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BUSINESS TOOLS FOR DECISION MAKING

16CCCCM8

(Problems & Solutions)

B.COM IV SEMESTER

TIME SERIES

SEMI AVERAGE

Odd Number

Problem 1:

Draw a trend line by the method of Semi Averages:

Year	2009	2010	2011	2012	2013	2014	2015
Production (in tonnes)	102	107	115	110	107	120	112

Solution:

Trend line by the method of Semi Averages

Year	Production (in tonnes)	Semi Total	Semi Average
2009	102		
2010	107	324	324/3=108
2011	115		
2012	110	Omitted	
2013	107		
2014	120	339	339/3=113
2015	112		

Even Number

Problem 2:

Fit a trend line by the method of Semi Average

Year	2010	2011	2012	2013	2014	2015
Sales (in tonnes)	120	150	162	220	212	240

Solution:

Trend line by the method of Semi Averages

Year	Sales (in tonnes)	Semi Total	Semi Average
2010	120		
2011	150	432	432/3=144
2012	162		
2013	220		
2014	212	672	672/3=224
2015	240		

METHOD OF LEAST SQUARE

Problem 3:

Below are given the figures of production (in thousand quintals) of a sugar factory

Year	2009	2010	2011	2012	2013	2014	2015
Production (in'000 quintals)	80	90	92	83	94	99	92

a) **Fit a straight line trend by the method of least square.**

b) **Estimate the production for 2016.**

Solution:

Fitting the straight line trend

Year	Production ('000 qtil.)	(X-2012) X	X^2	XY	Trend Values Yc*
2009	80	-3	9	-240	84
2010	90	-2	4	-180	86
2011	92	-1	1	-92	88
2012	83	0	0	0	90
2013	94	1	1	94	92
2014	99	2	4	198	94
2015	92	3	9	276	96
N=7	ΣY=630	ΣX=0	ΣX^2=28	ΣXY=56	

(i). The equation of the straight line trends is

$Yc = a+bx$

$a= \Sigma Y/N$,

$b= \Sigma XY/ \Sigma X^2$

Here, $\Sigma Y= 630$, $N = 7$, $\Sigma XY=56$, $\Sigma X^2=28$.

$a= \Sigma Y/N$, $630/7=90$.

and

$b= \Sigma XY/ \Sigma X^2$, $56/28=2$.

(ii) Hence,the equation of the straight line trends is

Yc=90 + 2Xwhere X denotes deviations from middle year

***Trend value for 2009 = 90 + 2(-3) = 84**

Trend value for 2010 = 90 + 2(-2) = 86

Trend value for 2011 = 90 + 2(-1) = 88

Trend value for 2012 = 90 + 2(0) = 90

Trend value for 2013 = 90 + 2(1) = 92

Trend value for 2014 = 90 + 2(2) = 94

Trend value for 2015 = 90 + 2(3) = 96

Trend value for 2016 = 90 + 2(4) = 98

Problem 4 :

Using the method of least squares , find out the trend values.

Year	2010	2011	2012	2013	2014
Sales (in tonnes)	100	120	140	160	180

Solution:

Fitting the straight line trend

Year	Profit Y	Deviation from Middle year (X-2012) X	X^2	XY	Trend Values Yc*
2010	100	-2	4	-200	100
2011	120	-1	1	-120	120
2012	140	0	0	0	140
2013	160	1	1	160	160
2014	180	2	4	360	180
N=5	ΣY=700	ΣX=0	$\Sigma X^2=10$	ΣXY=200	

(i). The equation of the straight line trends is

Yc = a+bX

a= ΣY/N,

b= ΣXY/ ΣX^2

Here, ΣY= 700, N = 5, ΣXY=200, $\Sigma X^2=10$.

a= ΣY/N , = 700/5=140.

and

b= ΣXY/ $\Sigma X^{2,}$ = 200/10=20.

(ii) Hence,the equation of the straight line trends is

Yc=140 + 20X where X denotes deviations from middle year

***Trend value for 2010 = 140 + 20(-2) = 100**

Trend value for 2011 = 140 + 20(-1) = 120

Trend value for 2012 = 140 + 20(0) = 140

Trend value for 2013 = 140 + 20(1) = 160

Trend value for 2014 = 140 + 20(2) = 180

MOVING AVERAGE METHOD

Problem 5:

Using three yearly moving average determine the trend and short term fluctuations

Year	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
Production (000 tons)	21	22	23	25	24	22	25	26	27	28

Solution:

Calculation of 3 yearly Moving Average

Year	Production (in tones) (y)	3 Yearly Moving Total	3 Yearly Moving Average (yt)	Short term fluctuations (y-yt)
2005	21	-	-	-
2006	22	66 / 3	22.00	0
2007	23	70 / 3	23.33	-0.33
2008	25	72 / 3	24.00	1.00
2009	24	71 / 3	23.67	0.33
2010	22	71 / 3	23.67	-1.67
2011	25	73 / 3	24.33	0.67
2012	26	78 / 3	26.00	0
2013	27	79 / 3	26.33	0.67
2014	28	-	-	-

Problem 6:

Using five yearly moving average and find out trend and short term fluctuations

Year	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
Sales (in tons)	332	317	357	392	402	407	412	427	405	436

Solution:

Calculation of 5 yearly Moving Average

Year	Production (in tones) (y)	5 Yearly Moving Total	5 Yearly Moving Average (yt)	Short term fluctuations (y-yt)
2005	332	-	-	-
2006	317	-	-	-
2007	357	1800 / 5	360.0	-3.00
2008	392	1875 / 5	375.0	17.00
2009	402	1970 / 5	394.0	8.00
2010	407	2040 / 5	408.0	-1.00
2011	412	2053 / 5	410.0	1.40
2012	427	2087 / 5	417.4	9.60
2013	405	-	-	-
2014	436	-	-	-

Problem 7 :

Using four yearly moving average determine the trend and short term fluctuations

Year	2006	2007	2008	2009	2010	2011	2012	2013	2014
Production (000 tons)	464	515	518	467	502	540	557	571	586

Solution:

Calculation of 4 yearly Moving Average

Year	Production (in tones) (y)	4 Yearly Moving Total	4 Yearly Moving Average	4 Yearly Moving Average centered (yt)	Short term fluctuations (y-yt)
2006	464	-	-		
2007	515	-	-		
		1964 / 4	491.00		
2008	518			495.80	22.20
		2002 / 4	500.50		
2009	467			503.60	-36.6
		2027 / 4	506.75		
2010	502			511.60	-9.6
		2066 / 4	516.50		
2011	540			529.50	10.5
		2170 / 4	542.50		
2012	557			553	4.0
		2254 / 4	563.50		
2013	571	-	-		
2014	586	-	-		-

INDEX NUMBERS

SIMPLE AGGREGATIVE METHOD

Problem 8:

Calculate price index number from the data given below and comment on the value

Commodity	A	B	C	D	E	F
Price in 2014	20	25	10	12	30	20
Price in 2015	22	30	10	15	33	30

Solution:
Let prices of 2014 be denoted by p_0 and of 2015 by p_1.

Calculation of Price Index

Commodities	Prices2014 P_0	Prices2015 P_1
A	20	22
B	25	30
C	10	10
D	12	15
E	30	33
F	20	30
Total	$\Sigma p_0=117$	$\Sigma p_1=140$

Price Index = $\Sigma p_1 / \Sigma p_0$ x 100 = 140 / 117 x 100 = 119.66

SIMPLE AVERAGE OF PRICE RELATIVES METHOD

Problem 9:

From the following data, construct an index for 2015 ,taking 2014 as base , by the average of price relatives method , using both arithmetic mean and geometric mean methods:

Commodity	A	B	C	D	E
Price in (Rs) 2014	10	12	18	30	5
Price in (Rs) 2015	12	15	18	45	4

Solution:

Calculation of Price Index

Commodities	Price 2014 P_o	Price 2015 P_1	Price Relative $P= P_1/ P_o x 100$	Log p
A	10	12	120	2.0792
B	12	15	125	2.0969
C	18	18	100	2.0000
D	30	45	150	2.1761
E	5	4	80	1.9031
Total	$\Sigma p_o=117$	$\Sigma p_1=140$		$\Sigma p_1=140$

Arithmetic mean of price relatives = $\dfrac{\Sigma \left[p_1 / p_o \times 100 \right]}{N}$

$= \Sigma P / N \quad = 575 / 5 \quad = 115$

$P_{01} = AL \left[\Sigma \log P / N \right]$

$= AL \left[10.2553 / 5 \right]$

$= AL \ 2.0511$

$= 112.5$

WEIGHTED AGGREGATIVE METHOD

Problem 10 :

Calculate laspeyre's, Paasche's and Fisher's ideal index numbers for the following data

Items	Base Period		Current period	
	Quantity	Price	Quantity	Price
A	12	10	15	12
B	15	07	20	05
C	24	05	20	09
D	05	16	05	14

Solution:

Calculation of Price Index

Items	Base Period		Current Period		$p_1 q_0$	$p_0 q_0$	$p_1 q_1$	$p_0 q_1$
	q_0	P_0	q_0	P_1				
A	12	10	15	12	144	120	180	150
B	15	7	20	5	75	105	140	140
C	24	5	20	9	216	120	100	100
D	5	16	5	14	70	80	80	80
					$\Sigma p_1 q_0 = 505$	$\Sigma p_0 q_0 = 425$	$\Sigma p_1 q_1 = 530$	$\Sigma p_0 q_1 = 470$

Laspeyre's Index : $P_{01} = \Sigma p_1 q_0 / \Sigma p_0 q_0 \times 100$
$= 505 / 425 \times 100$
$= 118.82$

Paasche's Index : $P_{01} = \Sigma p_1 q_1 / \Sigma p_0 q_1 \times 100$
$= 530 / 470 \times 100$
$= 112.77$

Fisher's Index : $P_{01} = \sqrt{L \times P} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} \times 100$

(L = Laspeyre's Index ; P = Paasche's Index)

$$= \sqrt{\frac{505}{425} \times \frac{530}{470}} \times 100$$

$$= \sqrt{118.82} \times \sqrt{112.77}$$
$$= 10.900 \times 10.619$$
$$= 115.76$$

Problem 11 :

Construct index numbers from the following data by using :

a) Laspeyre's Method

b) Paasche's Method

c) Bowley's Method

d) Fisher's Ideal Method

e) Marshall Edgeworth Method

Items	2014		2015	
	Price	Quantity	Price	Quantity
A	5	2	6	2
B	6	4	8	5
C	3	8	4	8
D	10	5	9	12

Solution:

Calculation of Price Index

Items	2014		2015		$p_1 q_0$	$p_0 q_0$	$p_1 q_1$	$p_0 q_1$
	p_0	q_0	P_1	q_1				
A	5	2	6	2	12	10	12	10
B	6	4	8	5	32	24	40	30
C	3	8	4	8	32	24	32	24
D	10	5	9	12	45	50	108	120
					$\Sigma p_1 q_0 = 121$	$\Sigma p_0 q_0 = 108$	$\Sigma p_1 q_1 = 192$	$\Sigma p_0 q_1 = 184$

Laspeyre's Index : $P_{01} = \Sigma p_1 q_0 / \Sigma p_0 q_0 \times 100$
$= 121 / 108 \times 100$
$= 112.04$

Paasche's Index : $P_{01} = \Sigma p_1 q_1 / \Sigma p_0 q_1 \times 100$
$= 192 / 184 \times 100$
$= 104.35$

Bowley's Index : $P_{01} = L + P / 2$
(L = Laspeyre's Index ; P = Paasche's Index)
$= 112.04 + 104.35 / 2$
$= 108.2$

Fisher's Index : $P_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} \times 100$
$= \sqrt{\frac{121}{108} \times \frac{192}{184}} \times 100$
$= \sqrt{1.120} \times \sqrt{1.043}$
$= 1.058 \times 1.021 \times 100$
$= 1.082 \times 100$
$= 108.02$

Marshall - Edgeworth Index:

$$P_{01} = \frac{\Sigma p_1 q_0 + \Sigma p_1 q_1}{\Sigma p_0 q_0 + \Sigma p_0 q_1} \times 100$$
$$= \frac{121 + 192}{108 + 184} \times 100$$
$$= \frac{313}{292} \times 100$$
$$= 107.19$$

Problem 12:

Calculate by suitable method the index number of quantity from the following

Items	2014		2015	
	Total Value	Price	Total Value	Price
A	80	8	110	10
B	90	10	108	12
C	256	16	340	20

Solution:

Lets us calculate Fisher's Quantity Index Number to the given data:

Quantity for 2014

$$A\ q_0 = \frac{\text{Total Value}}{\text{Price}}, \frac{80}{8} = 10,$$

$$B\ q_0 = \frac{\text{Total Value}}{\text{Price}}, \frac{90}{10} = 9,$$

$$C\ q_0 = \frac{\text{Total Value}}{\text{Price}}, \frac{256}{16} = 16,$$

Quantity for 2015

$$A\ q_1 = \frac{\text{Total Value}}{\text{Price}}, \frac{110}{10} = 11,$$

$$B\ q_1 = \frac{\text{Total Value}}{\text{Price}}, \frac{108}{12} = 9,$$

$$C\ q_1 = \frac{\text{Total Value}}{\text{Price}}, \frac{340}{20} = 17,$$

Commodity	p_0	q_0	P_1	q_1	$p_1 q_0$	$p_0 q_0$	$p_1 q_1$	$p_0 q_1$
A	8	10	10	11	100	80	110	88
B	10	9	12	9	108	90	108	90
C	16	16	20	17	320	256	340	272
					$\Sigma p_1 q_0 = 528$	$\Sigma p_0 q_0 = 426$	$\Sigma p_1 q_1 = 558$	$\Sigma p_0 q_1 = 450$

Fisher's Quantity Index : $= \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} \times 100$

$= \sqrt{\frac{450}{426} \times \frac{558}{528}} \times 100$

$= \sqrt{1.056} \times \sqrt{1.056} \times 100$

$= 1.027 \times 1.027 \times 100$

$= 1.054 \times 100$

$= 105.66$

Problem 13:

Calculate Fisher's ideal index and prove that it satisfies both the time reversal and factor reversal tests.

Items	2014		2015	
	Quantity	Price	Quantity	Price
A	8	4	8	5
B	10	5	12	6
C	6	3	7	4
D	5	8	4	10

Solution:

Calculation of Price Index

Commodity	p_0	q_0	P_1	q_1	$p_1 q_0$	$p_0 q_0$	$p_1 q_1$	$p_0 q_1$
A	4	8	5	8	40	32	40	32
B	5	10	6	12	60	50	72	60
C	3	6	4	7	24	18	28	21
D	8	5	10	4	50	40	40	32
					$\Sigma p_1 q_0 = 174$	$\Sigma p_0 q_0 = 140$	$\Sigma p_1 q_1 = 180$	$\Sigma p_0 q_1 = 145$

Fisher's Ideal Index : p_{01} $= \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} \times 100$

$$= \sqrt{\frac{174}{140} \times \frac{180}{145}} \times 100$$
$$= \sqrt{1.242} \times \sqrt{1.241} \times 100$$
$$= 1.114 \times 1.114 \times 100$$
$$= 1.241 \times 100$$
$$= 124.14$$

Time Reversal Test :

Time Reversal Test is satisfied if $p_{01} \times p_{10} = 1$

$$p_{01} \times p_{10} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times \frac{\Sigma p_0 q_1}{\Sigma p_1 q_1} \times \frac{\Sigma p_0 q_0}{\Sigma p_1 q_0}}$$

$$= \sqrt{\frac{174}{140} \times \frac{180}{145} \times \frac{145}{180} \times \frac{140}{174}}$$
$$= \sqrt{1}$$
$$= 1$$

Hence, Time Reversal Test is satisfied.

Factor Reversal Test :

Factor Reversal Test is satisfied when:

$$P_{01} \times Q_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$$

$$Q_{01} = \sqrt{\frac{\Sigma q_1 p_0}{\Sigma q_0 p_0} \times \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1}}$$

$$P_{01} \times Q_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times \frac{\Sigma q_0 p_1}{\Sigma q_0 p_0} \times \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1}}$$

$$= \sqrt{\frac{174}{140} \times \frac{180}{145} \times \frac{145}{140} \times \frac{180}{174}}$$
$$= \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$$
$$= 180 / 140$$
$$= 1.285$$

Hence, Factor Reversal Test is satisfied.

Problem 14:

Using the following data calculate Fisher's ideal index and show that it satisfies both the time reversal and factor reversal tests.

Items	Price Per Unit		No . of Unit	
	Base Year	Current Year	Base Year	Current Year
P	6	10	50	56
Q	2	2	100	120
R	4	6	60	60
S	10	12	30	24
T	8	12	40	36

Solution:

Calculation of Price Index

Commodity	p_0	q_0	P_1	q_1	$p_1 q_0$	$p_0 q_0$	$p_1 q_1$	$p_0 q_1$
P	6	50	10	56	500	300	560	336
Q	2	100	2	120	200	200	240	240
R	4	60	6	60	360	240	360	240
S	10	30	12	24	360	300	288	240
T	8	40	12	36	480	320	432	288
					$\Sigma p_1 q_0 = 1900$	$\Sigma p_0 q_0 = 1360$	$\Sigma p_1 q_1 = 1880$	$\Sigma p_0 q_1 = 1344$

Fisher's Ideal Index : p_{01}

$$= \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} \times 100$$

$$= \sqrt{\frac{1900}{1360} \times \frac{1880}{1344}} \times 100$$

$$= \sqrt{1.397} \times \sqrt{1.398} \times 100$$

$$= \sqrt{1.953} \times 100$$

$$= 1.398 \times 100$$

$$= 139.8$$

Time Reversal Test :

Time Reversal Test is satisfied if $p_{01} \times p_{10} = 1$

$$p_{01} \times p_{10} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times \frac{\Sigma p_0 q_1}{\Sigma p_1 q_1} \times \frac{\Sigma p_0 q_0}{\Sigma p_1 q_0}}$$

$$= \sqrt{\frac{1900}{1360} \times \frac{1880}{1344} \times \frac{1344}{1880} \times \frac{1360}{1900}}$$

$$= \sqrt{1}$$

$$= 1$$

Hence, Time Reversal Test is satisfied.

Factor Reversal Test :

Factor Reversal Test is satisfied when:

$$P_{01} \times Q_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$$

$$Q_{01} = \sqrt{\frac{\Sigma q_1 p_0}{\Sigma q_0 p_0} \times \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1}}$$

$$P_{01} \times Q_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times \frac{\Sigma q_0 p_1}{\Sigma q_0 p_0} \times \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1}}$$

$$= \sqrt{\frac{1900}{1360} \times \frac{1880}{1344} \times \frac{1344}{1360} \times \frac{1880}{1900}}$$

$$= \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$$

$$= 1880 / 1360$$

$$= 1.382$$

Hence, Factor Reversal Test is satisfied.

CHAIN BASE INDEX NUMBER

Problem 15 :

Construct chain base index number from the link relatives given below:

Year	2010	2011	2012	2013	2014
Link Index	100	105	95	115	102

Solution:

Apply the formula for Chain base Index

Link relative of the Current year X Previous Year Chain Index / 100

Construct of Chain Indices

Year	Link Relatives	Chain Indices (2010 = 100)
2010	100	100
2011	105	105 x 100 / 100 = 105
2012	95	95 x 105 / 100 = 99.75
2013	115	115 x 99.75 / 100 = 114.71
2014	102	102 x 114.71 / 100 = 117.00

COST OF LIVING INDEX NUMBER

Problem 16:

Calculate the cost of living index number from the data:

Items	Base Year Price	Current Year Price	Weights
Food	30	47	4
Fuel	8	12	2
Clothes	14	18	3
Rent	22	15	2
Miscellaneous	25	30	1

Solution:

Calculation of Cost of Living Index

Items	P_0	P_1	$P = P_1/P_0 \times 100$	W	PW
Food	30	47	47/30 x 100 =156.67	4	626.68
Fuel	8	12	12/8 x 100 = 150.00	2	300.00
Clothes	14	18	18/14 x 100 = 128.57	3	385.71
Rent	22	15	15/22 x 100 = 68.18	2	136.71
Miscellaneous	25	30	30/ 25 x 100 =120.00	1	120.00
			623.42	12	1568.75

Cost of Living Index number = $\Sigma PW / \Sigma W$
= 1,568.75 / 12
= 130.73

Problem 17:

In calculating a certain cost of living index number ,the following weight were used : Food 15 ,Clothing 3, Rent 4 , Fuel and Light 2 , Miscellaneous 1. Calculate the index for the data when the percentage increase in prices of items in the various groups over the base peroid were 32,54,47,78 and 58 respectively.

Solution:

Calculation of Cost of Living Index

Items	Percentage Increase in Price	Current Index* I	Weight W	WI
Food	32	132	15	1980
Fuel	54	154	3	462
Clothes	46	147	4	588
Rent	78	178	2	356
Miscellaneous	58	158	1	158
			ΣW= 25	ΣWI = 3544

Cost of Living Index number = ΣWI / ΣW

= 3,544 / 25

= 141.76

Problem 18:

Calculate the cost of living index number using Family budget method:

Commodity	A	B	C	D	E	F	G	H
Quantity in Base year(Units)	200	50	50	20	40	50	60	40
Price in Base year(Rs)	10	30	40	200	25	100	20	150
Price in Current year(Rs)	12	35	50	300	50	150	25	180

Solution:

Calculation of Cost of Living Index

Commodity	q_0	p_0	p_1	$P = P1/P0 \times 100$	W ($p_0 q_0$)	PW
A	200	10	12	120.00	2000	240000
B	50	30	35	116.67	1500	175005
C	50	40	50	125.00	2000	250000
D	20	200	300	150.00	4000	600000
E	40	25	50	200.00	1000	200000
F	50	100	150	150.00	5000	750000
G	60	20	25	125.00	1200	150000
H	40	150	180	120.00	6000	720000
					ΣW= 22700	ΣPW= 3085005

Cost of Living Index number = ΣPW / ΣW

= 3085005 / 22700

= 135.90