I <u>2 mark</u>

1) find the distance between the points (4,3,-6) and (-2,1,-3)

<u>soln:</u>

$A(4,3,-6)$, $B(-2,1,-3)$

$x_1 = 4$, $y_1 = 3$, $z_1 = -6$

$x_2 = -2$, $y_2 = 1$, $z_2 = -3$

$$\text{Distance} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

$$= \sqrt{(-2-4)^2+(1-3)^2+(-3+6)^2}$$

$$= \sqrt{36+4+9} = \sqrt{49} = 7$$

$$\boxed{\text{Distance} = 7}$$

② find the value of x if the distance between 2points (3,-5,4) and $(x,-8,4)$ is 5.

soln:

$A(3,-5,4)$, $B(x,-8,4)$

$x_1 = 3$, $x_2 = x$, $z_1 = 4$

$y_1 = -5$, $y_2 = -8$, $z_2 = 4$

$$\text{distance} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

$$5 = \sqrt{(x-3)^2+(-8+5)^2+(4-4)^2}$$

$$5 = \sqrt{(x-3)^2+9}$$

square on both side ②

$5^2 = (x-3)^2 + 9$

$25 - 9 = x^2 - 6x + 9$

$x^2 - 6x + 9 = 16$

$x^2 - 6x + 9 - 16 = 0$

$(x+1)(x-7) = 0$

$x+1 = 0,\ x-7 = 0$

$\boxed{x = -1}, \boxed{x = 7}$

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③ find the ratio in which the line joining the points $(4,4,-10)$ and $(-2,2,4)$ is divised by the line ~~y~~z-plane.

Soln:

$m:n = K:1$

$A(4,4,-10),\ B(-2,2,4)$

$x_1 = 4,\ x_2 = -2$
$y_1 = 4,\ y_2 = 2$
$z_1 = -10,\ z_2 = 4$

$$= \left[\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right]$$

$$= \left[\frac{K(-2)+1(4)}{K+1}, \frac{K(2)+1(4)}{K+1}, \frac{K(4)+1(-10)}{K+1}\right]$$

$$= \left[\frac{-2K+4}{K+1}, \frac{2K+4}{K+1}, \frac{4K-10}{K+1}\right]$$

$$\frac{-2K+4}{K+1} = 0 \Rightarrow -2K+4 = 0$$

$$-2K = -4$$

$$K = 4/2$$

$$\boxed{K = 2}$$

$$\boxed{m:n = 2:1}$$

④ find the Co-ordinate of point which divides the point (1, 3, 7) and (6, 3, 2) in the ratio 2:3

Soln:

$$m:n = 2:3$$

$$\text{divides} = \left[\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right]$$

$x_1 = 1$, $x_2 = 6$

$y_1 = 3$; $y_2 = 3$

$z_1 = 7$; $z_2 = 2$

$$= \left[\frac{2(6)-3(1)}{2-3}, \frac{2(3)-3(3)}{2-3}, \frac{2(2)-3(7)}{2-3}\right]$$

$$= \left[\frac{12-3}{-1}, \frac{6-9}{-1}, \frac{4-21}{-1}\right]$$

$$= [-9, 3, 17]$$

⑤ find the coordinates of the point which divide the line Joining the points (2, -4, 3) and (-4, 5, -6) in the ratio 2:1

Soln: 4

$m:n = 2:1$

$A(2,-4,3)$, $B(-4,5,-6)$

$x_1 = 2$, $y_1 = -4$, $z_1 = 3$

$x_2 = -4$, $y_2 = 5$, $z_2 = -6$

$$= \left[\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right]$$

$$= \left[\frac{2(-4)+1(2)}{2+1}, \frac{2(5)+(-4)}{2+1}, \frac{2(-6)+(3)}{3}\right]$$

$$= \left[\frac{-6}{3}, \frac{6}{3}, \frac{-9}{3}\right] = [-2, 2, -3]$$

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(5) Find the co-ordinate of Point which divides the points $(-4,1,-2)$ and $(3,8,5)$ in the ratio $5:2$

$m:n = 5:2$

$x_1 = -4$, $x_2 = 3$

$y_1 = 1$, $y_2 = 8$

$z_1 = -2$, $z_2 = 5$

$$= \left[\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}, \frac{mz_2-nz_1}{m-n}\right]$$

$$= \left[\frac{5(3)-(2)(-4)}{5-2}, \frac{5(8)-2(1)}{5-2}, \frac{5(5)-2(-2)}{5-2}\right]$$

$$= \left[\frac{23}{3}, \frac{38}{3}, \frac{29}{3}\right]$$

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(7) find the ratio in which the line xy plane divides (5)
The Join of $(-3,4,-8)$ and $(5,-6,4)$

Soln:

$$=\left[\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}, \frac{mz_2-nz_1}{m-n}\right]$$

$$=\left[\frac{K(5)-1(-3)}{K-1}, \frac{K(-6)-1(4)}{K-1}, \frac{K(4)-1(-8)}{K-1}\right]$$

$$=\left[\frac{5K+3}{K-1}, \frac{-6K-4}{K-1}, \frac{4K+8}{K-1}\right]$$

$$\frac{4K+8}{K-1}=0$$

$$4K+8=0$$

$$4K=-8 \Rightarrow \boxed{K=-2}$$

$\boxed{\text{The ratio is } -2:1}$

(8) Prove that $l^2+m^2+n^2=1$

Proof:- Let us consider any line in space direction cosines l, m, n

Let (x,y,z) be the co-ordinates points to P.

$$x=lr,\ y=mr,\ z=nr$$

$$x^2+y^2+z^2=(lr)^2+(mr)^2+(nr)^2$$

$$=l^2r^2+m^2r^2+n^2r^2$$

$$x^2+y^2+z^2=r^2$$

$$r^2=r^2[l^2+m^2+n^2]$$

$$l^2+m^2+n^2=1$$

Hence proved.

9) Direction ratios of a line (3, 4, 12) what are its directional cosines.

Soln:

$a = 3,\ b = 4,\ c = 12$

To find l, m, n

$$l = \frac{a}{\sqrt{a^2+b^2+c^2}} = \frac{3}{\sqrt{9+16+144}} = 3/13$$

$$m = \frac{b}{\sqrt{a^2+b^2+c^2}} = \frac{4}{\sqrt{169}} = 4/13$$

$$n = \frac{c}{\sqrt{a^2+b^2+c^2}} = \frac{12}{\sqrt{169}} = 12/13$$

$\therefore$ directional cosines $(3/13, 4/13, 12/13)$

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(10) Direction ratios of a line (3, 12, 4) what are its direction cosines.

Soln:

$a = 3,\ b = 12,\ c = 4$

To find l, m, n

$$l = \frac{3}{\sqrt{169}} = 3/13$$

$$m = \frac{12}{\sqrt{169}} = 12/13$$

$$n = \frac{4}{\sqrt{169}} = 4/13$$

II 5mark

① direction ratios of a line (4, 3, -5) and (-2, 1, -8) what are it's direction all cosines

Soln:

$(4, 3, -5)\ (-2, 1, -8)$

$\Rightarrow x_1 - x_2, y_1 - y_2, z_1 - z_2$

$\Rightarrow 4+2,\ 3-1,\ -5+8$

$\Rightarrow (6, 2, 3)$

$a = 6,\ b = 2,\ c = 3$

To find l, m, n

$l = \frac{6}{\sqrt{49}} = 6/7$

$m = \frac{2}{\sqrt{49}} = 2/7$

$n = \frac{3}{\sqrt{49}} = 3/7$

$\therefore$ directional cosines $(6/7, 2/7, 3/7)$

---

② S.T the four points $(0, -1, -1)$, $(4, 5, 1)$, $(3, 9, 4)$ and $(-4, 4, 4)$ lie on the plane

Solo:

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

$x_1 = 0$, $y_1 = -1$, $z_1 = -1$ | $x_2 = 4$, $y_2 = 5$, $z_2 = 1$ | $x_3 = 3$, $y_3 = 9$, $z_3 = 4$

$$\begin{vmatrix} x & y+1 & z+1 \\ 4 & 6 & 2 \\ 3 & 10 & 5 \end{vmatrix} = 0$$

$\Rightarrow x(30-20) - (y+1)(20-6) + (z+1)(40-18) = 0$

$\Rightarrow 10x - 14y - 14 + 22z + 22 = 0$

$10x - 14y + 22z + 8 = 0$

lie on the point $(-4, 4, 4)$

$10(-4) - 14(4) + 22(4) + 8 = 0$

$-40 - 56 + 88 + 8 = 0$

$-96 + 96 = 0$

$\therefore$ The Four points are lie in the plane.

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③ To find the angle b/w the plane $2x+4y-6z=11$ and $3x+6y+5z+4=0$

soln:

$$\cos\theta = \frac{\pm a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

① $\Rightarrow 2x + 4y - 6z = 11$

② $\Rightarrow 3x + 6y + 5z = -4$

$$\cos\theta = \pm\frac{(2)(3) + (4)(6) + (-6)(5)}{\sqrt{4+16+36}\sqrt{9+36+25}}$$

$$= \pm\frac{6+24-30}{\sqrt{56}\sqrt{70}}$$

$\cos\theta = 0$

$\cos 90° = \pi/2 = 0$

$\boxed{\theta = \pi/2}$

⑥ ④ find the equation of the passing throught the ⑦
Point $(2,-4,5)$ and Parallel to the $4x+2y-7z+6=0$

Soln:

The parallel to the $4x+2y-7z+6=0$

The required to the

$$4x+2y-7z+k=0 \rightarrow ①$$

Passing through the point $(2,-4,5)$

$$4(2)+2(-4)-7(5)+k=0$$

$$8-8-35+k=0$$

$$\boxed{k=35}$$

$k=35$ in equal ①

$$4x+2y-7z+k=0$$

The requried equation of the plane

$$4x+2y-7z+35=0$$

---

⑤ find the equation of the plane Passing through the point $(1,-2,2)$ and $(-3,1,-2)$ and Perpendicular to the plane $2x+y-z+6=0$

Soln:

Equation of the plane

$$a(x-x_1)+b(y-y_1)+c(z-z_1)=0$$

passing to the Point $(1,-2,2)$

$$a(x-1)+b(y+2)+c(z-2)=0 \rightarrow ①$$

Passing to another point $(-3,1,-2)$

$$a(-3-1)+b(1+2)+c(-2-2)=0$$

$$a(-4)+b(3)+c(-4)=0$$

$$-4a+3b-4c=0 \rightarrow ②$$

Perpendiontar - to the plane $2x+y-z+b=0$

$2a+b-c=0 \rightarrow 3$

$$\begin{array}{cccc} & a & b & c \\ 3 & -4 & -4 & 3 \\ 1 & -1 & 2 & 1 \end{array}$$

$$\frac{a}{-3+4} = \frac{b}{-8-4} = \frac{c}{-4-6}$$

$$\frac{a}{1} = \frac{b}{-12} = \frac{c}{-10}$$

$\boxed{a=1}$, $\boxed{b=-12}$, $\boxed{c=-10}$

sub the value of A, B, C in equal 1

$a(x-1)+b(y+2)+c(z-2)=0$

$(x-1)-12(y+2)-10(z-2)=0$

$x-1-12y-24-10z+20=0$

$x-12y-10z-5=0$

the requred plane $x-12y-10z-5=0$

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(6) find the equation of the plane passing through the point $(1,-1,-2)$ and perpendicular to the plane $2x+3y-2z=5$ and $x+2y-3z=8$

Soln:

equation of the plane

$a(x-x_1)+b(y-y_1)+c(z-z_1)=0$

passing to the point $(1,-1,-2)$

$a(x-1)+b(y+1)+c(z+2)=0 \rightarrow 1$

perpendicular to the plane $2x+3y-2z=0$

$2a+3b-2c=0 \rightarrow 2$

$x+2y-3z=0$

$a+2b-3c=0 \rightarrow 3$

a b c

3 -2 2 3

2 -3 1 2

$$\frac{a}{-9+4} = \frac{b}{6-2} = \frac{c}{4-3}$$

$\boxed{a=-5}$, $\boxed{b=4}$, $\boxed{c=1}$

sub the value of the a,b,c in ~~as~~ equal ①

$a(x-1)+b(y+1)+c(z-2)=0$

$-5(x-1)+4(y+1)+(z-2)=0$

$-5x+5+4y+4+z-2=0$

$5x-4y-z\cancel{+7}-7=0$

The required plane $5x-4y-z-7=0$

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⑦ find the equation of the plane passing through the point $(3,-3,1)$ and normal to the line joining the point $(3,4,-1)$ and $(2,-1,5)$

soln:

equation of the plane

$a(x-x_1)+b(y-y_1)+c(z-z_1)=0$

$a(x-3)+b(y+3)+c(z-1)=0 \rightarrow$ ①

Normal to the plane $(3,4,-1)$ $(2,-1,5)$

$= x_2-x_1,\ y_2-y_1,\ z_2-z_1$

$=(2-3),\ (-1-4),\ (5+1)$

$=-1,\ -5,\ 6$

$a=-1,\ b=-5,\ c=6$

sub the value of the a,b,c in equal

$a(x-3)+b(y+3)+c(z-1)=0$

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$1(-x+3)+(-5)(y+3)+6(z-1)=0$

$-x+3-5y-15+6z-6=0$

$-x-5y+6z-18=0$

$x+5y-6z+18=0$

2) find the equation of the plane which passes through (2,-1,1) and is Paralle to the plane $3x+7y-10z=5$

Soln:

Let the equation of the plane

$a(x-x_1)+b(y-y_1)+z(z-z_1)=0 \rightarrow ①$

passing through the point (2,-1,1)

$a(x-2)+b(y+1)+c(z-1)=0 \rightarrow ②$

$3x+7y-10z=5$

$3a+7b-10c=5$

$3(x-2)+7(y+1)-10(z-1)=0$

$3x-6+7y+7-10z+10=0$

$3x+7y-10z+11=0$

To find the equation of the plane passing through the line of intersection two given plane.

Soln:

$u=a_1x+b_1y+c_1z+d_1=0$

$v=a_2x+b_2y+c_2z+d_2=0$

$u+kv=(a_1x+b_1y+c_1z+d_1)+k(a_2x+b_2y+c_2z+d_2)$

$=0+k(0)$

$=0$

10 find the equ of the plane of the intersection of the plane $3x-y+2z-4=0$ and $x+y+z-2=0$ and passing through the point $(2,2,1)$

Soln:

$$u = 3x-y+2z-4=0$$
$$v = x+y+z-2=0$$

intersection of planes $u+kv=0$

$$u+kv \Rightarrow (3x-y+2z-4)+k(x+y+z-2)=0 \rightarrow (1)$$

Passing through the point $(2,2,1)$

$$\Rightarrow (3(2)-(2)+2(1)-4)+k(2+2+1-2)=0$$
$$(6-2+2-4)+k(3)=0$$
$$2+3k=0$$
$$3k=-2$$
$$\boxed{k=-2/3}$$

subsuit in equal (1)

$$(3x-y+2z-4)-2/3(x+y+z-2)=0$$
$$9x-3y+6z-12-2x-2y-2z+4=0$$
$$\boxed{7x-5y+4z-8=0}$$

equation of the plane $7x-5y+4z-8=0$

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11 find the equ of the plane throw $(3,4,5)$ $\parallel^{el}$ to plane $2x+3y-z=0$

Soln: The equation of any plane $\parallel^{el}$ to th

$$2x+3y-z+k=0$$

Given the point $(3,4,5)$

$$2x+3y-z+k=0$$

$2x+3y-z+k=0$

$2(3)+3(4)-5+k=0$

$6+12-5+k=0$

$18-5+k=0$

$\boxed{k=-13}$

Hence the equ of the required plane

$2x+3y-z-13=0$

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find the distance of the origin from the plane $6x-3y+2z-14=0$

Soln'.

The equation from the plane in normal form $lx+my+nz=p$

Given the plane

$6x-3y+2z-14=0$

$l/6 = m/-3 = n/2 = p/14$

$p/14 = \frac{1}{\sqrt{a^2+b^2+c^2}} = \frac{1}{\sqrt{6^2+3^2+2^2}} = \frac{1}{7}$

$p/14 = 1/7$

$p = 14/7 \Rightarrow \boxed{p=2}$

III 10 mark

15

1) find the distance of the point (3, 2, 4), (4, 8, 1) from the plane $3x + 4y + 12z = 3$. Are these points on the same side of the plane.

Soln:

The given equation of the plane

$$3x - 4y + 12z = 3$$

$$3x - 4y + 12z - 3 = 0$$

$\therefore$ length of $\perp$ from (3, 2, 4) to the plane

$$\Rightarrow \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}$$

$$= \frac{3(3) + (-4)(2) + 12(4) + (-3)}{\sqrt{3^2 + (-4)^2 + 12^2}}$$

$$= \frac{9 - 8 + 48 - 3}{\sqrt{169}} = \frac{46}{\sqrt{169}} = \frac{46}{13}$$

Length of $\perp$ from (4, 8, 1) to the plane

$$\Rightarrow \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}$$

$$= \frac{3(4) + (-4)(8) + (12)(1) + (-3)}{\sqrt{3^2 + (-4)^2 + 12^2}}$$

$$= \frac{24 - 35}{\sqrt{169}} = \frac{-11}{13}$$

(3, 2, 4) in plane $3x - 4y + 12z - 3 = 0$

$$3(3) - 4(2) + 12(4) - 3 = 0$$

$$9 - 8 + 48 - 3 = 0$$

$$57 - 11 = 0$$

$$= 46 \ (+ve)$$

sub the second point (4, 8, 1) in $3x-4y+12z-3=0$

$$3(4)-4(8)+12(1)-3=0$$

$$12-32+12-3=0$$

$$=-11(-ve)$$

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(8) find the equation of the plane of the intersection of the plane $2x-5y+z=3$ and $x+y+4z=5$ and parallel to the plane $x+3y+6z=1$

Soln:

$$u=2x-5y+z-3=0$$

$$v=x+y+4z-5=0$$

intersection of the plane $u+kv=0$

$$(2x-5y+z-3)+k(x+y+4z-5)=0 \rightarrow (1)$$

$$2x-5y+z-3+kx+yk+4zk-5k=0$$

$$x(2+k)+y(-5+k)+z(1+4)-5k-3=0$$

The plane is parallel to the plane $x+3y+6z=1$

$$\frac{a}{l}=\frac{b}{m}=\frac{c}{n}$$

$$\frac{x(2+k)}{1}=\frac{y(k-5)}{3}=\frac{1+4k}{6}$$

$$\frac{2+k}{1}=\frac{k-5}{3}=\frac{1+4k}{6}$$

$$\frac{2+k}{1}=\frac{k-5}{3}$$

$$6+3k=k-5$$

$$3k=k-5-6$$

$$3k=-11+k$$

$$3k-k=-11$$

$$2k=-11 \Rightarrow \boxed{k=-11/2}$$

$(2x-5y+z-3)-{}^{11}/_{2}(x+y+4z-5)=0$

$4x-10y+2z-6-11x-11y-44z+55=0$

$-7x-21y-42z+47=0$

$\div(-)$ $7x+21y+42z-47=0$

$\div(7)$ $x+3y+6z-7=0$

The equation of the plane

$x+3y+6z-7=0$

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(2) find the equation of the plane passing throught the line of Intersection of the plane $2x+y+3z-4=0$ and $4x-y+5z-7=0$ and is perpendicular to the plane $x+3y-4z+6=0$

Soln:

$u=2x+y+3z-4=0$

$v=4x-y+5z-7=0$

Intersection of plane

$u+kv=0$

$(2x+y+3z-4)+k(4x-y+5z-7)=0 \rightarrow (1)$

$x(2+4k)+(1-k)y+(3+5k)z-7k-4=0 \rightarrow$

Perpendicular to the plane $x+3y-4z+6$

$x=1, y=3, z=-4$ in equal (2)

$(2+4k)+(1-k)3+(3+5k)(-4)-7k-4=0$

$-4k-23k-7k+5-16=0$

$-26k-11=0$

$\boxed{k=-11/26}$

sub $k=-{}^{11}/_{26}$ equal (1)

$(2x+y+3z-4) - \frac{11}{26}(4x-y+5z-7) = 0$

$52x+26y+78z-104 + -44x+11y-55z+77=0$

$\boxed{8x+37y-23z-27=0}$

(4) find the distance b/w two Paralle Plane $2x-2y+z-6=0$ to $4x-4y+2z-7=0$

Soln:

$$2x-2y+z-6=0$$
$$2x-2y+z=6$$

Put $y=0, z=0$

$$2x = 6$$
$$\boxed{x=3}$$

The Points are $(3,0,0)$

$$\Rightarrow 4x-4y+2z-7=0$$
$$ax+by+cz+d=0$$
$$\Rightarrow \frac{ax+by+cz+d}{\sqrt{a^2+b^2+c^2}}$$

Here, $x=3, y=0, z=0, d=-7$

$$= \frac{4(3)+4(0)+2(0)+(-7)}{\sqrt{4^2+(-4)^2+2^2}} = \frac{12-7}{\sqrt{16+16+4}}$$

$$= \frac{5}{\sqrt{36}} = 5/6$$

(5) find the angle b/w the plane $2x-y+z=0$, $x+y+2z=0$

Soln:-

Given the equation

$$2x-y+z=0$$
$$x+y+2z=0$$

The equation of the Plane

$a_1x + b_1y + c_1z + d_1 = 0$

$a_2x + b_2y + c_2z + d_2 = 0$

angle b/w the plane

$$\cos\theta = \pm \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\cos\theta = \pm \frac{2-1+2}{\sqrt{(6)(6)}} = \pm \frac{3}{\sqrt{36}} = \pm 3/6$$

$$= \pm 1/2$$

$\cos\theta = 1/2$

$\theta = \cos^{-1}(1/2)$

$\theta = \cos(60)$

$\boxed{\theta = \pi/3}$

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unit - III

I 2 mark

① find the equation of the sphere with centre (-1, 2, -3) and radius 3 units.

Soln:

Given :-

centre = (-1, 2, -3)

radius = 3

$(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2 = r^2$

$(x+1)^2 + (y-2)^2 + (z+3)^2 = 3^2$

$x^2 + 2x + 1 + y^2 - 4y + 4 + z^2 + 6z + 9 = 9$

$x^2 + y^2 + z^2 + 2x - 4y + 6z + 14 = 9$

$x^2 + y^2 + z^2 + 2x - 4y + 6z + 5 = 0$

② find the radius the centre of the sphere
$x^2+y^2+z^2-2x+4y-6z=2$

Soln:

Centre $(-u,-v,-w)$

radius $= \sqrt{u^2+v^2+w^2-d}$

$x^2+y^2+z^2-2x+4y-6z=2$

General equation of the spher

$2ux=-2x$ | $2v=4$ | $2w=-6$

$\boxed{u=-1}$ | $\boxed{v=2}$ | $\boxed{w=-3}$

centre $(-1,2,-3)=(1,-2,3)$

$d=-2$

radius $= \sqrt{u^2+v^2+w^2-d} = \sqrt{1+4+9+2}$

$= \sqrt{16} = 4$

③ find the radius and centre of sphere
$x^2+y^2+z^2-2x+4y-6z=2$

Soln:

Centre $=(-u,-v,-w)$

Given sphere

$x^2+y^2+z^2-2x+4y-6z-2=0$

General equ of the sphere,

$x^2+y^2+z^2+2ux+2vy+2wz+d=0$

$2u=-2$ | $2v=4$ | $2w=-6$ | $d=-2$

$\boxed{u=-1}$ | $\boxed{v=2}$ | $\boxed{w=-3}$

Centre $=(1,-2,3)$

radius $= \sqrt{u^2+v^2+w^2-d}$

$= \sqrt{1+4+9+2} = \sqrt{16} = 4$

$\boxed{\text{radius} = 4}$

II 5mark

1) find the co-ordinate of the centre and radius of the sphere $2x^2+2y^2+2z^2-2x+4y+2z-15=0$

Soln:

$$2x^2+2y^2+2z^2-2x+4y+2z-15=0$$

$$2(x^2+y^2+z^2-x+2y+z-15/2)=0$$

General equ of the sphere

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0$$

$2u=-1$ | $2v=2$ | $2w=1$

$\boxed{u=-1/2}$ | $\boxed{v=1}$ | $\boxed{w=1/2}$

$$d=-15/2$$

Centre $(1/2,-1,-1/2)$

radius $=\sqrt{1/4+1+1/4+15/2}$

$=\sqrt{\frac{1+4+1+30}{4}}=\sqrt{\frac{36}{4}}=\sqrt{9}=3$

radius $=3$

2) find the equ of the sphere which has its centre at a points $(6,-1,2)$ and duch the plan $2x-y+2z-2=0$

Soln:

radius of the sphere

$$=\frac{ax+by+cz+d}{\sqrt{a^2+b^2+c^2}}=\frac{2x-y+2z-2}{\sqrt{4+1+4}}$$

$$=\frac{2x-y+2z-2}{3}$$

from centre $(6,-1,2)$

$$r=\frac{2x-y+2z-2}{3}$$

$$= \frac{2(6)-(-1)+2(2)-2}{3} = \frac{12+1+4-2}{3}$$

$$= 15/3 = 5$$

$$\boxed{r=5}$$

centre $(6,-1,2)$, radius $=5$

equ of the sphere

$$(x-x_1)^2+(y-y_1)^2+(z-z_1)^2=r^2$$

$$(x-6)^2+(y+1)^2+(z-2)^2=5^2$$

$$x^2+y^2+z^2-12x+2y-4z+16=0$$

A sphere of the constant radius k passes through the origin and meets the axis in A, B, C P.T The centroid of the Δ^{le} A, B, C lies on the sphere

$$9(x^2+y^2+z^2)=4k^2$$

Soln:

Equation of the sphere

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0$$

origin $(0,0,0) \Rightarrow \boxed{d=0} \rightarrow$ (1)

Radius of the sphere k

$$k=\sqrt{u^2+v^2+w^2-d}$$

$$k=\sqrt{u^2+v^2+w^2}$$

$$k^2=u^2+v^2+w^2 \rightarrow (2)$$

x-axis	y-axis	z-axis
$x^2+2ux=0$	$y^2+2vy=0$	$z^2+2wz=0$
$x(x+2u)=0$	$y(y+2v)=0$	$z(z+2w)=0$
$\boxed{x=-2u}$	$\boxed{y=-2v}$	$\boxed{z=-2w}$
$(-2u,0,0)$	$(0,-2v,0)$	$(0,0,-2w)$

Centroid of the sphere

$$x = \left(\frac{x_1+x_2+x_3}{3}\right), y = \left(\frac{y_1+y_2+y_3}{3}\right), z = \left(\frac{z_1+z_2+z_3}{3}\right)$$

$$x = \frac{-2u}{3}, y = \frac{-2v}{3}, z = \frac{-2w}{3}$$

$$\boxed{\frac{-3}{2}x = u}, \boxed{-\frac{3}{2}y = v}, \boxed{-\frac{3}{2}z = w}$$

subs ② $\Rightarrow k^2 = u^2+v^2+w^2$

$$k^2 = \left(-\frac{3}{2}x\right)^2 + \left(-\frac{3}{2}y\right)^2 + \left(-\frac{3}{2}z\right)^2$$

$$k^2 = \frac{9}{4}x^2 + \frac{9}{4}y^2 + \frac{9}{4}z^2$$

$$4k^2 = 9(x^2+y^2+z^2)$$

① A plane passes through the fixed point (a,b,c) cuts co-ordinate axis in the point (A,B,C). show that the locus of the centre of the sphere OABC is

$$\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$$

Soln:

Let the equation of the plane $lx+my+nz = p \rightarrow (*)$

passing through the point (a,b,c)

$$(*) \Rightarrow la+mb+nc = p \rightarrow ①$$

x-axis	y-axis	z-axis
$lx = p$	$my = p$	$nz = p$
$x = p/l$	$y = p/m$	$z = p/n$

$\therefore$ The point $(p/l, 0, 0), (0, p/m, 0), (0, 0, p/n)$

equation of the sphere

$$x^2+y^2+z^2+2ux+2vy+2wz+d = 0$$

centre of the sphere,

$2u = p/l \Rightarrow \boxed{u = \frac{p}{2l}}$

comparing "y"

$2v = p/m \Rightarrow \boxed{v = p/2m}$

comparing "z"

$2w = p/n$

$\boxed{w = p/2n}$

$\Rightarrow x = \frac{p}{2l}, \; y = \frac{p}{2m}, \; z = p/2n$

$l = \frac{p}{2x}, \; m = \frac{p}{2y}, \; n = \frac{p}{2z}$

sub in ①

$\frac{pa}{2x} + \frac{pb}{2y} + \frac{pc}{2z} = p$

$p/2 \, (a/x + b/y + c/z) = p$

$\frac{a}{x} + \frac{b}{y} + c/z = 2$

Hence Proved.

⑤ find the eqn of the sphere through the circle $x^2+y^2+z^2=9$, $2x+3y+4z=5$ and the point $(1,2,3)$

Soln:

$u + k.v = 0$

$u = x^2+y^2+y^2-9$

$v = 2x+3y+4z-5$

$(x^2+y^2+z^2-9) + k(2x+3y+4z-5) = 0$ —①

$x^2+y^2+z^2-9+k(2(1)+3(2)+4(3)-5) = 0$

$x^2+y^2+z^2-9+k(2+6+12-5)=0$

$1+4+9-9+k(15)=0$

$5+k(15)=0$

$15k=-5$

$\boxed{k=-1/3}$

$k=-1/3$ in eqn ①

$(x^2+y^2+z^2-9)-1/3(2x+3y+4z-5)=0$

$3x^2+3y^2+3z^2-27-2x-3y-4z+5=0$

$3x^2+3y^2+3z^2-2x-3y-4z-22=0$

6) find the Polar line of $\frac{x-1}{2}, \frac{y-2}{3}, \frac{z-3}{4}$ with respect to the sphere $x^2+y^2+z^2=16$

Soln:

$\frac{x-1}{2}, \frac{y-2}{3}, \frac{z-3}{4}=r$, $d=16$

$\frac{x-1}{2}=r$	$\frac{y-2}{3}=r$	$\frac{z-3}{4}=r$
$x-1=2r$	$y-2=3r$	$z-3=4r$
$\boxed{x_1=2r+1}$	$\boxed{y_1=3r+2}$	$\boxed{z_1=4r+3}$

$\Rightarrow xx_1+yy_1+zz_1=d$

$x(2r+1)+y(3r+2)+z(4r+3)=16$

$2rx+x+3ry+2y+4rz+3z-16=0$

$(x+2y+3z-16)+r(2x+3y+4z)=0$

Polar lines

$x+2y+3z-16 \Leftrightarrow 2x+3y+4z=0$

III 10 Marks

1. find the equ of the sphere through the 4 Points $(2,3,1)$, $(5,-1,2)$, $(4,3,-1)$ and $(2,5,3)$

Soln:

Equa of the sphere

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0$$

$\Rightarrow (2,3,1)$

$$4+9+1+4u+6v+2w+d=0$$

$$4u+6v+2w+14+d=0$$

$$4u+6v+2w+d=-14 \rightarrow (1)$$

$\Rightarrow (5,-1,2)$

$$25+1+4+10u-2v+4w+d=0$$

$$10u-2v+4w+d=-30 \rightarrow (2)$$

$\Rightarrow (4,3,-1)$

$$16+9+2+8u+6v-2w+d=0$$

$$8u+6v-2w+d=-26 \rightarrow (3)$$

$\Rightarrow (2,5,3)$

$$4+25+9+4u+10v+6w+d=0$$

$$4u+10v+6w+d=-38 \rightarrow (4)$$

solve (1) & (3)

$$4u+6v+2w+d=-14$$

$$\underline{\underset{-}{8u}\ \underset{-}{+6v}\ \underset{+}{-2w}\ \underset{-}{+d} = \underset{+}{-26}}$$

$$-4u+4w=12$$

$$-4(u-w)=12$$

$$-u+w=12/4$$

$$-u+w=3 \rightarrow (5)$$

Solve (1) & (2)

$$4u+6v+2w+d=-14$$
$$10u-2v+4w+d=-30$$
$$- \quad + \quad - \quad - \qquad +$$

$$-6u+8v-2w=16$$
$$-6u+8v-2w=16$$
$$2(-3u+4v-w)=16$$
$$-3u+4v-w=8 \rightarrow (6)$$

Solve (3) & (4)

$$8u+6v-2w+d=-26$$
$$4u+10v+6w+d=-38$$
$$- \quad - \quad - \quad - \qquad +$$

$$4u-4v-8w=12$$
$$4(u-v-2w)=12$$
$$u-v-2w=3 \rightarrow (7)$$

Solve (6) & (7)

$$-3u+4v-w=8$$
$$4u-4v-8w=12$$

$$u-9w=20 \rightarrow (8)$$

Solve (5) & (8)

$$-u+w=3$$
$$u-9w=20$$

$$-8w=23$$
$$\boxed{w=-\frac{23}{8}}$$

$w=-\frac{23}{8}$ in equ (5)

$$-u+w=3$$
$$-u-\frac{23}{8}=3$$

$$-u = 3 + \frac{23}{8}$$

$$-u = \frac{24+23}{8}$$

$$-u = \frac{47}{8}$$

$$u = -\frac{47}{8}$$

sub $u = -\frac{47}{8}$, $w = -\frac{23}{8}$ in equ (7)

$$u - v - 2w = 3$$

$$-\frac{47}{8} - v - \cancel{2}\left(\frac{-23}{\cancel{8}4}\right) = 3$$

$$-\frac{47}{8} + \frac{23}{4} - v = 3$$

$$\frac{-47+46}{8} - v = 3$$

$$\frac{-1}{8} - v = 3 \Rightarrow -v = 3 + 1/8$$

$$-v = \frac{24+1}{8} \Rightarrow -v = \frac{25}{8}$$

$$\boxed{v = -\frac{25}{8}}$$

$u = \frac{-47}{8}$, $v = \frac{-25}{8}$, $w = \frac{-23}{8}$ in equ (1)

$$4u + 6v + 2w + d = -14$$

$$\cancel{4}\left(\frac{-47}{\cancel{8}_2}\right) + \cancel{6}^3\left(\frac{-25}{\cancel{8}_4}\right) + \cancel{2}\left(-\frac{23}{\cancel{8}_4}\right) + d = -14$$

$$\frac{-47}{2} - \frac{75}{4} - \frac{23}{4} + d = -14$$

$$\frac{-94-75-23}{4} + d = -14$$

$$\frac{-192}{4} + d = -14 \Rightarrow d = -14 + \frac{192}{4}$$

(31)

$d = -14 + \frac{192}{4}$

$\boxed{d = 34}$

Equ of the sphere

$x^2+y^2+z^2+2ux+2vy+2wz+d=0$

$4x^2+4y^2+4z^2-47x-25y-23z+136=0$

(2) find the equation of the sphere (0,0,0) (0,1,-1) (-1,2,0) (1,2,3)

Soln:

equation of the sphere

$x^2+y^2+z^2+2ux+2vy+2wz+d=0$

(0,0,0)

$0+0+d=0$

$d=0 \rightarrow (1)$

(0,1,-1)

$0+1+1+0+2v-2w=0$

$2v-2w=-2 \rightarrow (2)$

(-1,2,0)

$1+4+0-2u+4v+0=0$

$-2u+4v=-5 \rightarrow (3)$

(1,2,3)

$1+4+9+2u+4v+6w=0$

$2u+4v+6w=-14 \rightarrow (4)$

Solve (3) & (4)

$-2u+4v=-5$

$\cancel{-2u}+4v+6w=-14$

$8v+6w=-19 \rightarrow (5)$

Solve (2) & (5) $8v-8w=-2$

(2)×4 ⇒ $8v+6w=-19$

$-14w=11 \Rightarrow \boxed{w=-11/14}$

Sub $w = -11/14$ in equ (3)

$$2v - 2w = -2$$

$$2v - 2(-11/14) = -2$$

$$2v + \frac{22}{14} = -2 \Rightarrow 2v = -2 - \frac{22}{14}$$

$$2v = -\frac{50}{14}$$

$$\boxed{v = -\frac{25}{14}}$$

Sub $w = -\frac{11}{14}$, $v = -25/14$ equ (1)

$$2u + 4v + 6w = -14$$

$$2u + 4\left(\frac{-25}{14}\right) + 6\left(\frac{-11}{14}\right) = -14$$

$$2u - \frac{100 - 66}{14} = -14$$

$$2u = \frac{-30}{14}$$

$$\boxed{u = -15/14}$$

Equ of the sphere

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0$$

$$x^2+y^2+z^2 - \frac{15}{7}x - \frac{25}{7}y - 11/7\, z = 0$$

$$7x^2+7y^2+7z^2 - 15x - 25y - 11z = 0$$

Find the equation of the sphere having the circul $x^2+y^2+z^2-2x+4y-6z+7=0$, $2x-y+2z=5$ for a great circle.

Soln:-

$$u + kv = 0$$

where,

$$(x^2+y^2+z^2-2x+4y-6z+7)+k(2x-y+2z-5)=0$$

$x^2+y^2+z^2+x(2k-2)+y(4-k)+z(2k-6)-5k+7=0$

equ of the sphere

$x^2+y^2+z^2+2ux+2vy+2wz+d=0$

$2u = 2k-2$ | $2v = 4-k$ | $2w = 2k-6$

$u = \frac{2(k-1)}{2}$ | $v = \frac{4-k}{2}$ | $w = \frac{2k-6)}{2}$

$u = \frac{2k-2}{2}$ | | $w = \frac{2k-6}{2}$

centre $(-u, -v, -w)$

$= \left(\frac{-2k+2}{2}, \frac{-k-4}{2}, \frac{-2k+6}{2} \right)$

for a great circle

$2x-y+2z-5=0$

$2\left(\frac{-2k+2}{2}\right)-\left(\frac{k-4}{2}\right)+2\left(\frac{-2k+6}{2}\right)-5=0$

$-2k+2-\frac{k+4}{2}-2k+6-5=0$

$\frac{-4k+4-k+4-4k+12-10}{2}=0$

$\frac{-9k+10}{2}=0$

$-9k+10=2 \Rightarrow -9k=-10$

$\boxed{k=10/9}$

$(x^2+y^2+z^2-2x+4y-6z+7)+10/9(2x-y+2z-5)=0$

$9x^2+9y^2+9z^2+2x+26y-34z+13=0$

S.T the two circles $x^2+y^2+z^2-y+2z=0$, $x-y+z-2=0$, $x^2+y^2+z^2+x-3y+z-5=0$, $2x-y+4z-1=0$, and lie on the same sphere and find its equation

Sol:

$u+kv=0$

$(x^2+y^2+z^2-y+2z)+k(x-y+z-2)=0$

$x^2+y^2+z^2-y+2k+kx-ky+kz-2k=0$

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$x^2+y^2+z^2+1x+3y+x+3$

$x^2+y^2+z^2+kx-y(1+k)+z(2+k)-2k=0 \rightarrow (1)$

$u+k_1v=0$

$(x^2+y^2+z^2+x-3y+z-5)+k_1(2x-y+4z-1)=0$

$x^2+y^2+z^2+x(1+2k_1)-y(3+k_1)+z(1+4k_1)-5-k_1=0 \rightarrow (2)$

x - co

$k=1+2k_1$

$k-2k_1-1=0 \rightarrow (3)$

y - co

$1+k=3+k_1$

$k-k_1-2=0 \rightarrow (4)$

z - co

$2+k=1+4k_1$

$k-4k_1+1=0 \rightarrow (5)$

Solve (3) & (4)

$$\cancel{k}-2k_1-1=0$$
$$\cancel{k}-k_1-2=0$$
$$-\quad+\quad+$$
$$-k_1+1=0$$
$$\boxed{k_1=1}$$

$k_1=1$ in sub (3)

$k-2k_1-1=0$

$k-2-1=0$

$\boxed{k=3}$

$\boxed{k=3}$ in sub equ (1)

$x^2+y^2+z^2+(3)x-y(4)+z(5)-2(3)=0$

$x^2+y^2+z^2+3x-4y+5z-6=0$

$\boxed{k_1=1}$ in sub equ (1)

$x^2+y^2+z^2+3x-4y+5z-6=0$

$\therefore$ The two sphere lie on the plane is same

(5) find the equ of the sphere, passes through the circle, $x^2+y^2+z^2-2x-4y=0$, $x^2+2y+3z=8$, and touches the plane, $4x+3y=25$

Soln:

$$u = x^2+y^2+z^2-2x-4y$$

$$v = x^2+2y+3z-8$$

$$u+kv=0$$

$$x^2+y^2+z^2+x(k-2)+y(2k-4)+3kz-8k=0. \rightarrow (1)$$

Centre of the sphere

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0,$$

Centre $(-u,-v,-w)$

$$2u=k-2 \quad\Big|\quad 2v=2k-4 \quad\Big|\quad 2w=3k$$

$$u=\frac{k-2}{2} \quad\Big|\quad v=\frac{2k-4}{2} \quad\Big|\quad w=\frac{3k}{2}$$

$$\text{centre } (-u,-v,-w) = \left(\frac{2-k}{2}, \frac{4-2k}{2}, \frac{-3k}{2}\right)$$

$$d=-8k$$

$$\text{radius} = \sqrt{u^2+v^2+w^2-d}$$

$$= \sqrt{\left(\frac{2-k}{2}\right)^2+\left(\frac{4-2k}{2}\right)^2+\left(\frac{-3k}{2}\right)^2+8k}$$

$$= \sqrt{\frac{4-4k+k^2}{4}+\frac{16-16k+4k^2}{4}+\frac{9k^2}{4}+8k}$$

$$= \sqrt{\frac{14k^2-20k+32k+20}{4}}$$

$$r = \sqrt{\frac{\cancel{2}(7k^2+6k+10)}{\cancel{2}(2)}}$$

$$= \left(\frac{7k^2+6k+10}{2}\right)^{1/2}$$

sphere touches the plane $4x+3y=25$; $\perp^r$ distance from the sphere to the plane, is equal to the radius of sphere.

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$$\frac{ax+by+cz+d}{\sqrt{a^2+b^2+c^2}} = \text{radius of the sphere}$$

$$\frac{4x+3y-25}{\sqrt{4^2+3^2}} = \left(\frac{7k^2+6k+10}{2}\right)^{1/2}$$

centre $\left(\frac{2-k}{2}, \frac{4-2k}{2}, \frac{-3k}{2}\right)$

$$\Rightarrow \frac{4\left(\frac{2-k}{2}\right)+3\left(\frac{4-2k}{2}\right)-25}{\sqrt{25}} = \left\{\left(\frac{7k^2+6k+10}{2}\right)^{1/2}\right\}$$

$$\frac{-10k-30}{10} = \left\{\frac{7k^2+6k+10}{2}\right\}^{1/2}$$

$$(-(k+3))^2 = \frac{7k^2+6k+10}{2}$$

$$(k+3)^2 = \frac{7k^2+6k+10}{2}$$

$$k^2+6k+9 = \frac{7k^2+6k+10}{2}$$

$$2k^2+12k+18 = 7k^2+6k+10$$

$$-5k^2+6k+8 = 0$$

$$5k^2-6k-8 = 0$$

$$(k-2)(5k+4) = 0$$

$$k=2, \quad k=-4/5$$

$k=2$ in equal ①

$$x^2+y^2+z^2+x(2-2)+y(4-4)+6z-8(2)=0$$

$$x^2+y^2+z^2+6z-16=0$$

$k=-4/5$ in equal ①

$$x^2+y^2+z^2+x(-4/2-2)+y(-4/2-4)+6(-4/5)z-8(-4/5)$$

$\Rightarrow x^2+y^2+z^2+x\left(-\frac{19}{5}\right)+y\left(-\frac{24}{5}\right)-\frac{12}{5}z+\frac{32}{5}=0$ (59)

$\Rightarrow 5x^2+5y^2+5z^2-19x-24y-12z+32=0$

(6) find the equ of the sphere through the circle $x^2+y^2+z^2=1$, $2x+4y+5z=6$ and touching the plane $z=0$

Soln:

$u+kv=0$

$x^2+y^2+z^2-1+2kx+4ky+5kz-6k=0$

$x^2+y^2+z^2+2kx+4ky+5kz-6k-1=0 \rightarrow (1)$

equ of the sphere

$x^2+y^2+z^2+2ux+2vy+2wz+d=0$

x - co	y - co	z - co
$2u=2k$	$2v=4k$	$2w=5k$
$\boxed{u=k}$	$\boxed{v=2k}$	$\boxed{w=\frac{5k}{2}}$

centre $(-u,-v,-w)=\left(-k,-2k,-\frac{5k}{2}\right)$

$d=-6k$

radius $=\sqrt{u^2+v^2+w^2-d}$

$=\sqrt{k^2+4k^2+\frac{25k^2}{4}+6k}$

$=\sqrt{\frac{4k^2+16k^2+25k^2+24k}{4}}$

$=\left(\frac{45k^2+24k}{4}\right)^{1/2}$

sphere touch to the plane $z=0$

$z=0$, $0=\left(\frac{45k^2+24k}{4}\right)^{1/2} \Rightarrow 0^2=\frac{45k^2+24k}{4}$

$0=45k^2+24k$

$$k(15k+24) = 0$$

$$15k+24 = 0$$

$$15k = -24$$

$$k = \frac{-24}{45} \Rightarrow \boxed{k = -\frac{8}{15}}$$

The sphere

$$x^2+y^2+z^2-\frac{16}{15}x-\frac{32}{15}y-\frac{40}{15}z-6k-1=0$$

$$\frac{15x^2+15y^2+15z^2-16x-32y-40z-48-15}{15}=0$$

$$15x^2+15y^2+15z^2-16x-32y-40z-63=0$$

Find the equ of the two tangent plane to the sphere $x^2+y^2+z^2=9$ which passes through $x+y=6$, $x-2z=3$

Soln:

Given sphere

$$x^2+y^2+z^2=9$$

$$x^2+y^2+z^2=3^2$$

$$\boxed{r=3}$$

$$(x+y-6)+\lambda(x-2z-3)=0 \rightarrow (1)$$

$$x+y-6+\lambda x-2\lambda z-3\lambda=0$$

$$x(1+\lambda)+y-2\lambda z-3\lambda-6=0 \rightarrow (2)$$

$$\frac{d}{\sqrt{a^2+b^2+c^2}}=r^2$$

$$\frac{-6-3\lambda}{\sqrt{(1+\lambda)^2+1^2+(-2\lambda)^2}}^2=3$$

$$\Rightarrow \frac{-6-3\lambda}{\sqrt{5\lambda^2+2\lambda+2}} = 3$$

$$-6-3\lambda = 3\sqrt{5\lambda^2+2\lambda+2}$$

Squaring on both

$$(-6-3\lambda)^2 = 9\sqrt{(5\lambda^2+2\lambda+2)}$$

$$(6+3\lambda)^2 = 9(5\lambda^2+2\lambda+2)$$

$$= 45\lambda^2+18\lambda+18-36\lambda+9\lambda^2-36$$

$$\Rightarrow 36\lambda - 18\lambda - 18 = 0$$

$$2\lambda^2-\lambda-1=0$$

$$(2\lambda+1)(\lambda-1)=0$$

$2\lambda = -1$ | $\lambda - 1 = 0$

$\boxed{\lambda = -1/2}$ | $\boxed{\lambda = 1}$

$\lambda = -1/2$ equ (2)

$$x(1+\lambda)+y-2\lambda z-3\lambda-6=0$$

$$x(1/2)+y+z+3/2-6=0$$

$$\frac{x}{2}+y+z+\frac{3}{2}-6=0$$

$$\frac{x+2y+2z+3-12}{2}=0$$

$$\boxed{x+2y+2z-15=0}$$

$\lambda=1$ in equ (2)

$$x(2)+y-2z-3-6=0$$

$$\boxed{2x+y-2z-9=0}$$

(38)

Q. 17 The sphere $x^2+y^2+z^2=64$, and $x^2+y^2+z^2-12x+4y-6z+48=0$. Touch intinally and find the Paint of contanct.

Soln:

Radius of the first sphere $\Rightarrow r^2=64$

$$\boxed{r=8}$$

$$\text{radius} = \sqrt{36+4+9-48} = \sqrt{49-48} = 1$$

difference of radius $= 8-1 = 7$

centre of the first sphere $= (-u,-v,-w) = (0,0,0)$

centre of the second sphere $= (-u,-v,-w) = (6,-2,3)$

Let (α,β,γ) paint to contanct

$$-6\alpha+2\beta-3\gamma=-64K$$

$$\frac{\alpha-6}{\alpha} = \frac{\beta+2}{\beta} = \frac{\gamma-3}{\gamma} = K$$

$$\frac{\alpha-6}{\alpha}=K$$

$$\alpha-6=\alpha K$$

$$-6=\alpha(K-1)$$

$$-6=-\alpha(1-K)$$

$$\frac{6}{1-K}=\alpha$$

$$\frac{\beta+2}{\beta}=K$$

$$\beta+2=\beta K$$

$$2=\beta(K-1)$$

$$2=-\beta(1-K)$$

$$\frac{-2}{1-K}=\beta$$

$$\frac{\gamma-3}{\gamma}=K$$

$$\gamma-3=\gamma K$$

$$-3=\gamma(K-1)$$

$$-3=-\gamma(1-K)$$

$$\frac{3}{1-K}=\gamma$$

Sub in equ 1

$$-6\alpha+2\beta-3\gamma+48=-64K$$

$$-6\left(\frac{6}{1-K}\right)+2\left(\frac{-2}{1-K}\right)-3\left(\frac{3}{1-K}\right)+48=-64K$$

$$\frac{-36}{1-K}-\frac{4}{1-K}-\frac{9}{1-K}+48=-64K$$

Scanned with CamScanner

$-36-4-9+48(1-k) = -64k(1-k)$

$+1-48k = 64k(k-1)$

$+1-48k-64k^2+64=0$

$-64k^2+8k+8k-1=0$

$8k(8k-1)-1(8k-1)=0$

$(8k-1)=0,\ 8k-1=0$

$\boxed{k=1/8},\ \boxed{k=1/8}$

$$= \left(\frac{6}{1-1/8}, \frac{-2}{1-1/8}, \frac{3}{1-1/8}\right)$$

$$= \left(\frac{48}{7}, \frac{-16}{7}, \frac{24}{7}\right)$$

$$\alpha = \frac{48}{7},\ \beta = \frac{-16}{7},\ \gamma = \frac{24}{7}$$

---

3) find the limilling point of the co-oxal system defined by the spheres $x^2+y^2+z^2+3x-3y+6=0$, $x^2+y^2+z^2-6y-6z-6=0$

Soln: The equ of the plane of the circle throught the two given sphere

$(u_1-u_2)+(v_1-v_2)+(w_1-w_2)=0$

$(3x-0)+(-3y+6y)+(0+6z)=0$

$3x+3y+6z=0$

$x+y+2z=0$

Then the equa of the co-onal system

$(x^2+y^2+z^2+3x-3y+6)+\lambda(x+y+2z)=0 \rightarrow (1)$

$x^2+y^2+z^2+x(3+\lambda)-y(3-\lambda)+2\lambda z+6=0$ (40)

u-axis

$2u=3+\lambda$

$\boxed{u=\frac{3+\lambda}{2}}$

v-axis

$2v=3-\lambda$

$\boxed{v=\frac{3-\lambda}{2}}$

w-axis

$2w=2\lambda$

$\boxed{w=\lambda}$

Centre $(-u,-v,-w)=\left(\frac{\lambda-3}{2},\frac{\lambda-3}{2},-\lambda\right)$

$\boxed{d=6}$

Radius $=\sqrt{u^2+v^2+w^2-d}$

$$0=\left[\left(\frac{3+\lambda}{2}\right)^2+\left(\frac{3-\lambda}{2}\right)^2+\lambda^2-6\right]^{1/2}$$

$$=\frac{9+6\lambda+\lambda^2+\lambda^2-6\lambda+9+4\lambda^2-24}{4}$$

$$0=6\lambda^2+18-24$$

$$0=6\lambda^2-6$$

$$6\lambda^2=6$$

$$\lambda^2=1$$

$$\boxed{\lambda=\pm1}$$

$\boxed{\lambda=1}$

$u=\frac{3+\lambda}{2}$

$\boxed{u=2}$

$v=\frac{3-\lambda}{2}$

$\boxed{v=1}$

$w=\lambda$

$\boxed{w=1}$

$\boxed{\lambda=-1}$

$u=\frac{3-1}{2}$

$\boxed{u=1}$

$v=\frac{3+1}{2}$

$\boxed{v=2}$

$w=-1$

$\boxed{w=-1}$

unit - 5

5 mark

① condition for the plane $lx+my+nz=0$ touch the quadratic cone $ax^2+by^2+cz^2+2fyz+2gzx+2hxy=0$

Soln:

Let (x_1, y_1, z_1) be the point of contact the tangent plane at (x_1, y_1, z_1)

$x(ax_1+hy_1+gz_1)+y(hx_1+by_1+fz_1)+z(gx_1+fy_1+cz_1)=0$

This is identical with the plane $lx+my+nz=0$

$$\frac{ax_1+hy_1+gz_1}{l} = \frac{hx_1+by_1+fz_1}{m} = \frac{gx_1+fy_1+cz_1}{n}$$

If each ratio is $\left|\frac{ax_1+hy_1+gz_1}{l}\right| = k$

$$ax_1+hy_1+gz_1-kl=0$$

$$\Rightarrow \frac{hx_1+by_1+fz_1}{m} = k$$

$$hx_1+by_1+fz_1-km=0$$

$$\Rightarrow \frac{gx_1+fy_1+cz_1}{n} = k$$

$$gx_1+fy_1+cz_1-kn=0$$

We get $\Rightarrow \begin{vmatrix} a & h & g & l \\ h & b & f & m \\ g & f & c & n \\ l & m & n & 0 \end{vmatrix} = 0$

We got $\Rightarrow lx_1+my_1+nz_1+0=0$

Simplification:-

$$Al^2+Bm^2+Cn^2+2Fmn+2Gln+2Hlm=0$$

where, A, B, C, H, F, G are the co-factor of a, b, c, f

g, h in the determinal $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$

$\Rightarrow \Delta x_1 = K(Al + Hm + Gn)$

$\Rightarrow \Delta y_1 = K(Hl + Bm + Fn)$

$\Rightarrow \Delta z_1 = K(Gl + Fm + Cn)$

The Point of the contact is given by

$$\frac{x_1}{Al + Hm + Gn} = \frac{y_1}{Hl + Bm + Fn} = \frac{z_1}{Gl + Fm + Cn}$$

is $\perp^r$ to the plane

$$\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$$

$lx + my + nz = 0$

$Ax^2 + By^2 + Cz^2 + 2Fyz + 2Gzx + 2Hxy = 0$

The determinal

$$\Delta = \begin{vmatrix} A & H & G \\ H & B & F \\ G & F & C \end{vmatrix}$$

$A' = BC - F^2 = a\Delta'$

$B' = CA - G^2 = b\Delta'$

$C' = AB - H^2 = c\Delta'$

$F' = GH - AF = f\Delta'$

$G' = HF - BG = g\Delta'$

$H' = FG - CH = h'$

$\perp^r$ to the length plane to the cone

$A'x^2 + B'y^2 + C'z^2 + 2F'yz + 2G'zx + 2H'xy = 0$

(i,e) $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$

The cone ① & ② ③ are said to be reciprodca