac{\sin\theta}{\Theta}\cdot\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right)+\frac{1}{\Phi}\cdot\frac{\partial^2\Phi}{\partial\phi^2}+\frac{2IE}{\hbar^2}\sin^2\theta=0$$

$$\frac{\sin\theta}{\Theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right)+\frac{2IE}{\hbar^2}\sin^2\theta=-\frac{1}{\Phi}\frac{\partial^2\Phi}{\partial\phi^2}$$

$$\frac{1}{\Phi}\cdot\frac{\partial^2\Phi}{\partial\phi^2}=-m^2$$

reduced mass. The mass can be varied w.r.t to ϕ.

$$\frac{\partial^2\Phi}{\partial\phi^2}=-m\Phi$$

$$\frac{\partial^2\Phi}{\partial\phi^2}+m^2\Phi=0$$

The soln of Φ is :- $\Phi = Ae^{\pm im\phi}$

Where,

$A \to$ arbitary constants,

$\pm i \to$ imaginary,

$m \to$ reduced mass,

$\phi \to$ angle.

Normalisation Condition:-

$$\int\Phi\,\Phi^*\,d\Phi=1$$

$$A^2\int e^{im\phi}\,e^{-im\phi}\,d\Phi=1$$

$$A^2(2\pi)=1$$

$$A^2=\frac{1}{2\pi}$$

$$\boxed{A=\frac{1}{\sqrt{2\pi}}}$$

$Ae^{\pm im\phi}$

$\Phi = Ae^{im\phi}$

$\Phi^* = Ae^{-im\phi}$

Substitute the value of A

$$\Phi=\frac{1}{\sqrt{2\pi}}e^{\pm im\phi}.$$

$$\frac{\sin\theta}{\Theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right)+\frac{2IE}{\hbar^2}\sin^2\theta=m^2.$$

$(\times)\ \frac{\Theta}{\sin^2\theta}$ on both sides:-

$$\psi_{lm}(\theta,\phi)=\left\{\frac{2l+1}{2}\cdot\frac{l-|m|!}{l+|m|!}\right\}^{1/2}P_l^m(\cos x)e^{im\phi}$$

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right)+\frac{2IE}{\hbar^2}\Theta=\frac{m^2\,\Theta}{\sin^2\theta}$$

$$\frac{1}{\sin\theta}\cdot\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right)+\left(\frac{2IE}{\hbar^2}-\frac{m^2}{\sin^2\theta}\right)\Theta=0.$$

here:-

$$\lambda=\frac{2IE}{\hbar^2}$$

$$\frac{1}{\sin\theta}\cdot\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right)+\left(\lambda-\frac{m^2}{\sin^2\theta}\right)\Theta=0$$

here:-

$x=\cos\theta$, $\cancel{dx = \cos\theta\, d\theta}$ $-\sin\theta\, d\theta = dx$.

$dx = -\sin\theta\, d\Theta / dx$

$$1\quad \frac{\partial}{}\quad = \quad \frac{1}{}\cdot\frac{d}{}\quad=\quad\frac{-d}{}$$

$x = \cos\theta$, $d\theta \cdot (-\sin\theta) = dx$ ~~$dx = \cos\theta\, d\theta$~~

$dx = -\sin\theta\, d\Theta / dx$

$$\frac{1}{\sin\theta}\cdot\frac{\partial}{\partial\theta} = \frac{1}{\sin\theta}\cdot\frac{d}{d\theta} = \frac{-d}{dx}$$

$= (1-\cos^2\theta)^{1/2}$ $= (\sin^2\theta)^{1/2}$ neglect 1/2

$$\sin\theta\,\frac{\partial \Theta}{\partial\theta} = \frac{\sin^2\theta}{\sin\theta}\,\frac{d\Theta}{\partial\theta} = -(1-x^2)\frac{d\Theta}{dx}$$

$$\sin^2\theta = (1-x^2).$$

Substitute these value in above equation :-

$$\frac{+d}{dx}\left(+(1-x^2)\frac{d\Theta}{dx}\right) + \left(\lambda - \frac{m^2}{(1-x^2)}\right)\Theta\Big) = 0$$

$$(1-x^2)\frac{d^2\Theta}{dx^2} + \left(\lambda - \frac{m^2}{(1-x^2)}\right)\Theta = 0.$$

from legendre palynomial :-

$$P_l^m(x) = (1-x^2)^{m/2}\frac{d^m}{dx^m}P_l(x)$$

$$\Theta_l^m(\theta) = B\,P_l^m(x)\cos\theta$$

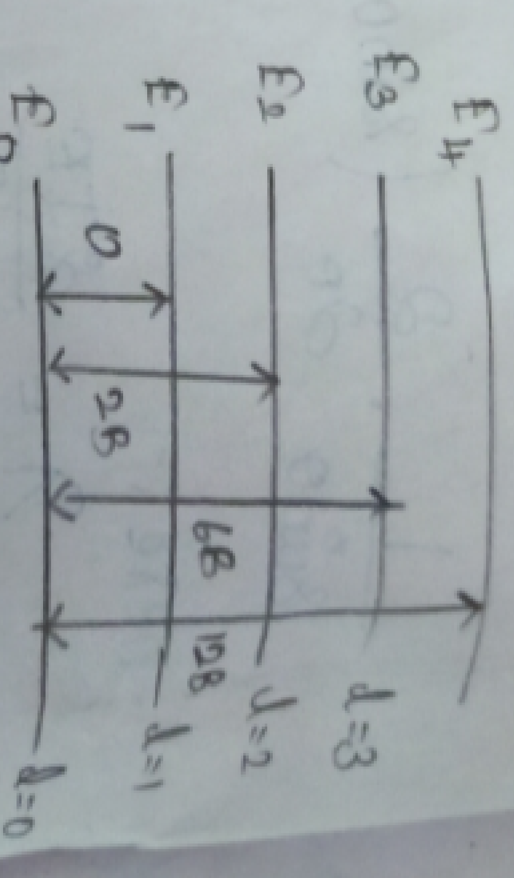
The equations both are equal in may be.

$$B = \left\{\frac{(2l+1)}{2}\,\frac{l-|m|!}{l+|m|!}\right\}^{1/2}$$

Eigen value :-

$$\lambda = \frac{2IE}{\hbar^2} = (l+1)l$$ → orbital quantum number.

$$E_n = \frac{l(l+1)\hbar^2}{2I}$$



Where,

$n = 0, 1, 2, \dots$

$l = 0, 1, 2, \dots$

$E_0 = 0$

$E_1 = \frac{\hbar^2}{2I}\times 2 = 2B$

$E_2 = \frac{\hbar^2}{2I}\times 6 = 6B$

$E_3 = \frac{\hbar^2}{2I}\times 12 = 12B$

3/16 Hydrogen atom:

Potential energy enclosed

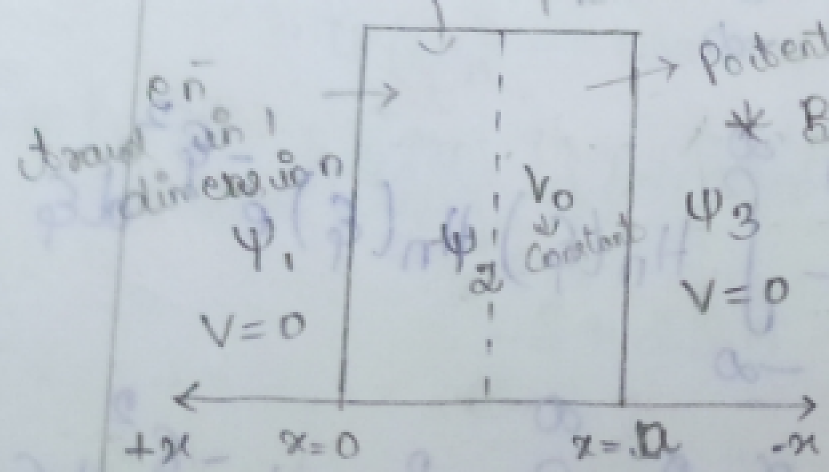
11/3/16 Rectangular Potential barrier (or) Penetration of (Tunnel effect). Potential Barrier

no. of particles present * easily Penetrate, additional of Photons

→ Potential energy is exist. en Create to break a junction — e hole pair — Tunnel effect

* Barrier ↳ after the limit, the particle cannot travel.

* Stream of particle collide allow to Rectangular barrier continuous line of particle — depth → α — length



From Schroedinger time independent equation,

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

For Region I: (Sometimes reflection may be occured) (Reflection Region)

$\psi = \psi_1$

$V = 0$

$$\frac{d^2\psi_1}{dx^2} + \frac{2m}{\hbar^2}E\psi_1 = 0 \longrightarrow (1)$$

$$\frac{d^2\psi_1}{dx^2} + \alpha^2\psi_1 = 0$$

here:

α

The sol...

$\psi_1(x) = Ae^{i\alpha...}$

where:

A, B → arbitrary

For Region-II: (Pot...

$\psi = \psi_2$

$V = V_0$

$$\frac{d^2\psi_2}{dx^2} + \ldots$$

$$\frac{d^2\psi_2}{dx^2} - \ldots$$

here:

$$\beta^2 = \frac{2m}{\hbar^2}(\ldots$$

$$\frac{d^2\psi_2}{dx^2} - \ldots$$

Chamber ↓ maintains Constant temp

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here:

$$\alpha^2 = \frac{2mE}{\hbar^2}$$

The soln is :- $\xrightarrow{\text{reflection}}$ $\xrightarrow{\text{penetration}}$

$$\psi_1(x) = Ae^{i\alpha x} + Be^{-i\alpha x} \rightarrow (2)$$

rays: α (+ve), β (−ve), γ (neutral)

Reflection.

$\angle i = \angle r$ incident ray(i) and emergence ray(r) are exist in same medium.

$i \neq r \rightarrow$ Scattering

refraction ↓ exist in different medium.

where:

$A, B \rightarrow$ arbitrary Constant.

for Region-II : (Potential barrier)

$\psi = \psi_2$ $\quad \dfrac{d^2\psi_2}{dx^2} + \dfrac{2m}{\hbar^2}(E - V_0)\psi_2 = 0$

$V = V_0$

$$\frac{d^2\psi_2}{dx^2} - \frac{2m}{\hbar^2}(V_0 - E)\psi_2 = 0$$

Chamber ↓ maintains Constant temp

here:

$$\beta^2 = \frac{2m}{\hbar^2}(V_0 - E)$$

The soln is

$$\psi_2(x) = Ce^{\beta x} + De^{-\beta x}$$

$$\frac{d^2\psi_2}{dx^2} - \beta^2\psi_2 = 0.$$

Also

Continuous in 3 page

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16/8/16 for Region-III : (Rectangular ... (does not reflected occur in there Region)

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0$$

here:

$\psi = \psi_3$ $\quad \dfrac{d^2\psi_3}{dx^2} + \dfrac{2m}{\hbar^2}E\psi_3 = 0$

$V = 0$

$$\frac{d^2\psi_3}{dx^2} + \alpha^2\psi_3 = 0.$$

The soln is :-

$$\psi_3 = Fe^{i\alpha x} + \underbrace{Ge^{-i\alpha x}}_{=0} \rightarrow \text{because no reflection.}$$

$$\therefore \boxed{\psi_3 = Fe^{i\alpha x}}$$

when, wave function (ψ) is Continuous,

$$\psi_1 = \psi_2$$

$$\left(\frac{\partial^2\psi_1}{\partial x^2}\right)_{x=0} = \left(\frac{\partial^2\psi_2}{\partial x^2}\right)_{x=0}$$

$e^{nx} = ne^x$

$e^x = 1$

$Ae^{i\alpha x} + Be^{-i\alpha x} = Ce^{\beta x} + De^{-\beta x}$

↓ diffe

$i\alpha A(e^x) - i\alpha Be^{-x} \ldots$

$$A + B = C + D$$

$$i\alpha A - i\alpha B = C\beta - D\beta$$

By fourier series, 1 to seperate a needed.

$$\therefore \boxed{\psi_3 = Fe^{i\alpha x}}$$

when,

wave function (ψ) is continuous,

$$\psi_1 = \psi_2$$

$$\left(\frac{\partial^2 \psi_1}{\partial x^2}\right)_{x=0} = \left(\frac{\partial^2 \psi_2}{\partial x^2}\right)_{x=0}$$

$$A + B = C + D$$

$$i\alpha A - i\alpha B = C\beta - D\beta$$

By fourier series, (to seperate a needed particles)

$$A = \left(1 - \frac{i\beta}{\alpha}\right)\frac{C}{2} + \left(1 + \frac{i\beta}{\alpha}\right)D/2$$

Penetrate reflection

$$B = \left(1 + \frac{i\beta}{\alpha}\right)\frac{C}{2} + \left(1 - \frac{i\beta}{\alpha}\right)D/2 .$$

$$\psi_2 = \psi_3$$

$$\left(\frac{\partial^2 \psi_2}{\partial x^2}\right)_{x=a} = \left(\frac{\partial^2 \psi_3}{\partial x^2}\right)_{x=a}$$

put: $x = a$

$$C + D = F$$

$$Ce^{\beta x} + De^{-\beta x} = Fe^{i\alpha x}.$$

$$\cancel{\beta C - \beta D = i\alpha F}$$

$$Ce^{\beta a} + De^{-\beta a} = Fe^{i\alpha a}$$

$$C\beta e^{a} - D\beta e^{-a} = i\alpha F e^{a}$$

$$C = \left(1 + \frac{i\alpha}{\beta}\right)F/2\, e^{-\beta a} e^{i\alpha a}$$

$$C = \left(1 + \frac{i\alpha}{\beta}\right)F/2\, e^{(i\alpha - \beta)a}$$

$$D = \left(1 - \frac{i\alpha}{\beta}\right)F/2\, e^{(i\alpha + \beta)a}$$

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when,

the thickness box is maximum, The particle does not penestrate inside a box.

$$A = \left(1 + \frac{i\beta}{\alpha}\right) D/2$$

(from fourier series $\psi_1 = \psi_2$)

sub D value in above eqn:

$$A = \left(1 + \frac{i\beta}{\alpha}\right)\left(1 - \frac{i\alpha}{\beta}\right) F/4\, e^{(i\alpha + \beta)a}$$

Transmission co-efficient

$$T = \frac{|F|^2}{|A|^2}$$ (intensity of transmitted wave) / (incident wave).

Reflection co-efficient

$$R = \frac{|B|^2}{|A|^2}$$

Damped (Uncontinuous oscillator)

Undamped - (equal all force) (Continuous oscillation)

Restoring force → regain to original position (with respect to external force).

$F \propto -x$, displacement

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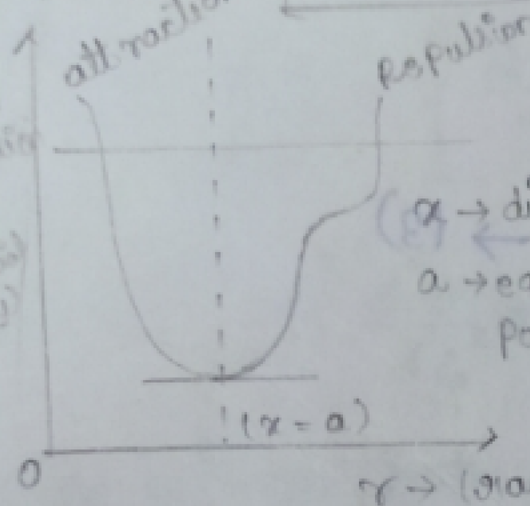
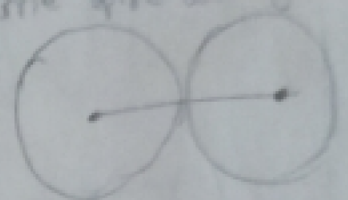
Linear Harmonic oscillator:

Mathematical simplification

$F = +kx$

Two: The two harmonic oscillator vibrate same frequency with





radial distance → Centre of 1st atom to Centre of 2nd atom

Square series: $E = \frac{k}{2}(x-a)^2 \to (1)$ → kinetic energy

Power V

$x = a$, $x - a = 0$.

Acemtotic hormonic equation:: Asymptotic

from restoring force

$$F \propto -x$$

$$F = -kx$$

Potential with respect to distance.

Potential energy $V = V(x) = \frac{1}{2} kx^2 \to (2)$

from Schroedinger

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0$$

→ dimension equation with respect to one.

Substitute equ (2) in above equation.

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left(E - \tfrac{1}{2}kx^2\right)\psi = 0$$

Convert these equ as a dimensionless by introducing

here:

$$\boxed{\xi \propto x}$$

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$\xi \to$ dimensionless quantity

$$\boxed{\xi = \alpha x}$$

$\alpha \to$ proportionality Constant.

differentiate with respect to above equation

$$\xi = \alpha x$$

$$\frac{d\xi}{dx} = \alpha \to (3)$$

$$\frac{d\psi}{dx} = \frac{d\psi}{d\xi} \times \frac{d\xi}{dx}.$$

$$\boxed{\frac{d\psi}{dx} = \frac{d\psi}{d\xi}\alpha}$$

Again differentiate

$$\frac{d^2\psi}{dx^2} = \frac{d}{dx}\left(\frac{d\psi}{dx}\right)$$

$$= \frac{d}{dx}\left(\frac{d\psi}{d\xi}\alpha\right)$$

$$\frac{d^2\psi}{dx^2} = \frac{d\xi}{dx}\left(\frac{d\psi}{d\xi^2}\cdot\alpha\right).$$

Multiply & divide (ξ)

$$\frac{d^2\psi}{dx^2} = \frac{d\xi}{dx}\left(\frac{d\psi}{d\xi^2}\right)\alpha$$

$$= \frac{d^2\psi}{d\xi^2}\left(\frac{d\xi}{dx}\right)\alpha$$

Put in equ (3)

$$= \frac{d^2\psi}{d\xi^2}(\alpha)(\alpha)$$

$$\frac{d^2\psi}{dx^2} = \left(\frac{d^2\psi}{d\xi^2}\right)\alpha^2 \longrightarrow (4)$$

8/3/16 Substitute equ (4) in these equation:

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left(E - \tfrac{1}{2}kx^2\right)\psi = 0 \qquad \xi = \alpha x, \quad x = \xi/\alpha$$

$$\alpha^2\frac{d^2\psi}{d\xi^2} + \frac{2m}{\hbar^2}\left(E - \tfrac{1}{2}k\,\frac{\xi^2}{\alpha^2}\right)\psi = 0 \rightarrow (5)$$

$$\alpha^2\frac{d^2\psi}{d\xi^2} + \left[\frac{2mE}{\hbar^2} - \frac{2mk\xi^2}{2\hbar^2\alpha^2}\right]\psi = 0$$

$$\alpha^2\frac{d^2\psi}{d\xi^2} + \left[\frac{2mE}{\hbar^2} - \frac{mk\xi^2}{\hbar^2\alpha^2}\right]\psi = 0$$

divide α^2 on both sides

$$\frac{d^2\psi}{d\xi^2} + \Bigg[\underbrace{\frac{2mE}{\hbar^2\alpha^2}}_{\lambda} - \underbrace{\frac{mk\xi^2}{\hbar^2\alpha^4}}_{1}\Bigg]\psi = 0.$$

here:

Assume that $\frac{mk}{\hbar^2\alpha^4} = 1$

$$\frac{d^2\psi}{d\xi^2} + \left[\frac{2mE}{\hbar^2\alpha^2} - \xi^2\right]\psi = 0 \rightarrow (5a)$$

$$\frac{mk}{\hbar^2\alpha^4} = 1 \Rightarrow \frac{mk}{\hbar^2} = \alpha^4 \Rightarrow \alpha = \left(\frac{mk}{\hbar^2}\right)^{1/4}$$

$$\alpha^2 = \left(\frac{mk}{\hbar^2}\right)^{1/2}$$

Put: $\lambda = \frac{2mE}{\hbar^2\alpha^2}$

$$\lambda = \frac{2mE}{\hbar^2}\left[\frac{\hbar^2}{mk}\right]^{1/2} \rightarrow (6)$$

$$= \frac{2mE}{\hbar^2} \times (\hbar^2)^{1/2} \times \left(\frac{1}{mk}\right)^{1/2}$$

$$= \frac{2mE}{\hbar}\left(\frac{1}{mk}\right)^{1/2}$$

$$= \frac{2mE}{\hbar}\left(\frac{1}{mk}\right)^{1/2}$$

$$= \frac{2E}{\hbar} \times \frac{\sqrt{m}\times\sqrt{m}}{\sqrt{m}}\left(\frac{1}{k}\right)^{1/2}$$

$$\lambda = \frac{2E}{\hbar}\sqrt{\frac{m}{k}} \rightarrow (7).$$

equ (5a) becomes:

$$\lambda = \frac{2mE}{\hbar^2\alpha^2}$$

$$\frac{d^2\psi}{d\xi^2} + [\lambda - \xi^2]\psi = 0 \rightarrow (8).$$

~~Acemitatic~~ form: Asymptotic:

Polynomial series:-

$$\psi(\xi) = H(\xi)e^{-\xi^2/2}$$

Hermition

↳ The value may be +ve (or) -ve fraction value

$$\frac{d\psi}{d\xi} = H'(\xi)e^{-\xi^2/2}\left(-\xi^2/2\right)(-2\xi)$$

$$\frac{d\psi}{d\xi} = H'(\xi)e^{-\xi^2/2} + e^{-\xi^2/2}H(\xi)\left(\frac{-2\xi}{2}\right)$$

$$\frac{d\psi}{d\xi} = H'(\xi)e^{-\xi^2/2} - H(\xi)e^{-\xi^2/2}(\xi)$$

$$\frac{d^2\psi}{d\xi^2} = H''(\xi)e^{-\xi^2/2} + H'(\xi)e^{-\xi^2/2}\left(-\frac{2\xi}{2}\right) - \left[H'(\xi)e^{-\xi^2/2}(\xi) + H(\xi)e^{-\xi^2/2}(\xi)\left(\frac{-2\xi}{2}\right)\right]$$

$$= H''(\xi)e^{-\xi^2/2} - H'(\xi)e^{-\xi^2/2}(\xi) - H'(\xi)e^{-\xi^2/2}(\xi) + H(\xi)e^{-\xi^2/2}(\xi)^2 - H(\xi)e^{-\xi^2/2}$$

again differentiate

$$\frac{d^2\psi}{d\xi^2} = e^{-\xi^2/2}\left(H''(\xi) - 2H'(\xi)\times\xi + (\xi^2-1)H(\xi)\right)$$

9/3/16 Put the above eqn in equ (8) here: $\psi = H(\xi)e^{-\xi^2/2}$

neglect higher order term.

$$(8) \Rightarrow \frac{d^2\psi}{d\xi^2} + [\lambda - \xi^2]\psi = 0$$

Add

$$e^{-\xi^2/2}\left(H''(\xi) - 2\xi H'(\xi) + (\xi^2-1)H\xi + (\lambda-\xi^2)H(\xi)\right) = 0$$

$$e^{-\xi^2/2}\left[H''(\xi) - 2\xi H'(\xi) + (\lambda-1)H(\xi)\right] = 0$$

Multiply one Hermitian form.

$$H'' - 2\xi H' + (\lambda-1)H = 0 \rightarrow (9)$$

The equ (9) is Called a Acemitatic form.

Energy level:

Hermitic Polynomial equation. (9) can

$$H(\xi) = \xi^s(a_0 + a_1\xi^1 + a_2\xi^2 + \dots)$$

$$= \xi^s\sum_{v=0}^{\infty} a_v\xi^v$$

$$\boxed{H(\xi) = \sum_{v=0} a_v\xi^{v+s}}$$

$\xi^s(a_v\xi^v)$ $v = 0, 1, \dots$ $\xi^s(a_0\xi^0 + a_1\xi + a_2\xi^2 \dots)$

here: $\xi^s \rightarrow$ dimensionless quantity with series

Diff the above equation.

$$\frac{dH}{d\xi} = \sum_{v} a_v (s+v) \xi^{s+v-1}$$

$x^n = nx^{n-1}$

Again differentiate

$$\frac{d^2H}{d\xi^2} = \sum_{v} a_v (s+v)(s+v-1) \xi^{s+v-2}$$

Compond - combination of more than one simple ...

Put the value of H, $dH/d\xi$ and $d^2H/d\xi^2$ in equ (9)

$$(9) \Rightarrow H'' - 2\xi H' + (\lambda - 1) H = 0$$

$$\sum_{v} a_v (s+v)(s+v-1) \xi^{s+v-2} - 2\xi \sum_{v} a_v (s+v) \xi^{s+v-1} + (\lambda-1) \sum_{v} a_v \xi^{s+v} = 0$$

$$\sum_{v} a_v (s+v)(s+v-1) \xi^{s+v-2} - 2 \sum_{v} a_v (s+v) \xi^{s+v} + (\lambda-1) \sum_{v} a_v \xi^{s+v} = 0$$

Put, $v = v+2$ in 1st term above equ

$$\sum_{v} a_{v+2} (s+v+2)(s+v+1) \xi^{s+v+2-2} - 2 \sum_{v} a_v (s+v) \xi^{s+v} + (\lambda-1) \sum_{v} a_v \xi^{s+v} = 0$$

$$a_{v+2} (s+v+2)(s+v+1) - (2s - \lambda + 2v + 1) a_v = 0$$

$v \to$ integer

$$\boxed{a_{v+2} = \frac{(2s - \lambda + 2v + 1)}{(s+v+2)(s+v+1)} a_v}$$

Angular acceleration

10/3/16

Recursion formula

This above equation is called energy-level value Recursion formula.

$2s - \lambda + 2v + 1 = 0$

$\lambda = 2s + 2v + 1$

from a_{v+2} value

$$\lambda = 2s + 2v + 1$$

we don't want 2nd series value so $\boxed{s = 0}$

$s = 0$,

$\lambda = 2v + 1$

$$\boxed{\lambda = 2n + 1} \to (10)$$

Equating Equation (7) and Eq (10)

$$(8) \Rightarrow \lambda = \frac{2E}{\hbar} \sqrt{\frac{m}{k}}$$

$$\frac{2E}{\hbar} \sqrt{\frac{m}{k}} = 2n + 1$$

$$E_n = (2n+1) \frac{\hbar}{2} \sqrt{\frac{k}{m}}$$

here;

$$\omega_c = \sqrt{k/m}$$

$$E_n = (n + 1/2) \hbar \sqrt{\frac{k}{m}}$$

$$\boxed{E_n = (n + 1/2) \hbar \omega_c} \to (11)$$

Where,

$\omega_c \to$ classical angular frequency value in classical mechanics of the oscillator

$E_n \to$ energy level.

$E_n \rightarrow$ energy level.

$n \rightarrow$ no. of oscillation.

Eigen value for LHO.

These eqn (11) is called energy-level equation

10/3/16 Case (i): Zero point energy (or) Ground state energy.

from energy level equation:

$$E_n = (n+\tfrac{1}{2})\hbar\omega_c$$

Energy diagram: E_2 — $\tfrac{5}{2}\hbar\omega_c$; E_1 — $\tfrac{3}{2}\hbar\omega_c$; E_0 — $\tfrac{1}{2}\hbar\omega_c$; 0

Puts.
$n=0 \quad E_0 = \tfrac{1}{2}\hbar\omega_c$

$n=1 \quad E_1 = \tfrac{3}{2}\hbar\omega_c$

$n=2 \quad E_2 = \tfrac{5}{2}\hbar\omega_c$

Polynomial ↓ many no. of combine single value.

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from acemtotic equation:

$$(9) \Rightarrow H'' - 2\xi H' + (\lambda-1)H = 0$$

$$H_n'' - 2\xi H_n' + (\lambda-1)H_n = 0$$

Put $\lambda = 2n+1$ in above eqn

$$H_n'' - 2\xi H_n' + (2n+\cancel{1}-\cancel{1})H_n = 0$$

$$H_n'' - 2\xi H_n' + 2nH_n = 0$$

Hermite polynomial generating function is:

$$e^{-s^2+2\xi s}$$

Wave function of LHO:

from Acemtotic form:

$$\psi(\xi) = H(\xi)\, e^{-\xi^2/2}$$

$$\psi_n(\xi) = N_n H_n(\xi)\, e^{-\xi^2/2} \rightarrow (1).$$

Where:

$N_n \rightarrow$ normalized quantity

$\psi(\xi) \rightarrow$ wave function of dimensionless quantity

Generating formula for Hermite func:-

$$e^{-s^2+2s\xi} = \frac{H_n(\xi)}{n!} s^n \rightarrow (2)$$

$$e^{-t^2+2t\xi} = \frac{H_m(\xi)}{m!} t^m.$$

here: $S = t$, $n = m$

Multiply the above (2) equations:-

$$e^{-s^2+2s\xi} \cdot e^{-t^2+2t\xi} = \frac{H_n(\xi)}{n!} s^n$$

Taking integral on both sides $\int(2)e^{-\xi^2}d\xi$ on both side

$$\int_{-\infty}^{\infty} e^{-s^2+2s\xi}\, e^{-t^2+2t\xi}\, e^{-\xi^2} d\xi = \int_{-\infty}^{\infty} \frac{H_n(\xi)}{n!} s^n \cdot \frac{H_m(\xi)}{m!} t^m\, e^{-\xi^2} d\xi.$$

$$\int_{-\infty}^{\infty} e^{(-s^2-t^2-\xi^2+2s\xi+2t\xi)} d\xi = \frac{s^n t^m}{n!\,m!} \int_{-\infty}^{\infty} H_n(\xi) H_m(\xi)\, e^{-\xi^2} d\xi$$

L.H.S:

$$e^{2st} \int_{-\infty}^{\infty} e^{(-s-t-\xi)^2} d\xi$$

$2s\xi + 2t\xi$

$\xi \rightarrow$ common term

$2 \rightarrow$ we take only one variable.

$$= e^{2st}\, \pi^{1/2}$$

Taking integral on both sides

$$\int_{-\infty}^{\infty} e^{-s^2+2s\xi}\, e^{-t^2+2t\xi}\, e^{-\xi^2} d\xi = \int_{-\infty}^{\infty} \frac{H_n(\xi)}{n!} s^n \cdot \frac{H_m(\xi)}{m!} t^m\, e^{-\xi^2} d\xi .$$

$$\int_{-\infty}^{\infty} e^{(-s^2-t^2-\xi^2+2s\xi+2t\xi)} d\xi = \frac{s^n t^m}{n!\,m!} \int_{-\infty}^{\infty} H_n(\xi) H_m(\xi)\, e^{-\xi^2} d\xi$$

L.H.S :

$$e^{2st} \int_{-\infty}^{\infty} e^{(-s-t-\xi)^2} d\xi$$

$2s\xi + 2t\xi$

$\xi \rightarrow$ Common term

$2 \rightarrow$ we take only one variable.

$$= e^{2st}\, \pi^{1/2}$$

$$= \pi^{1/2} \left[\frac{2st}{1!} + \frac{(2st)^2}{2!} + \dots \right]$$

$e^x = \frac{x}{1!} + \frac{x^2}{2!} +$

$$= \pi^{1/2} \sum_{n=0}^{\infty} \frac{(2st)^n}{n!}$$

$$\pi^{1/2} \sum_{n=0}^{\infty} \frac{(2st)^n}{n!} = \frac{s^n t^m}{n!\,m!} \int_{-\infty}^{\infty} H_n(\xi) H_m(\xi)\, e^{-\xi^2} d\xi$$

Put, $m = n$

$$\pi^{1/2} \sum_{n=0}^{\infty} \frac{(2st)^n}{n!} = \frac{s^n t^n}{n!\,n!} \int_{-\infty}^{\infty} H_n(\xi) H_n(\xi)\, e^{-\xi^2} d\xi$$

$$\pi^{1/2} \sum_{n=0}^{\infty} \frac{2^n \cancel{s^n} \cancel{t^n}}{\cancel{n!}} = \frac{\cancel{s^n} \cancel{t^n}}{\cancel{n!}\, n!} \int_{-\infty}^{\infty} H_n^2(\xi)\, e^{-\xi^2} d\xi$$

$$\int_{-\infty}^{\infty} H_n^2(\xi)\, e^{-\xi^2} d\xi = \pi^{1/2} \sum_{n=0}^{\infty} (n!)\, 2^n$$

Put, normalization value,

$\xi = \alpha x$

$$= \frac{N_n^2}{\alpha} \pi^{1/2} \sum_{n=0}^{\infty} (n!)\, 2^n$$

$$N_n^2 = \left[\frac{\alpha}{\pi^{1/2} (n!)\, 2^n} \right]$$

$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$$

$$N_n = \left[\frac{\alpha}{\pi^{1/2} (n!)\, 2^n} \right]^{1/2}$$

$$\frac{N_n^2}{\alpha} \int_{-\infty}^{\infty} H_n^2\, e^{-\xi^2/2}$$

Put N_n value in equ (1)

$$\frac{N_n^2}{\alpha} 2^n (n!)$$

$$(1) \Rightarrow \psi_n(\xi) = N_n H_n(\xi)\, e^{-\xi^2/2}$$

$N_n^2 =$

$$\psi_n(\xi) = \left[\frac{\alpha}{\pi^{1/2} (n!)\, 2^n} \right]^{1/2} H_n(\xi)\, e^{-\xi^2/2}$$

The above equ is called Linear Har oscillator wave function equation.

Potential energy enclosed

$$= \pi^{1/2} \sum_{n=0}^{\infty} \frac{(2st)^n}{n!}$$

$$\pi^{1/2} \sum_{n=0}^{\infty} \frac{(2st)^n}{n!} = \frac{s^n t^m}{n!\, m!} \int_{-\infty}^{\infty} H_n(\xi) H_m(\xi) e^{-\xi^2} d\xi$$

Put, $m = n$

$$\pi^{1/2} \sum_{n=0}^{\infty} \frac{(2st)^n}{n!} = \frac{s^n t^n}{n!\, n!} \int_{-\infty}^{\infty} H_n(\xi) H_n(\xi) e^{-\xi^2} d\xi$$

$$\pi^{1/2} \sum_{n=0}^{\infty} \frac{2^n \cancel{s^n} \cancel{t^n}}{\cancel{n!}} = \frac{\cancel{s^n} \cancel{t^n}}{\cancel{n!}\, n!} \int_{-\infty}^{\infty} H_n^2(\xi) e^{-\xi^2} d\xi$$

$$\int_{-\infty}^{\infty} H_n^2(\xi) e^{-\xi^2} d\xi = \pi^{1/2} \sum_{n=0}^{\infty} (n!) 2^n$$

Put, normalization value,

$\xi = \alpha x$

$$= \frac{N_n^2}{\alpha} \pi^{1/2} \sum_{n=0}^{\infty} (n!) 2^n$$

$$N_n^2 = \left[\frac{\alpha}{\pi^{1/2} (n!) 2^n} \right] \qquad \int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$$

$$N_n = \left[\frac{\alpha}{\pi^{1/2} (n!) 2^n} \right]^{1/2} \qquad \frac{N_n^2}{\alpha} \int_{-\infty}^{\infty} H_n^2 e^{-\xi^2/2} d\xi =$$

Put N_n value in equ (1) $\quad \frac{N_n^2}{\alpha} 2^n (n!) \pi^{1/2} = 1$

$$(1) \Rightarrow \psi_n(\xi) = N_n H_n(\xi) e^{-\xi^2/2} \qquad N_n^2 = \frac{\alpha}{2^n (n!) \pi^{1/2}}$$

$$\psi_n(\xi) = \left[\frac{\alpha}{\pi^{1/2} (n!) 2^n} \right]^{1/2} H_n(\xi) e^{-\xi^2/2}$$

The above equ is called Linear Harmonic oscillator wave function equation.

Potential energy enclosed