Name:

PAVENDAR BHARATHIDASAN COLLAGE OF ARTS & SCIENCE.

DEPT : MATHEMATICAL

SUBJECT : DYNAMICS

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SUBCODE : 16SCCMM14

Asst prof in mathematics

UNIT - I 2 MARKS

1. uniform speed:

The speed of a particle is said to be uniform when its describes equal length of its path in equal interval of time

2. Average speed:

The average speed of a particle in any time interval is got by dividing the distance travelled in that time interval by the time interval

3. displacement:

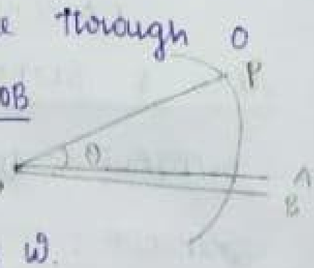
The displacement of a moving point in any interval of time is its change of position it is a vector quantity so it has both magnitude & direction.

4. velocity:

The velocity of moving point is the rate of its displacement the velocity has both magnitude and direction it is vector quantity.

Angular velocity:

If a particle P be moving along any path in a plane and if O be a fixed point in a plane OA is a fixed straight line through O the rate at which the angle $\angle AOB$ increase is called the angular velocity of P about O. its denoted by ω.



6. Change of velocity :

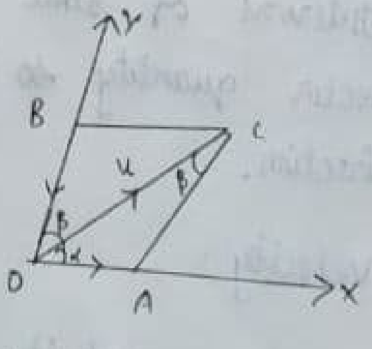
Since a velocity has both magnitude and direction. it will be changed if one of these changes or both changes.

7. Acceleration :

The acceleration of a moving point is the rate of change of its velocity. It is a vector quantity.

5 marks.

8. Components of a velocity along two given direction :



Let OC represent the given velocity u and OX, OY be two lines making angles α and β with OC.

draw CA parallel to OY. CB parallel to OX making the parallegram OACB.

Then OA and OB are the parallegram component of the velocity OC. along OX and OY respectively. from ΔOAC

$$\frac{OA}{\sin \angle OCA} = \frac{AC}{\sin \angle AOC} = \frac{OC}{\sin \angle OAC}$$

$$\frac{OA}{\sin\beta} = \frac{AC}{\sin\alpha} = \frac{OC}{\sin(180^\circ-(\alpha+\beta))}$$

$$\frac{OA}{\sin\beta} = \frac{AC}{\sin\alpha} = \frac{OC}{\sin(\alpha+\beta)}$$

equating the term we have

$$\frac{OA}{\sin\beta} = \frac{OC}{\sin(\alpha+\beta)}$$

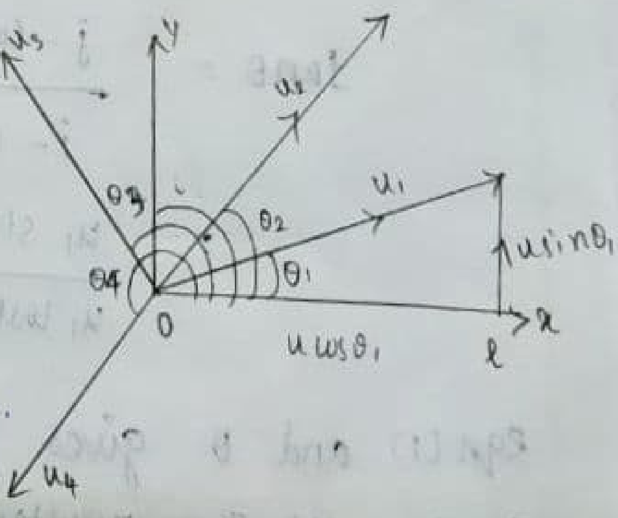
$$OA = \frac{OC.\sin\beta}{\sin(\alpha+\beta)} = \frac{u.\sin\beta}{\sin(\alpha+\beta)}$$

$$\frac{AC}{\sin\alpha} = \frac{OC}{\sin(\alpha+\beta)}$$

$$AC = \frac{u.\sin\alpha}{\sin(\alpha+\beta)}$$

$$AC = OB = \frac{u\sin\alpha}{\sin(\alpha+\beta)}.$$

9. **Resultant of several simultaneous coplaner velocity of a particle.**



Let a point O have several simultanous velocities represented by vectors u_1, u_2, u_3 .... etc. In direction inclient an angle $\theta_1, \theta_2, \theta_3$ ... etc. to a fixed line OX. and OY be perpendicular to OX.

Let i and j be a unit vectors along ox and oy.

$$\vec{OA_1} = u_1$$

from A_1, draw $A_1 L \perp$ to ox

$$u_1 = \vec{OA_1} = \vec{OL} + \vec{LA_1}$$

$$= u_1 \cos\theta_1 \vec{i} + u_1 \sin\theta_1 \vec{j}$$

ly $\quad u_2 = u_2 \cos\theta_2 \vec{i} + u_2 \sin\theta_2 \vec{j}$ and so on.

Let v be the vector representing the resultant velocity.

$$V = u_1 + u_2 + u_3 + \cdots$$

$$V = (u_1 \cos\theta_1 i + u_1 \sin\theta_1 j) + (u_2 \cos\theta_2 i + u_2 \sin\theta_2 j) + \cdots$$

$$= i(u_1 \cos\theta_1 + u_2 \cos\theta_2 + \cdots) + j(u_1 \sin\theta_1 + u_2 \sin\theta_2 + \cdots)$$

magnitude of the resultant velocity.

$$v = \sqrt{(u_1 \cos\theta_1 + u_2 \cos\theta_2 + \cdots)^2 + (u_1 \sin\theta_1 + u_2 \sin\theta_2 + \cdots)^2} \quad (1)$$

If the vector V. makes an angle θ with ox

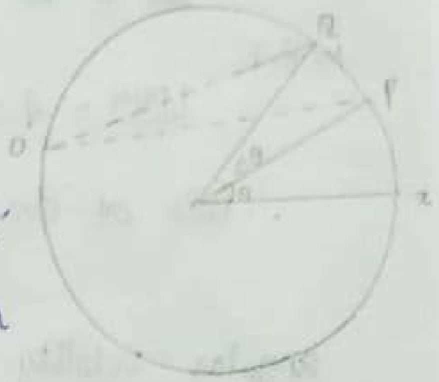
$$\tan\theta = \frac{j\text{- component of } v}{i\text{- component of } v}$$

$$= \frac{u_1 \sin\theta_1 + u_2 \sin\theta_2 + \cdots}{u_1 \cos\theta_1 + u_2 \cos\theta_2 + \cdots}$$

eqn (1) and θ gives the magnitude & direction of the resultant.

10. Angular velocity of a particle moving along a circle with uniform speed

Let a point move with uniform speed v along a circle centre O and radius r. Let P be its position at time 't' & 's' be the arc AP. measure from a fixed point A on the circle.



OA is a fixed direction and let triangle AOB = θ at time 't + Δt' seconds. Let the point B at Q.

Such that "APOQ = ΔQ" and "PQ = ΔS" then we know that

$$\Delta S = r.\Delta\theta$$

$$\frac{\Delta s}{\Delta t} = r.\frac{\Delta\theta}{\Delta t}$$

Taking limits

$$\lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = r.\lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

$$\frac{ds}{dt} = r.\frac{d\theta}{dt} \rightarrow (1)$$

now,

$\frac{ds}{dt}$ is the rate of which the length of the path is described and so it is the linear velocity v of the particle.

$\frac{d\theta}{dt}$ is the angular velocity ω.

so eqn (1) becomes $v = r\omega$

Corollary:

Let O be any point on the circumference

W.K.T

$\angle POQ = 2 \angle PO'Q$

$\therefore$ Rate of change of $\angle POQ = 2 \times$ rate of change of $\angle PO'Q$

angular velocity about the centre O.

$= 2 \times$ angular velocity about O'.

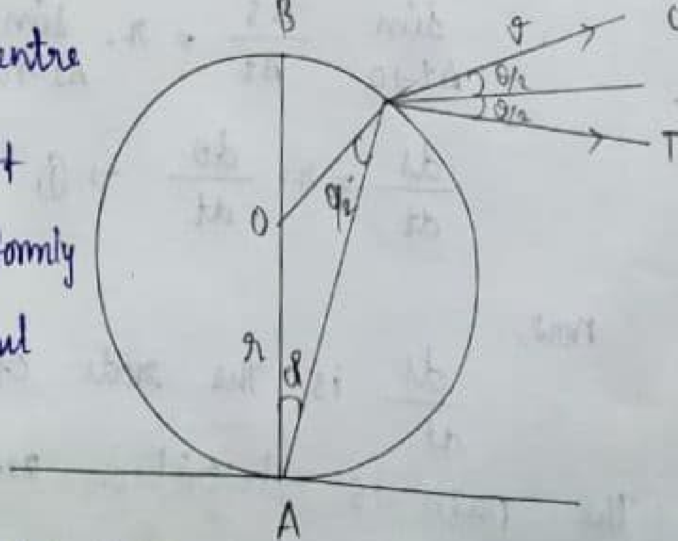
$\therefore$ angular velocity about $O' = \frac{1}{2} \times$ angular velocity about O.

$= \frac{1}{2}\omega$

$= \frac{1}{2} r$.

11. Angular velocity of any point on a wheel rolling uniformly: 10 marks

Let O be the centre and r the radius of the wheel rolling uniformly on the ground without sliding.



Let A be its point of contact with the ground at a certain instand B the highest point and v the velocity.

The wheel turns uniformly about its centre while the centre moves forward uniformly in a straight line since each point of the wheel in succession touch the ground it is clear that any point of the wheel discribes the perimeter of the wheel relative to the centre. while the centre moves forward through a distance equal to the permeter in the same time. hence the velocity of any point on a wheel relative to the centre is equal in magnitude to the velocity B of the centre.

Let P be any point of the wheel such that $\angle BOP = \theta$. This point has two velocities each equal to v. one its velocity relative to O. which is along PC. As PC and PT are respectively perpendicular to OB. and OP.

$$\angle TPC = \angle BOP = \theta$$

The resultant of this two equal velocities v is $2v\cos\theta/2$ in the direction PD bisecting $\angle TPC$ joint AP.

$$\angle OAP = \angle OPA = \theta/2$$

$$\angle APT = \angle OPT - \angle OPA$$

$$= 90^\circ - \theta/2$$

$$\angle APD = \angle APT + \angle TPD$$

$$a'^2 = R^2 + a^2 - 2Ra\cos\alpha$$

hence

$$a\cos\alpha = \frac{R^2 + a^2 - a'^2}{2R}$$

$$a'\cos\alpha' = \frac{R^2 - a^2 + a'^2}{2R}$$

angular velocity of AB :

$$\frac{(R^2 + a^2 - a'^2)\omega + (R^2 - a^2 + a'^2)\omega'}{2R^2}$$

13. motion in a straight line under uniform acceleration.

A particle moves along a straight line starting with velocity u and having a constant acceleration f in its direction of motion. if v is its velocity after time t and s is the distance described by it during that time, then

$$v = u + ft \rightarrow (1)$$

$$s = ut + \tfrac{1}{2} ft^2 \rightarrow (2)$$

$$v^2 = u^2 + 2fs \rightarrow (3)$$

These equations are established as follows:

we know that $\frac{ds}{dt}$ and $\frac{d^2 s}{dt^2}$ are respectively the formula for velocity and acceleration at time t.

$$\therefore \frac{v^2}{2} = fs + \frac{u^2}{2}$$

$$v^2 = u^2 + 2fs \rightarrow (3)$$

the eqn (1), (2), & (3)

Corollary:

when the particle starts from rest we have $u = 0$ and the above equations then become

$$v = ft \quad ; \quad s = \tfrac{1}{2} ft^2 \quad \& \quad v^2 = 2fs.$$

UNIT- II

2 Marks:

1. projectiles:

we shall consider motion of a particle projected into the air in any direction with any velocity such a particle is called a projectiles.

2. Angle of projection:

The angle of projection is the angle that the direction in which the particle is initially projected makes with the horizontal plane through the point of projection.

$$\therefore \frac{d^2s}{dt^2} = f$$

Integrating $\frac{ds}{dt} = ft + A$

as f is a constant $\rightarrow$ (i)

When $t = 0$; $\frac{ds}{dt} = u$

using this condition in (i) we have

$$\therefore \frac{ds}{dt} = ft + u \rightarrow \text{(ii)}$$

$$v = u + ft \rightarrow ①$$

Integrating (ii) further

$$s = \frac{ft^2}{2} + ut + B \rightarrow \text{(iii)}$$

when $t = 0$, $s = 0$ and applying this condition in (iii).

we have $B = 0$

$$\therefore s = ut + \frac{ft^2}{2} \rightarrow ②$$

acceleration is also given by the formula $v.\frac{dv}{ds}$.

$$\therefore v\frac{dv}{ds} = f$$

$$v.dv = f.ds$$

Integrating $\frac{v^2}{2} = fs + c \rightarrow \text{(iv)}$

when $s = 0$, $v = u$ and using this (iv)

$$c = \frac{u^2}{2}$$

3. Velocity of projection:

The velocity of projection is the velocity with the particle is projected. The trajectory is the path which the particle describes.

4. Range on a plane:

The range on a plane through the point of projection is the distance between the point of projection and the point where the trajectory meets that plane.

5. The time of Flight:

The time of flight is the interval of time that elapses from the instant of projection till the instant when particle again meets the horizontal plane through the point of projection.

6. Two Fundamental principles:

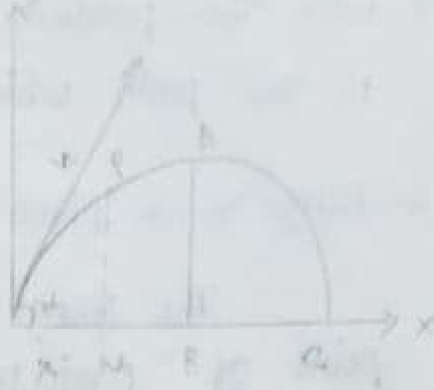
The horizontal velocity remains constant throughout the motion as there is no force to cause any acceleration in that direction.

The vertical component acceleration of the velocity will be subject to a retardation g.

These two main principles will help us to study the motion of a projectile.

5 marks:

7. To show that the path of a projectile is a parabola:



Let a particle be projected from O, with a velocity u at an angle α to the horizontal and the upward vertical through O, as axes of x and y respectively the initial velocity u can be split into two components which are,

$u\cos\alpha$ in the horizontal direction and $u\sin\alpha$ in the vertical direction.

The horizontal component $u\cos\alpha$ is constant throughout the motion as there is no horizontal acceleration.

The vertical component $u\sin\alpha$ is subject to an acceleration g downwards.

Let $P(x, y)$ be the position of the particle at time t secs. after projection.

x = horizontal distance described in t secs

$$= (u\cos\alpha) \cdot t \rightarrow (1)$$

Y = vertical distance described in t secs.

$$= (u\sin\alpha)t - \tfrac{1}{2} g t^2 \rightarrow (2)$$

① & ② can be taken as the parametric eqns of the trajectory the eqn to the path is got by eliminating t, between them from (1)

$t = x/u\cos\alpha$ and putting this in (2).

we get

$$y = u\sin\alpha \cdot \frac{x}{u\cos\alpha} - \frac{1}{2} g \left(\frac{x}{u\cos\alpha}\right)^2$$

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha} \rightarrow ③$$

multiplying (3) by $2u^2\cos^2\alpha$

$$2u^2\cos^2\alpha \cdot y = 2u^2\cos^2\alpha \cdot x \frac{\sin\alpha}{\cos\alpha} - gx^2$$

(ie)

$$x^2 - \frac{2u^2\sin\alpha\cos\alpha}{g} x = -\frac{2u^2\cos^2\alpha}{g} y$$

(or)

$$\left(x - \frac{u^2\sin\alpha\cos\alpha}{g}\right)^2 = \frac{u^4\sin^2\alpha\cos^2\alpha}{g^2} - \frac{2u^2\cos^2\alpha}{g} y$$

$$= -\frac{2u^2\cos^2\alpha}{g}\left(y - \frac{u^2\sin^2\alpha}{2g}\right)$$

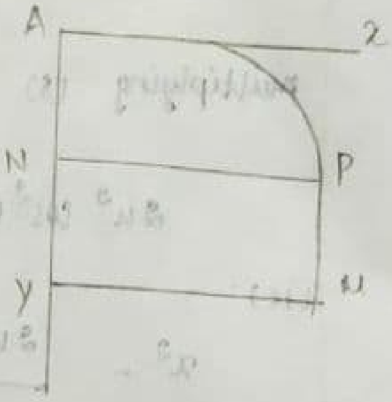
$$\left(\frac{u^2\sin\alpha\cos\alpha}{g}, \frac{u^2\sin^2\alpha}{2g}\right)$$

$$x^2 = \frac{-2u^2\cos^2\alpha}{g} \cdot y \rightarrow ④$$

(4) is clearly the eqn to a parabola of latus reaction $\frac{2u^2\cos^2\alpha}{g}$, whose axis is vertical and downwards and whose vertex is the point

$$\left(\frac{u^2\sin\alpha\cos\alpha}{g}, \frac{u^2\sin^2\alpha}{2g}\right)$$ //

8. A particle is projected horizontaly from a point at a certain height above the ground to S.T the path discribed by it is a parabola.



Let a particle be projected horizontally with a velocity u from a point A at a height h above the ground level.

Let it strike the ground at M. take A as origin the horizontal through A as x axis and the downward vertical throughout the motion

x = horizontal distance described in time $t = ut \to (1)$

y = vertical distance described in time $t = \frac{1}{2}gt^2 \to (2)$

eliminate t between (1) & (2)

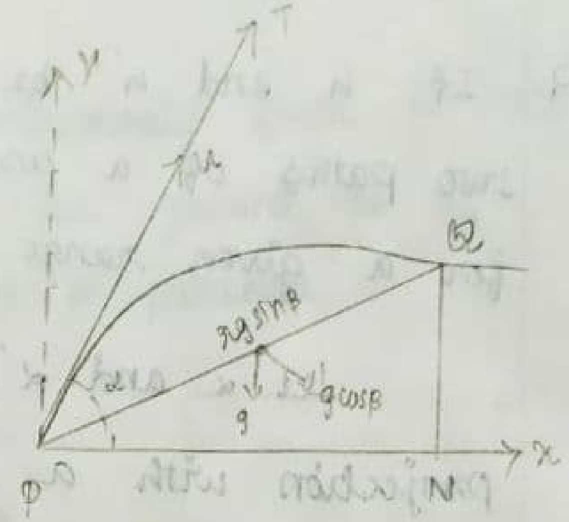
$$y = \frac{1}{2}g \cdot \frac{x^2}{u^2}$$

$$= \frac{u^2 \sin\alpha \cos\alpha}{2g} = R/4$$

using (2)

$R = 4\sqrt{hh'}$ //

10. Range on an incliened plane:



Let P be the point of projection and the particle strike the incliened plane at Q. The PQ is the range on the inclined plane.

Let $PQ = r$. Taking P as the origin and the horizontal and the vertical through P as the axes of x and y respectively.

$$Y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha} \quad \text{(1)}$$

drawn $QN \perp$ to the horizontal plane through P.

The co-ordinates of Q are $(r\cos\beta, r\sin\beta)$

$$r\sin\beta = r\cos\beta \cdot \tan\alpha - \frac{gr^2\cos^2\beta}{2u^2\cos^2\alpha}$$

multiply by $2u^2\cos^2\alpha$ and cancelling r throughout, we have,

$$2u^2\cos^2\alpha \sin\beta\, r = 2u^2\cos\beta\, r \sin\alpha\cos\alpha - gr\cos^2\beta$$

$$x^2 = \frac{2u^2}{g} \cdot y \rightarrow (3)$$

(3) S.T y is a quadratic funt of x so it represents a parabola with vertex at A and axis AN.

9. If h and h' be the greatest heights in the two paths of a projectile with a given velocity for a given range R, P.T $R = 4\sqrt{(hh')}$

Let α and α' be the two angles of projection with a given velocity u to get a given rang R.

Then W.K.T $\alpha + \alpha' = 90°$

$$\alpha' = 90° - \alpha \rightarrow (1)$$

Also,

$$R = \frac{2u^2 \sin\alpha \cos\alpha}{g} \rightarrow (2)$$

$$h = \frac{u^2 \sin^2\alpha}{2g} \rightarrow (3)$$

$$h' = \frac{u^2 \sin^2\alpha'}{2g} \rightarrow (4)$$

$$\therefore hh' = \frac{u^4 \sin^2\alpha \cdot \sin^2\alpha'}{2g}$$

$$= \frac{u^2 \sin\alpha \sin(90° - \alpha)}{2g}$$

$$\therefore x = \frac{2u^2 \cos\alpha \sin\alpha \cos\alpha - 2u^2 \cos^2\alpha \sin\beta}{g\cos^3\beta}$$

$$= \frac{2u^2 \cos\alpha (\sin\alpha \cos\beta) - \cos\alpha \sin\beta}{g\cos^2\beta}$$

$$x = \frac{2u^2 \cos\alpha \sin(\alpha-\beta)}{g\cos^2\beta}$$

10 marks:

11. Characteristics of the motion of a projectile:

(i) Greatest height attained by a projectile:

At A, the highest point, the particle will be moving only horizontally, having lost all its vertical velocity.

Let $AB = h$ => The greatest height reached considering vertical motion separately initial upward vertical velocity $= u\sin\alpha$ & the acceleration in this direction is $-g$

$A = 0$.

$$0 = (u\sin\alpha)^2 - 2g.h$$

$$h = \frac{u^2\sin^2\alpha}{2g}$$

The vertical of the parabola is the highest point of the path.

(ii) Time taken to reach the greatest height:

Let T be the time from O to A. then in time T, the initial vertical velocity $u \sin\alpha$ is reduced to zero acted on by an acceleration $-g$

$$0 = u\sin\alpha - gT$$

$$T = \frac{u\sin\alpha}{g}.$$

(iii) Time of flight the time taken to return to the same horizontal level as O.

When the particle arrives at O. The effective vertical distance it has described is zero.

Hence t is the time of flight considering vertical motion.

$$0 = u\sin\alpha\, t - \tfrac{1}{2} g t^2$$

$t = 0$ (or) $t = \dfrac{2u\sin\alpha}{g}$.

The time of flight $= \dfrac{2u\sin\alpha}{g}$ //.

(iv) The range on the horizontal plane through the point of projection

The time of flight $t = \dfrac{2u\sin\alpha}{g}$

OC = horizontal distance

$= u\cos\alpha . t = u\cos\alpha \cdot \frac{2u\sin\alpha}{g}$

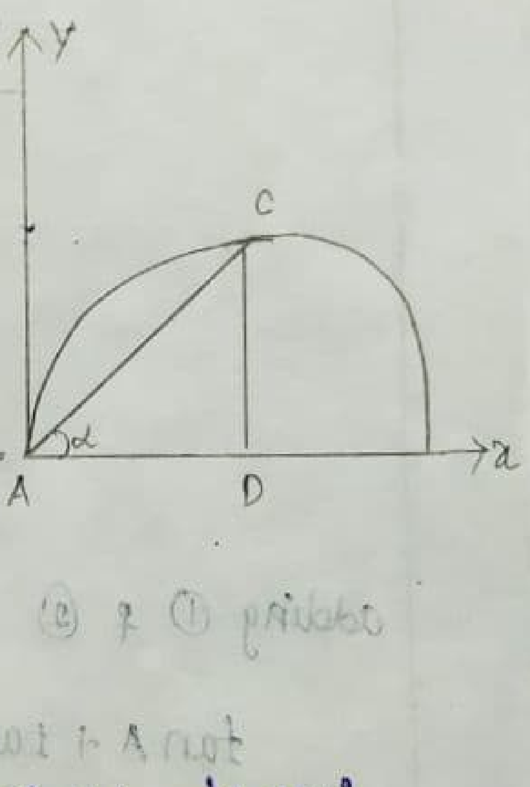
$= \frac{2u^2 \sin\alpha \cos\alpha}{g}$

$R = \frac{2u^2 \sin\alpha \cos\alpha}{g} = \frac{u^2 \sin 2\alpha}{g}$ //

12. $\tan\alpha = \tan A + \tan B$

Let u be the velocity α angle of projection. Let t secs, be the time from A to C

draw $CD \perp AB$ and let $CD = h$



$h = u\sin\alpha . t - \frac{1}{2} g t^2$

$AD = u\cos\alpha . t$

from $\triangle CAD$

$\tan A = \frac{CD}{AD} = \frac{h}{AD} = \frac{u\sin\alpha\, t - \frac{1}{2} g t^2}{u\cos\alpha . t}$

$= \tan\alpha - \frac{gt}{2u\cos\alpha} \rightarrow (1)$

AB = horizontal range

$= \frac{2u^2 \sin\alpha \cos\alpha}{g}$

$$DB = AB - AD = \frac{2u^2 \sin\alpha \cos\alpha}{g} - u\cos\alpha . t$$

from Δ CDB

$$\tan B = \frac{CD}{DB} = \frac{h}{\left(\frac{2u^2 \sin\alpha \cos\alpha}{g} - u\cos\alpha . t\right)}$$

$$= \frac{u \sin\alpha . t - \frac{1}{2} g t^2}{\left(\frac{2u^2 \sin\alpha \cos\alpha}{g} - u\cos\alpha . t\right)}$$

$$= \frac{gt (u\sin\alpha - \frac{1}{2} gt)}{u\cos\alpha (2u\sin\alpha - gt)}$$

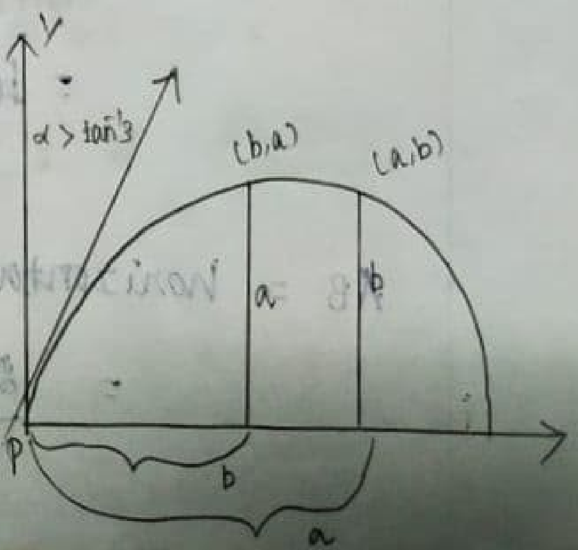
$$= \frac{gt . (2u \sin\alpha - gt)}{2u\cos\alpha (2u\sin\alpha - gt)}$$

$$= \frac{gt}{2u\cos\alpha} \rightarrow (2)$$

adding (1) & (2)

$$\tan A + \tan B = \tan\alpha$$ //.

13. $\tan^{-1} 3$



To show that the angle of projection exceeds $\tan^{-1} 3$.

To prove $\alpha > \tan^{-1} 3$.

$$\frac{a-bt}{b-at} = \frac{-gb^2/2u^2(1+t^2)}{-ga^2/2u^2(1+t^2)} = \frac{b^2}{a^2}$$

$$a^2(a-bt) = b^2(b-at)$$

$$a^3 - a^2bt = b^3 - b^2at$$

$$b^2at - a^2bt = b^3 - a^3$$

$$abt(b-a) = b^3 - a^3$$

$$t = \frac{b^3-a^3}{ab(b-a)} = \frac{(b-a)(b^2+ba+a^2)}{ab(b-a)}$$

$$t = \frac{b^2+ba+a^2}{ab} \rightarrow (5)$$

(4) & (5) $\pm 3ab$

$$t = \frac{b^2-3ab+a^2+ab+3ab}{ab}$$

$$= \frac{(a-b)^2}{ab} + \frac{3ab}{ab} = \frac{(a-b)^2}{ab} + 3$$

$t = +ve + 3$

$t > 3$

$\tan\alpha > 3$

$\alpha = \tan^{-1} 3$

(ii) To find range on the horizontal plane. To find R

$$\text{horizontal range} = \frac{2u^2 \sin\alpha \cos\alpha}{g}$$

$$= \frac{u^2 \sin 2\alpha}{g}$$

$$= \frac{u^2\, 2\tan\alpha}{g(1+\tan^2\alpha)} \qquad \left[\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}\right]$$

$\tan\alpha = t$

$$R = \frac{2u^2\, t}{g(1+t^2)} \rightarrow (6)$$

from (3)

$$a-bt = \frac{-gb^2}{2u^2}(1+t^2)$$

$$\frac{g(1+t^2)}{2u^2} = \frac{a-bt}{-b^2} = \frac{bt-a}{b^2}$$

$$= \frac{\frac{b(a^2+ab+b^2)}{ab} - a}{b^2} = \frac{a^2+ab+b^2-a^2}{ab^2}$$

$$= \frac{ab+b^2}{ab^2} = \frac{b(a+b)}{ab^2}$$

$$\frac{g(1+t^2)}{2u^2} = \frac{a+b}{ab}$$

$\frac{g(1+t^2)}{2u^2}$ in (6)

$$R = \frac{ab}{a+b} \cdot t$$

$$R = \frac{ab}{a+b}\left(\frac{a^2+ab+b^2}{ab}\right) = \frac{a^2+ab+b^2}{a+b}$$

u - initial velocity ; α - angle of projection

(b, a) } → points on the trajectory
(a, b)

The eqn of the path is

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha} \rightarrow (1)$$

put $\tan\alpha = t$

eqn (1)
$$y = xt - \frac{gx^2}{2u^2}(\sec^2\alpha)$$

$$y = xt - \frac{gx^2}{2u^2}(1+\tan^2\alpha)$$

$$y = xt - \frac{gx^2}{2u^2}(1+t^2) \rightarrow (2)$$

the points (b, a) & (a, b)

$x = b$; $y = a$

$$a = bt - \frac{gb^2}{2u^2}(1+t^2)$$

$$a - bt = \frac{-gb^2}{2u^2}(1+t^2) \rightarrow (3)$$

for (a, b) put $x = a$; $y = b$ eqn (2)

$$b = at - \frac{ga^2}{2u^2}(1+t^2)$$

$$b - at = \frac{-ga^2}{2u^2}(1+t^2) \rightarrow (4)$$

(3)/(4)

UNIT- III

2 marks:

1. Complession & Restitution:

When two elastic bodies impinge the time during where the impact lasts may be divides into two stages. during the first stage the bodies are sightly compressing one another and during the second stage they are recovering there shape.

2. Elasticity and inelasticity

The perpecty which causes a solid body to recover its shape is called elasticity

If a body doesnot tends to recover its shape it will cause no forces of resitution and such a body is said to be inelasticity.

3. perfectly elastic and perpectly inelastic:

When a body completely regains its shape after a collision its is said to be perfectly elastic

if it doesnot come to its original shape it is called perfectly ~~el~~ inelastic.

4. impinge directly:

Two bodies are said to impinge directly when the direction of motion of each before impact is along the common normal as the point where the touch.

5 marks:

5. Fundamendal of Laws of impact

(i) Newton's experimental law:

This law can be put symbolically as follows:

If u_1, u_2 are the components of the velocities of the impinging bodies along their common normal before impact and v_1, v_2 their component velocities.

e is the co-efficient of restitution then

$$\frac{v_2 - v_1}{u_2 - u_1} = -e$$

(ii) motion of two smooth bodies $\perp^r$ to the line of impact.

hence the velocity of either body resolve in a direction $\perp^r$ to the line of impact is not altered by impact.

(iii) principle of conservation of momentum:

The algebraic sum of the moment of the impinging bodies after impact is equal to the algebric sum of their moments before impact all moments being measured along the common normal.

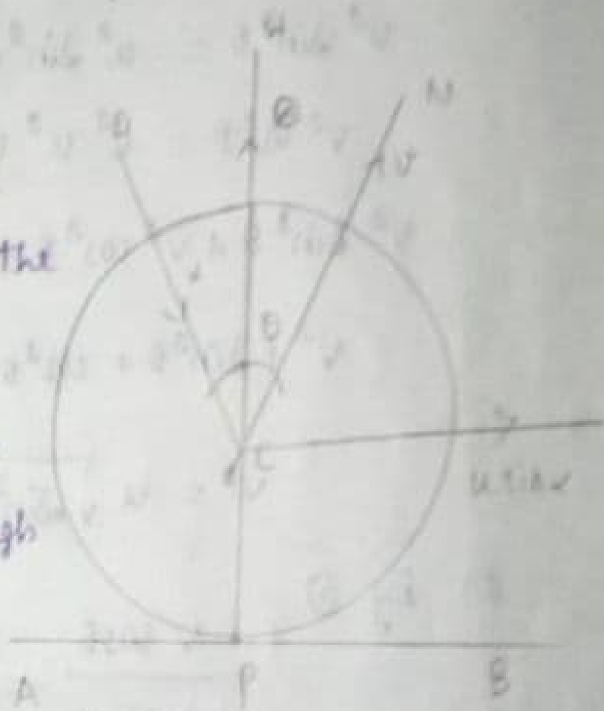
5. impact of a smooth space on a fixed smooth plane.

Stm:

A smooth sphere of particles where mass is 'm' where co-efficients of restitution is e on a smooth the fixed plane to find impinge obliquely its velocity 2 direction of motion after impact.

proof:

Let AB be the plane p be the point at which the sphere strikes it.



Let pc be the common normal at p passing through the centre of the sphere.

This is line of impact.

Let v be the velocity of the sphere an angle θ with CN.

Since the plane & sphere are smooth only forces.

hence the velocity of the sphere resolved in a direction parallel n the plane.

hence

$$v \sin\theta = u \sin\alpha \rightarrow (1)$$

by newton's experimental law (8)

$$\frac{V\cos\theta}{-u\cos\alpha - 0} = -e$$

$$V\cos\theta = -e(-u\cos\alpha - 0)$$

$$V\cos\theta = +eu\cos\alpha \rightarrow (2)$$

Squar (1) & (2)

$$V^2\sin^2\theta = u^2\sin^2\alpha$$

$$V^2\cos^2\theta = e^2u^2\cos^2\alpha$$

$$V^2\sin^2\theta + V^2\cos^2\theta = u^2\sin^2\alpha + e^2u^2\cos^2\alpha$$

$$V^2(\sin^2\theta + \cos^2\theta) = u^2(\sin^2\alpha + e^2\cos^2\alpha)$$

$$V^2 = u\sqrt{\sin^2\alpha + e^2\cos^2\alpha} \rightarrow (3)$$

(2) by (1)

$$\frac{V\cos\theta}{V\sin\theta} = \frac{eu\cos\alpha}{u\sin\alpha}$$

$$\cot\theta = e\cot\alpha \rightarrow (4)$$

So eqn (3) & (4) gives the velocity & direction of the motion after impact.

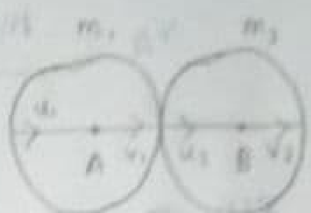
7. direction impact of two smooth sphere:

Stm:

A smooth sphere of mass m_1 impings directly with velocity u_1 on another smooth sphere of mass m_2 moving the same direction with velocity u_2. If the co-effcient of restitution is a to find their velocity after the impact.

proof:

AB is the ~~dump~~ impact common normal after impact, the sphere will move only in the direction AB. velocities v_1 & v_2



by newton experimental law.

$$\frac{v_2-v_1}{u_2-u_1} = -e$$

$$v_2 - v_1 = -e(u_2-u_1) \to ①$$

$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2 \to ②$$

② - ① $\times m_2$

$$m_1V_1 + m_2V_2 - (V_2-V_1) = m_1u_1 + m_2u_2 + e(u_1-u_2)\,m_2$$

$$m_1v_1 - v_2 + v_1 = m_1u_1 + m_2u_2 + em_2u_2 - em_2u_1$$

$$v_1(m_1+1) + v_2(m_2-1) = m_1u_1 + m_2u_2 + em_2(u_2-u_1)$$

$$(v_1+v_2)(m_1+1+m_2-1) = m_1u_1 + m_2u_2 + m_2(u_2-u_1)$$

$$v_1(m_1+m_2) = m_1u_1 + m_2u_2 + em_2(u_2-u_1)$$

$$= m_1u_1 + m_2u_2 + e m_2u_2 - e m_2u_1$$

$$v_1(m_1+m_2) = m_2u_2(1+e) + u_1(m_1 - e m_2)$$

$$v_1 = \frac{m_2u_2(1+e) + u_1(m_1 - e m_2)}{m_1+m_2} \rightarrow (3)$$

(1) $\times m_1$ gives (2)

$$v_2(m_1+m_2) = -e m_1(u_2-u_1) + m_1u_1 + m_2u_2$$

$$= -e m_1u_2 + e m_1u_1 + m_1u_1 + m_2u_2$$

$$= m_1u_1(1+e) + u_2(m_2 - e m_1)$$

$$v_2 = \frac{m_1u_1(1+e) + u_2(m_2 - e m_1)}{m_1+m_2} \rightarrow (4)$$

eqn (3) & (4) gives the velocity of sphere after impact.

10 marks:

8. Loss of kinetic energy due to direct impact of the two smooth sphere.

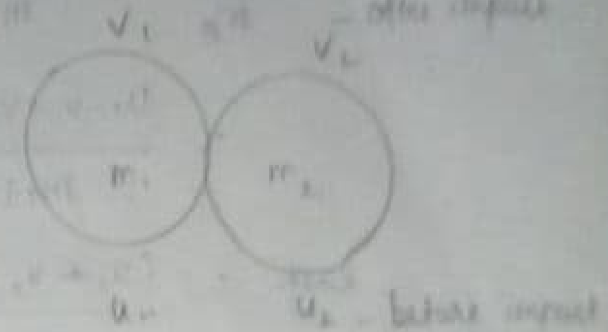
Sm:

Two sphere of gives of given masses with given velocity impinges directly to show that there is a loss of kinetic energy and to find the amount.

Let m_1, m_2 be the mass of sphere.

u_1, u_2 before impact

v_1, v_2 after impact



by newton's Law $v_2 - v_1 = -e(u_2 - u_1) \rightarrow ①$

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2 \rightarrow ②$$

Total kinetic energy before impact

$$= \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2$$

$$= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

Change in K.E = initial - final

$$= \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 - \frac{1}{2} m_1 v_1^2 - \frac{1}{2} m_2 v_2^2$$

$$= \frac{1}{2} m_1 (u_1 - v_1)(u_1 + v_1) + \frac{1}{2} m_2 (u_2 - v_2)(u_2 + v_2)$$

$$= \frac{1}{2} m_1 (u_1 - v_1)(u_1 + v_1) + \frac{1}{2} m (v_1 - u_1)(u_2 + v_2)$$

$$= \frac{1}{2} m_1 (u_1 - v_1) [u_1 + v_1 - (u_2 + v_2)]$$

$$= \frac{1}{2} m_1 (u_1 - v_1) [u_1 - u_2 - (v_2 - v_1)]$$

$$= \frac{1}{2} m_1 (u_1 - v_1) [u_1 - u_2 + e(u_2 - u_1)]$$

$$= \frac{1}{2} m (u_1 - v_1) [u_1 - u_2](1 - e) \rightarrow ③$$

from ②

$$m_1 (u_1 - v_1) = m_2 (v_2 - u_2)$$

$$\frac{u_1-v_1}{m_2} = \frac{v_2-u_2}{m_1}$$

$$= \frac{u_1-v_1+v_2-u_2}{m_1+m_2}$$

$$\text{each} = \frac{(u_1-u_2)+(v_2-v_1)}{m_1+m_2}$$

$$\frac{u_1-v_1}{m_2} = \frac{(u_1-u_2)(1+e)}{m_1+m_2}$$

$$u_1-v_1 = \frac{m_2(u_1-u_2)(1+e)}{m_1+m_2}$$

$$\text{Change K.E} = \frac{\frac{1}{2} m_1 m_2 (u_1-u_2)(1+e)(u_1-u_2)(1-e)}{m_1+m_2}$$

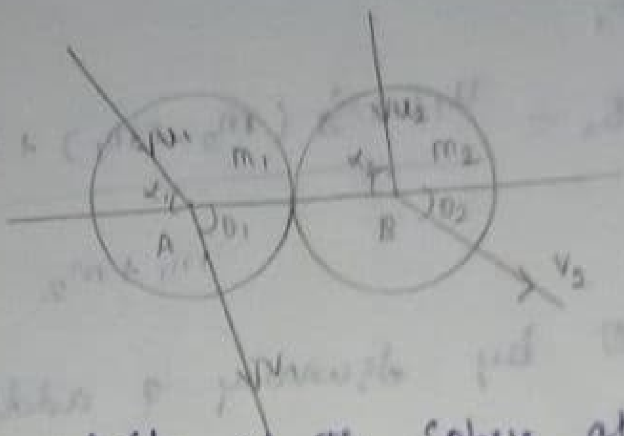
$$= \frac{1}{2}\frac{m_1 m_2 (u_1-u_2)^2(1-e^2)}{m_1+m_2} \rightarrow (5)$$

9. Oblique impact of two smooth sphere:-

Stm:-

A smooth sphere of mass m_1, impinges obliquely with velocity u_1 on another smooth sphere of mass m_2 moving with velocity u_2. If the direction of motion before impact make angles α_1 and α_2 respectively with the lines joining the centres of the sphere - to e. To find the velocities & direction of motion after impact.

proof:



Let the velocity of the sphere after impact be v_1 and v_2 in direction inclined at angles θ_1 & θ_2

For each sphere the velocities in the tangential direction are not affected by impact

$$v_1 \sin\theta_1 = u_1 \sin\alpha_1 \rightarrow (1)$$

$$v_2 \sin\theta_2 = u_2 \sin\alpha_2 \rightarrow (2)$$

velocities along the common normal AB.

$$v_2 - v_1 = -e(u_2 - u_1)$$

$$v_2 \cos\theta_2 = -v_1 \cos\theta_1$$

$$= -e(u_2 \cos\alpha_2 - u_1 \cos\alpha_1) \rightarrow (3)$$

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$$

$$m_2 v_2 \cos\theta_2 + m_1 v_1 \cos\theta_1 = m_2 u_2 \cos\alpha_2 + m_1 u_1 \cos\alpha_1 \rightarrow (4)$$

(4) - (3)

$$v_1 \cos\theta_1 (m_1 + m_2) = m_2 u_2 \cos\alpha_2 + m_1 u_1 \cos\alpha_1 +$$

$$e m_2 (u_2 \cos\alpha_2 - u_1 \cos\alpha_1)$$

$$v_1 \cos\theta_1 = \frac{u_1 \cos\alpha_1 (m_1 - e m_2) + m_2 u_2 (1+e)}{m_1 + m_2} \rightarrow (5)$$

(4) + (3) $\times m_1$

$$v_2 \cos\theta_2 = \frac{u_1 \cos\alpha_2 (m_2 - e m_1) + m_1 u_2 \cos\alpha_2 (m_2 - e m_1)}{m_1 + m_2} \rightarrow (6)$$

from (5) & (6) by squaring & adding

Corallary:

If the 2 spheres are perfectly elastic and equal mass then $e = 1$ and $m_1 = m_2$ eqn (5) & (6)

$$v_1 \cos\theta_1 = \frac{0 + m_1 u_2 \cos\alpha_2 \cdot 2}{2 m_1} = u_2 \cos\alpha_2$$

and

$$v_2 \cos\theta_2 = \frac{0 + m_1 u_1 \cos\alpha_1 \cdot 2}{2 m_1}$$

$$= u_1 \cos\alpha_1$$

Perfectly elastic spheres impinge they interchange their velocities in the directions of the line of centres.

4. Amplitude :

The distance through which the particle moves away from the centre of motion on either side of it is called the amplitude of oscillation.

5marks

5. General soln of the simple harmonic motion:

The simple harmonic motion eqn is $\frac{d^2x}{dt^2} = -\mu x$

$$\frac{d^2x}{dt^2} + \mu x = 0 \rightarrow (1)$$

$$x = A\cos\sqrt{\mu}t + B\sin\sqrt{\mu}t \rightarrow (2).$$

A & B arbitary constants.

$$x = C\cos(\sqrt{\mu}t + \epsilon) \rightarrow (3)$$

$$x = D\sin(\sqrt{\mu}t + \alpha) \rightarrow (4)$$

from (3) & (4)

$$x = a\cos(\sqrt{\mu}t + \epsilon) \rightarrow (5)$$

$$x = a\sin(\sqrt{\mu}t + \alpha) \rightarrow (6)$$

$$x = a\cos(\sqrt{\mu}t + \epsilon).$$

The quantity ϵ is called the epoch. eqn (5)

x is maximum. $\cos(\sqrt{\mu}t + \epsilon) = 1$

UNIT - IV

2 marks:

1. Simple harmonic motion :

Simple harmonic in a straight line when a particle moves a straight line - so that its acceleration is always directed towards a fixed point in the line and proportional to the distance from that point. its motion is called simple harmonic motion.

2 The period or the periodic time :

The period or the periodic time of a simple harmonic motion is the interval of time that elapses from any instant till a subsequent instant when the particle is again moving through the same position with the same velocity in same direction.

3. Frequency :

The frequency of the oscillation is the number of complex oscillation that the particle makes in one second. so frequency its the reciprocal of the period and is equal to $\sqrt{\mu}/2\pi$

If t_0 is the value of $\sqrt{\mu}\, t + \epsilon = 0$

$$t_0 = \frac{-\epsilon}{\sqrt{\mu}}$$

Hence phase at time $t = t - t_0$

$$= t + \frac{\epsilon}{\sqrt{\mu}}$$

$$= \frac{\sqrt{\mu}t + \epsilon}{\sqrt{\mu}} \quad //$$

6. A particle is moving with simple harmonic are x_1, x_2, x_3 P.T $\dfrac{2\pi}{\cos^{-1}\left(\frac{x_1+x_3}{2\pi}\right)}$

If a is a amplitude

$$x = a\cos\sqrt{\mu}t \rightarrow (1)$$

seconds t_1, t_1+1, t_1+2 the corresponding displacement x_1, x_2, x_3

$$x_1 = a\cos\sqrt{\mu}\,t_1 \rightarrow (2)$$

$$x_2 = a\cos\sqrt{\mu}\,(t_1+1) = a\cos(\sqrt{\mu}t_1 + \sqrt{\mu}) \rightarrow (3)$$

$$x_3 = a\cos\sqrt{\mu}\,(t_1+2) = a\cos(\sqrt{\mu}t_1 + 2\sqrt{\mu}) \rightarrow (4)$$

$$x_1 + x_3 = a\left[\cos\sqrt{\mu}t_1 + 2\sqrt{\mu} + \cos\sqrt{\mu}t_1\right]$$

$$= \frac{a \cdot 2\cos\sqrt{\mu}t_1 + 2\sqrt{\mu} + \sqrt{\mu}t_1}{2} \cdot \frac{\cos\sqrt{\mu}t_1 + 2\sqrt{\mu} - \sqrt{\mu}t_1}{2}$$

$$= 2a \cdot \cos(\sqrt{\mu}t_1 + \sqrt{\mu}) \cdot \cos\sqrt{\mu} = 2 \times 2\cos\sqrt{\mu}$$

$$\frac{x_1 + x_3}{2\times 2} = \cos\sqrt{\mu} \quad (\text{or}) \quad \sqrt{\mu} = \cos^{-1}\left(\frac{x_1+x_3}{2\times 2}\right)$$

$$\text{period},\ \frac{2\pi}{\sqrt{\mu}} = \frac{2\pi}{\cos^{-1}\left(\frac{x_1+x_3}{2\times 2}\right)} \quad //$$

7. $x = a\cos\omega t + b\sin\omega t$; $a = 3$, $b = 4$, ω

$$x = a\cos\omega t + b\sin\omega t \rightarrow (1)$$

$$\frac{dx}{dt} = a\omega\sin\omega t + b\omega\cos\omega t \rightarrow (2)$$

$$\frac{d^2x}{dt^2} = a\omega^2\cos\omega t - b\omega^2\sin\omega t$$

$$= -\omega^2(a\cos\omega t + b\sin\omega t)$$

$$= -\omega^2 x \rightarrow (3)$$

(3) S.T

$$\text{period} = \frac{2\pi}{\sqrt{\mu}} = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi \text{ sec.}$$

Amplitude is the greater value of x.

When x is maximum $\frac{dx}{dt} = 0$

$$\therefore -a\omega\sin\omega t + b\omega\cos\omega t = 0$$

$$a\sin\omega t = b\cos\omega t$$

$$\tan\omega t = b/a = 4/3$$

$$x = a_1 \cos(\sqrt{\mu}t + \varepsilon_1) + a_2 \cos(\sqrt{\mu}t + \varepsilon_2)$$

$$= \cos\sqrt{\mu}t\,(a_1 \cos\varepsilon_1 + a_2 \cos\varepsilon_2) - \sin\sqrt{\mu}t\,(a_1 \sin\varepsilon_1 + a_2 \sin\varepsilon_2)$$

$$= \cos\sqrt{\mu}t \cdot A\cos\varepsilon - \sin\sqrt{\mu}t \cdot A\sin\varepsilon \rightarrow (1)$$

$$A\cos\varepsilon = a_1\cos\varepsilon_1 + a_2\cos\varepsilon_2 \rightarrow (2)$$

and

$$A\sin\varepsilon = a_1\sin\varepsilon_1 + a_2\sin\varepsilon_2 \rightarrow (3)$$

Cont. A & ε equ (2) & (3)

$$A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos(\varepsilon_1 - \varepsilon_2) \rightarrow (4)$$

$\div$ (3) by (2)

$$\tan\varepsilon = \frac{a_1 \sin\varepsilon_1 + a_2 \sin\varepsilon_2}{a_1\cos\varepsilon_1 + a_2\cos\varepsilon_2} \rightarrow (5)$$

$$x = A(\cos\sqrt{\mu}t \cos\varepsilon - \sin\sqrt{\mu}t \cdot \sin\varepsilon)$$

$$= A\cos(\sqrt{\mu}t + \varepsilon) \rightarrow (6)$$

//

9. composition of two simple harmonic motion of the same period in two perpendicular direction.

If a particle possesses two simple harmonic motion in perpendicular direction and of the same period.

two perpendicular lines axis x & y.

$\tan\omega t = 4/3$ ∴ $\sin\omega t = 4/5$ & $\cos\omega t = 3/5$

$= a \times 3/5 + b \times 4/5$

$= \dfrac{3a+4b}{5} = \dfrac{3.3+4.4}{5} = \dfrac{9+16}{5}$

$= \dfrac{25}{5}$

$= 5$

amplitude $= 5$

10 marks:

8. Composition of 2 simple Harmonic motions of the same period and in the same straight line.

Since the period is dependent only on the constant μ, the two separate simple harmonic motion are expressed by the same diff eqn

$$\frac{d^2x}{dt^2} = -\mu x$$

Let x_1 & x_2 the displacement

$$x_1 = a_1 \cos(\sqrt{\mu}t + \varepsilon_1) \quad \&$$

$$x_2 = a_2 \cos(\sqrt{\mu}t + \varepsilon_2)$$

Then $x = x_1 + x_2$.

2 marks.

1. ~~Apses~~ Define Apses:

If there is a point A on a central orbit at which the velocity of the particle is perpendicular to the radius OA, then the point A is called Apses

2. Define Apsidal distance:

The length OA is the corresponding apsidal distance hence at an apse, the particle is moving at right angles to the radius vector.

3. Areal velocity:

The rate of description of the area traced out by the radius vector joining the particle to a fixed point is called areal velocity of the particle.

4. central force:

Suppose particle describes a path, acted on by an attractive force F towards a fixed point O. such a force is called a central force.

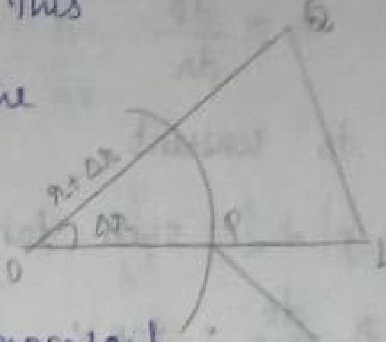
5. central orbit:

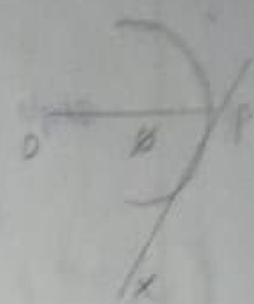
The path described by the particle is called central orbit. the fixed point is known as the centre of force.

5 Marks:

6. Note on the equiangular spiral:

Some quations in this Chapter will related the curve called the equiangular spiral





this curve has the important property that the tangent at any point P on it makes a constant angle with the radius vector OP.

Let $OP (= r)$ and $OQ (= r + \Delta r)$ be two consective radii vectors such that the included angle $POQ = \Delta\theta$

draw $QL \perp OP$

Then $$OL = (r+\Delta r) \cos \Delta\theta = r + \Delta r \text{ approximately}$$

hence $PL = OL - OP = \Delta r$

and $$LQ = (r + \Delta r) \sin \Delta\theta = (r + \Delta r)\Delta\theta$$

$$= r\Delta\theta + 0$$

the 1^{st} order of smallness

hence $$\tan \angle QPL = \frac{QL}{PL} = r\frac{\Delta\theta}{\Delta r}$$

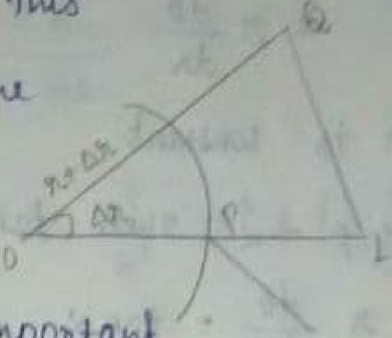
In the limit as $\Delta r + \Delta b$ both $\to 0$ the points tends to coinide with P.

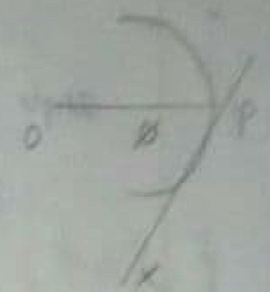
Then $$\phi = \lim_{Q \to P} \angle QPL$$

5 Marks:

6. Note on the equiangular spiral:

Some qualions in this Chapter will related the curve called the equiangular spiral





this curve has the important property that the Tangent at any point P on it makes a constant angle with the radius vector OP.

Let OP $(=r)$ and OQ $(=r+\Delta r)$ be two consective radii vectors such that the included angle POQ $=\Delta Q$

draw $QL \perp OP$

Then

$$OL = (r+\Delta r)\cos\Delta\theta = r+\Delta r \text{ approximately}$$

hence $PL = OL - OP = \Delta r$

and

$$LQ = (r+\Delta r)\sin\Delta\theta = (r+\Delta r)\Delta\theta$$

$$= r\Delta\theta + 0$$

the 1^{st} order of smallness

hence $\tan \underline{|QPL} = \frac{QL}{PL} = r\frac{\Delta\theta}{\Delta r}$

In the limit as $\Delta r + \Delta b$ both $\to 0$ The points tends to coinide with P.

Then

$$\phi = \lim_{Q\to P} \underline{|QPL}$$

$$\tan\phi = \lim_{Q \to P} \tan \angle QPL$$

$$= \lim_{\Delta r \to 0} r \frac{\Delta \theta}{\Delta r} = r \frac{d\theta}{dr}$$

$$= r \frac{d\theta}{dr}$$

angle ϕ is constant

Let $\phi = \alpha$ then $\tan\phi = \tan\alpha$

$$r \frac{d\theta}{dr} = \tan\alpha \quad (\text{or}) \quad \frac{dr}{r} = \cot\alpha . d\theta$$

$\int$ ing

$$\log r = \theta \cot\alpha$$

$$r = a e^{\theta \cot\alpha}.$$

This is the ~~shape~~ polar eqn to the equiangular spiral this is the shape of the curve.

7. pedal eqn of the central orbit:

In certain curves the relation b/w p and r is very simple such a relation is called the pedal eqn or the (p, r) eqn to the curve. we can get the (p, r) eqn to a central orbit as follows

$$\frac{1}{p^2} = u^2 + \left(\frac{du}{d\theta}\right)^2 \rightarrow ①$$

Diff both sides of (1) w.r.t θ.

$$\frac{-2}{p^3} \cdot \frac{dp}{d\theta} = 2u \cdot \frac{du}{d\theta} + 2 \frac{du}{d\theta} \cdot \frac{d^2u}{d\theta^2}$$

$$\frac{-2}{p^3} \cdot \frac{dp}{d\theta} = 2 \frac{du}{d\theta} \left[u + \frac{d^2u}{d\theta^2}\right) \rightarrow ②$$

Diff eqn polars

$$u + \frac{d^2u}{d\theta^2} = \frac{P}{h^2u^2}$$

hence (2) becomes

$$-\frac{1}{p^3} \cdot \frac{dp}{d\theta} = \frac{P}{h^2u^2} \cdot \frac{du}{d\theta}$$

$$-\frac{p}{p^3} \cdot dp = \frac{P}{h^2u^2} du = \frac{P}{h^2} r^2 d\left(\frac{1}{r}\right)$$

$\{u = 1/r$

$$-\frac{1}{p^3} dp = \frac{Pr^2}{h^2} \times -\frac{1}{r^2} \cdot \frac{dr}{d\theta}$$

$$= \frac{-P}{h^2} \frac{dr}{d\theta}$$

(or)

$$\frac{h^2}{p^3} \cdot \frac{dp}{dr} = P \rightarrow (3)$$

(3) is the (p.r) eqn or the pedal eqn to the central orbit.

8. Find the law of forces towards under which the curve $r^n = a^n \cos n\theta$

$$r^n = a^n \cos n\theta$$

Since $r = 1/u$.

The eqn is $u^n a^n \cos n\theta = 1 \rightarrow (1)$

Taking log

$$n \log u + n \log a + \log \cos n\theta = 0 \rightarrow (2)$$

diff (2) w.r to θ

$$n \cdot \frac{1}{u} \frac{du}{d\theta} - \frac{n \sin \theta}{\cos n\theta} = 0$$

$$\frac{du}{d\theta} = u \tan n\theta \rightarrow (3)$$

diff (3) w.r. to θ.

$$\frac{d^2u}{d\theta^2} = un \sec^2 n\theta + \tan n\theta \cdot \frac{du}{d\theta}$$

$$= nu \sec^2 n\theta + u \tan^2 n\theta \quad \text{(using 3)}$$

$$u + \frac{d^2u}{d\theta^2} = u + nu \sec^2 n\theta + u \tan^2 n\theta$$

$$= nu \sec^2 n\theta + u(1 + \tan^2 n\theta)$$

$$= nu \sec^2 n\theta + u \sec^2 n\theta$$

$$= (n+1)\, u \sec^2 n\theta$$

$(n+1)u \cdot u^{2n} \cdot a^{2n}$ using (1) substitude for $\sec^2 n\theta$

$(n+1)u \cdot u$ using (1) substitude for $\sec^2 n\theta$

$$= (n+1)a^{2n}\; u^{2n+1}$$

$$P = h^2u^2\left(u + \frac{d^2u}{d\theta^2}\right) = h^2u^2(n+1)\, a^{2n} \cdot u^{2n+1}$$

$$= (n+1)\, a^{2n}\, h^2\, u^{2n+3}$$

$$= (n+1)\, a^{2n} \cdot h^2 \cdot \frac{1}{r^{2n+3}} \rightarrow (4)$$

(ii)

$F \propto \frac{1}{r^{2n+3}}$ which means that the central acceleration varies inversely as the $(2n+3)^{rd}$ power of the distance.

10 marks:

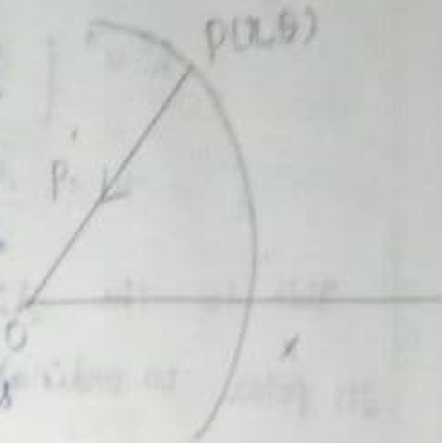
(4)

9. Differential eqn of central orbits

Stm:

A particle moves in a plane with an acceleration which is always directed to a fixed point o in the plane to obtain the differential eqn of its path.

proof:



Take 'o' as the pole and a fixed line through 'o' as the initial line. Let $p(r,\theta)$ be the polar co-ordinates of the particle at time 't' and 'm' be its mass.

The eqn of motion of the particle $m(\ddot{r}-r\dot{\theta}^2)=m$

$\ddot{r}-r\dot{\theta}^2=-P \to ①$ and $\frac{m}{r}\cdot\frac{d}{dt}(r^2\dot{\theta})=0$

$\frac{1}{r}\cdot\frac{d}{dt}(r^2\dot{\theta})=0 \to ②$

eqn ② from ②

$r^2\dot{\theta}=h$ (say) const $\to ③$

put $u=\frac{1}{r}$

from ③

$$\dot{\theta}=\frac{h}{r^2}=hu^2$$

Also $\dot{r}=\frac{dr}{dt}\cdot\frac{d}{dt}\left(\frac{1}{u}\right)=-\frac{1}{u^2}\frac{du}{dt}$

$$=-\frac{1}{u^2}\frac{du}{d\theta}\cdot\frac{d\theta}{dt}$$

$$\frac{du}{d\theta} = -\frac{e\sin\theta}{l} \text{ and } \frac{d^2u}{d\theta^2} = \frac{-e\sin\theta}{l}$$

now

$$u + \frac{d^2u}{d\theta^2} = \frac{1+e\cos\theta}{l} - \frac{e\cos\theta}{l}$$

$$u + \frac{d^2u}{d\theta^2} = 1/l$$

from the eqn of central orbit

$$\frac{F}{h^2u^2} = u + \frac{d^2u}{d\theta^2} = 1/l$$

$$F = \frac{h^2u^2}{l} = \frac{\mu}{r^2} \quad \text{where } \mu = h^2/l$$

$$F = \mu/r^2$$

(ii) W.K.T

$$1/p^2 = u^2 + \left(\frac{du}{d\theta}\right)^2$$

$$= \frac{(1+e\cos\theta)^2}{l} + \left(\frac{-e\sin\theta}{l}\right)^2$$

$$= \frac{1 + 2e\cos\theta + e^2}{l^2}$$

Also

$h = pv$, v is the linear velocity

$$v = h/p \; ; \; v^2 = h^2/p^2$$

$$v^2 = \frac{h^2}{p^2} = \frac{\mu l}{l^2}\left[1 + e^2 + 2\left(\frac{l}{r} - 1\right)\right]$$

$$= \frac{\mu}{l}\left[e^2 + \frac{2l}{r} - 1\right]$$

$$= \mu/l \left[\frac{2l}{r} - (1-e^2) \right]$$

$$= \mu \left[2/r - 1/l (1-e^2) \right] = \mu \left[2/r - 1/a \right]$$

Since $l = b^2/a^2$

$$= \frac{a^2(1-e^2)}{a} = a(1-e^2)$$

$v^2 = \mu(2/r - 1/a)$ gives the velocity v.

areal velocity in the orbit $= \frac{1}{2}h$ & this is constant the total area of the ellipse $= \pi ab$

periodic time $T = \frac{\text{area}}{\text{velocity}}$

$$\frac{\pi ab}{\frac{1}{2}h} = \frac{2\pi ab}{h}$$

$\therefore h = \sqrt{\mu l}$

$$= \frac{2\pi ab}{\sqrt{\mu l}} \quad (\text{since } \mu = h^2/l)$$

$$= \frac{2\pi ab}{\sqrt{\mu \cdot b^2/a}} \qquad \left[b^2 = a^2(1-e^2) \right]$$

$l = b^2/a$

$$= \frac{2\pi}{\sqrt{\mu}} a^{3/2} \qquad \frac{2\pi}{\mu} \text{ is constant.}$$