with the empty set of ${}^{1}S$. The total orderings ${}^{.75}O$ that are compatible with the pairs in the crisp relation ${}^{.75}S$ are

(ii) $${}^{.75}O = \{\langle z, w, x, y\rangle, \langle w, x, y, z\rangle, \langle w, z, x, y\rangle, \langle w, x, z, y\rangle, \langle z, w, \langle z, x, w, y\rangle, \langle x, w, y, z\rangle, \langle x, z, w, y\rangle, \langle x, w, z, y\rangle\}$$

Thus

$${}^{1}O \cap {}^{.75}O = {}^{.75}O$$

The orderings compatible with ${}^{.625}S$ are

$${}^{.625}O = \{\langle w, z, x, y\rangle, \langle w, z, y, x\rangle\}$$

and

$${}^{1}O \cap {}^{.75}O \cap {}^{.625}O = \{\langle w, z, x, y\rangle\}$$

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fuzzy group preference ordering relation S (where $n=8$) we arrive at the following fuzzy social preference relation :

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(10) \quad S = \begin{array}{c} \\ w \\ x \\ y \\ z \end{array}
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Solution:

The α-cuts of this fuzzy relation S are

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$^{.375}S = \{\langle z,w\rangle, \langle x,z\rangle, \langle y,z\rangle, \langle x,w\rangle, \langle w,x\rangle, \langle w,z\rangle, \langle z,x\rangle, \langle z,y\rangle, \langle w,y\rangle, \langle x,y\rangle\}$

$^{.25}S = \{\langle y,w\rangle, \langle y,x\rangle, \langle z,w\rangle, \langle x,z\rangle, \langle y,z\rangle, \langle x,w\rangle, \langle w,x\rangle, \langle w,z\rangle, \langle z,x\rangle, \langle z,y\rangle, \langle w,y\rangle, \langle x,y\rangle\}$

Now, we can apply the procedure to arive at the unique crisp ordering that constitutes the group choice. All total orderings on $X \times Y$ are, Compatible

Intersecting the classes of crisp total orderings that are compatible with the pairs in the α-cuts α_s for increasingly smaller values of α until a single crisp total ordering is achieved. In this, any pairs $\langle x_i, x_j \rangle$ is an intransitivity are removed. (a)

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Assume that each individual of a group of eight decision makers has a total preference ordering P_i $(i \in N_8)$ on a set of alternatives $X = \{w, x, y, z\}$ as follows :

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$$S(x_i, x_j) = \frac{N(x_i, x_j)}{n} \rightarrow (1)$$

$$S(x_i, x_j) = \begin{cases} 1, & \text{if } x_i \overset{k}{>} x_j \text{ for some individual } k \\ 0, & \text{otherwise} \end{cases}$$

Where $\overset{k}{>}$ represents the preference ordering of the one individual k.

The final nonfuzzy group preference can be determined by converting S into its resolution form

$$S = \bigcup_{\alpha \in [0,1]} \alpha\, {}^{\alpha}S$$

Which is the union of the crisp relations ${}^{\alpha}S$ comprising α-cuts of the fuzzy relation S, each scaled by α.

Each value α represents essentially the level of agreement between the individuals concerning the particular crisp ordering ${}^{\alpha}S$. One procedure that maximizes the final agreements level of

The job to be seen is $\hat{a} = 2$, this is the most desirable jobs among the four available jobs under the given goal G_1 and Constraints C_1, C_2. We can aggregate the goal and constraints as expressed by,

$$D(a) = \min \left[\inf_{i \in N_n} G_i(a), \inf_{j \in N_m} C_j(a) \right] \quad \forall a \in A.$$

01/08/20

Multiperson Decision Making :

Let, each member of a group of n individual decision makers is assumed to have a reflexive, antisymmetric and transitive preference ordering P_k, $k \in N_n$, which totally (or) partially orders a set X of alternatives. The social preference S may be defined as a fuzzy binary relation with membership grade function.

$$S : X \times X \rightarrow [0,1]$$

which assigns the membership grade $S(x_i, x_j)$ indicating the degree of group preference of alternative x_i over x_j.

The expression of this group preference requires some aggregating individual driving distance to be close, is expressed interms of the driving distance from home to work. We denote the fuzzy set expressing this constraints by C_2'. A possible of C_2' is given in diagram whose distances of the four jobs are.

(b)

$c_2(a_1) = 27$ miles

$c_2(a_2) = 7.5$ miles

$c_2(a_3) = 12$ miles

$c_2(a_4) = 2.5$ miles

By composing functions c_2 and c_2' we obtain the fuzzy set.

$$C_2 = .1/a_1 + .9/a_2 + .7/a_3 + .1/a_4$$

Which express the constraints interms of the set A.

Applying now,

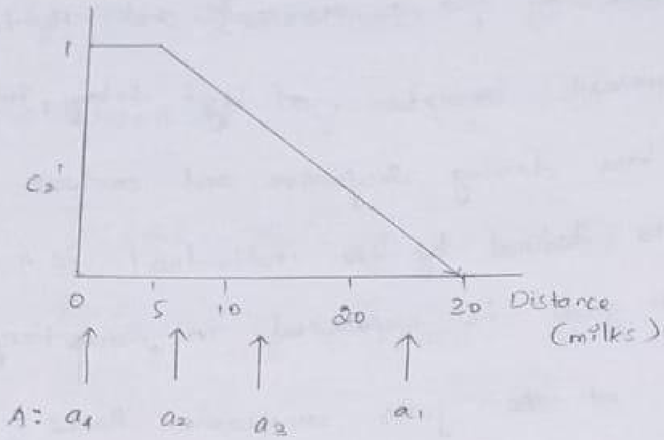
$$D(a) = \min\left[\inf_{i \in N_n} G_i(a), \inf_{j \in N_m} C_j(a)\right]$$

We obtain the fuzzy set,

$$D = .1/a_1 + .3/a_2 + .2/a_3 + .2/a_4$$

Which represents a fuzzy characterization of the concepts of desirable jobs.

(5)



(b) constant C_2' : Close driving distances

Now, the functions g and G', according to fig(a), We obtain the fuzzy set

$$G = {.11}/{a_1} + {.3}/{a_2} + {.48}/{a_3} + {.8}/{a_4}$$

Which express the goal interms of the available jobs in set A.

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Assume that the individual assigns to the four jobs in A the following membership functions grades in the fuzzy set of interesting jobs.

$$C_1 = {.4}/{a_1} + {.6}/{a_2} + {.2}/{a_3} + {.2}/{a_4}$$

The second constraint, requiring that the

In this case, $A = \{a_1, a_2, a_3, a_4\}$ and the fuzzy sets involved represent concepts of high salary, interesting job, and close driving subjective and context-dependent, and must be defined by the individual is a given context the goal is expressed in monetary terms,
(1) independent of the jobs available. Hence, according to our notation we denote the fuzzy set expressing the goal by G'. To express the goal in terms of set A, we need a function $g : A \to \mathbb{R}^+$ which assigns to each job the respective salary. Assume the following assignments:

$g(a_1) = \$ 40,000$

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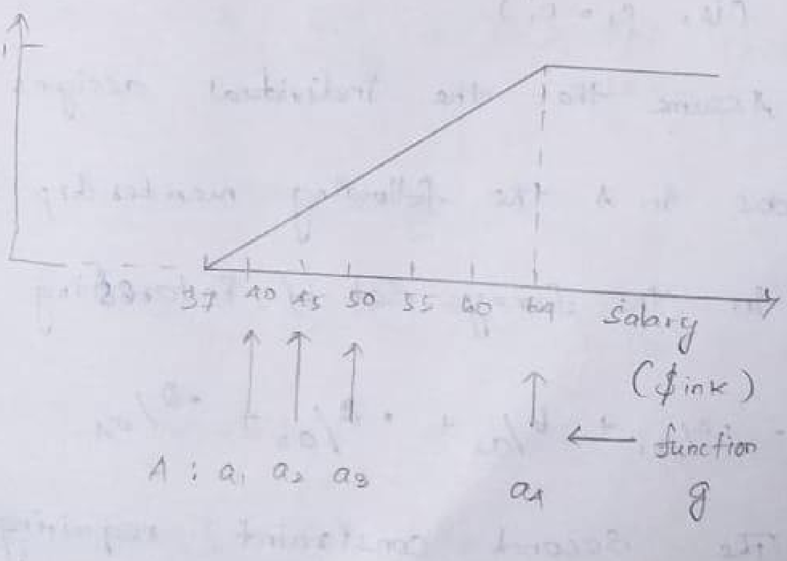
Solution:



(a) Goal G' : High Salary

C_j by the compositions, of g_i with G_i' and the compositions of C_j and C_j' that is,

$$G_i(a) = G_i'(g_i(a)) \rightarrow (1)$$

(3) $$C_j(a) = C_j'(C_j(a)) \quad \text{for each } a \in A \rightarrow (2)$$

Given a decision situation characterized by fuzzy sets A, G_i ($i \in N_n$) and C_j ($j \in N_m$) a fuzzy decision, D is connected as a fuzzy set on A that simultaneously satisfies the given goals G_i and constraints C_j. That is,

$$D(a) = \min\left[\inf_{i \in N_n} G_i(a), \inf_{i \in N_m} C_j(a)\right] \rightarrow (3)$$

for all $a \in A$, provided that the standard operator of fuzzy intersection is employed.)

Problem:

Suppose that an individual needs to decide which of four possible jobs, a_1, a_2, a_3 and a_4 to choose. His or her goal is to choose a job that offers a high salary under the constraints that the job is interesting and which within close driving distances.

are referred to as individual decision making and multiperson decision making, respectively.

Individual Decision Making :

A decision situation in this model is characterized by the following components :

* a set A of possible actions.
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Let G_i' and C_j' be fuzzy sets defined on sets X_i and Y_j respectively where $i \in N_n$ and $j \in N_m$. Assume that these fuzzy sets represents goals and constraints expressed by the decision maker.

Then for each $i \in N_n$ and each $j \in N_m$ the actions in set A in terms of sets X_i and Y_j by functions,

$$g_i : A \to X_i$$

$$c_j : A \to Y_j$$

and express goals G_i and constraints

UNIT - V

27/1/20

FUZZY DECISION MAKING:

(1) Decision making problems are made under conditions of certainty when the outcome for each action can be determined and ordered precisely.

The decision-making problem becomes an optimization problem of maximizing the expected utility.

When probabilities of the outcomes are not known (or) may not even be relevant, and outcomes for each action are characterized only approximately, the decisions are made under uncertainty. This is the prime domain for fuzzy decision making.

According to one criterion, decision problems are classified as those involving a single decision maker and those which involve several decision makers. These problem classes with the empty set of ${}^{1}S$. The total orderings ${}^{.75}O$ that are compatible with the pairs in the crisp relation ${}^{.75}S$ are

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Thus

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$$S = \begin{array}{c} \\ w \\ x \\ y \\ z \end{array} \begin{array}{c} \begin{array}{cccc} w & x & y & z \end{array} \\ \begin{bmatrix} 0 & .5 & .75 & .625 \\ .5 & 0 & .75 & .375 \\ .25 & .25 & 0 & .375 \\ .375 & .625 & .625 & 0 \end{bmatrix} \end{array} \qquad (10)$$

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The α-cuts of this fuzzy relation S are

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01/08/20

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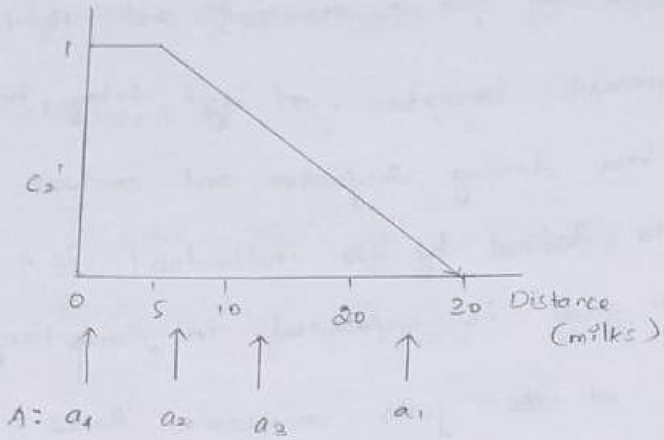

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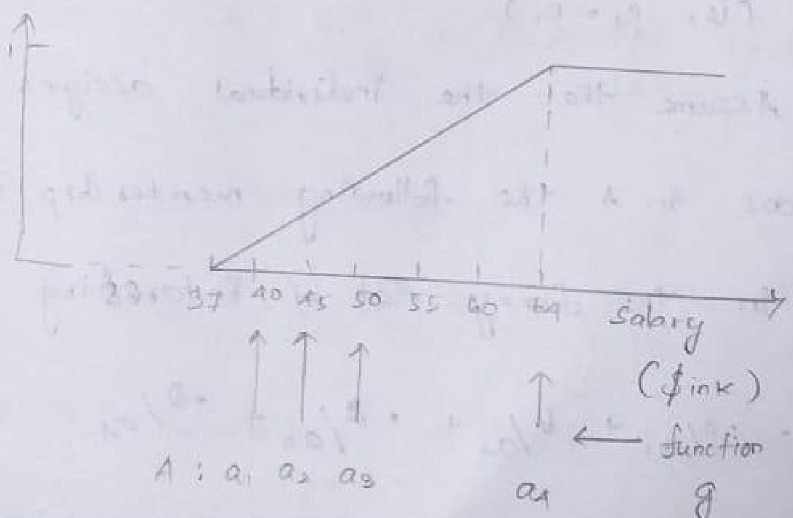

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UNIT - V

27/1/20

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