

I BSC. PHYSICS

Physics

ODE, PDE, LAPLACE TRANSFORM AND VECTOR ANALYSIS

SECTION - A = p + q solve

1. Solve $p^2 - 3p + 2 = 0$
2. Solve $y'' + p + p^2$
3. solve $\frac{d^2y}{dx^2} + 5 \frac{dy}{dx} + 4y = 0$
4. solve $D^2 - 3D + 5y = 0$
5. what is Clairaut's equation
6. solve $(D^2 + 5D + 6)y = e^{2x}$
7. solve $y = (x-a)p - p^2$
8. solve $(D^2 + 4D + 4)y = \cos x$
9. Eliminating arbitrary constant a and b form the equation $z = ax + by + a + b$
10. solve $z p + x = 0$
11. find the complete integral of the equation $\sqrt{p} + \sqrt{q} = \sqrt{x}$
12. Eliminate 'f' from $z = f(x+y)$

- 13. solve Lagrange's linear equation method of grouping $p z x + q z dy = r z$
- 14. solve $p + q = r$
- 15. Form the partial differential equation by eliminating arbitrary constants a and b from $z = axe^y + \frac{1}{2}a^2 e^{2y} + b$
- 16. solve $pe^y = qe^x$
- 17. Prove that $L(\cos at) = \frac{s}{s^2 + a^2}$
- 18. Evaluate $L(t^2 + 2t + 3)$
- 19. Find $L(\cosh at)$
- 20. find $L(t^2 + 2t + 3)$
- 21. $L(\cos at) = \frac{1}{a} f\left(\frac{s}{a}\right)$ where $L(\cos t) = \frac{s}{s^2 + 1}$
- 22. find $L(e^{-at})$
- 23. find $L(\sin^2 2t)$
- 24. $L\{f'(t)\} = s L\{f(t)\} - f(0)$
- 25. Evaluate $\int_0^\infty \frac{e^{-t} - e^{-2t}}{t} dt$
- 26. find $L(t \cos^2 t)$
- 27. find $L\{f(t)\}$ where $f(t) = 0$, where $0 < t \leq 2$ and $f(t) = 3$ where $t > 2$
- 28. find $L\{t^2 e^{3t}\}$

29. Find $L^{-1} \left(\frac{s}{s^2 + 2s + 5} \right)$

30. Find $L^{-1} \left\{ \log \left(\frac{s+1}{s} \right) \right\}$

31. Find $L^{-1} \left(\frac{s}{(s+3)^2 + 4} \right)$

32. Find $L^{-1} \left(\frac{1}{(s-3)^5} \right)$

33. Find $L^{-1} \left(\frac{2s-1}{s^2(s-1)^2} \right)$

34. Find $L^{-1} \left(\frac{s+3}{(s^2+6s+13)^2} \right)$

35. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ prove that $\nabla \times \frac{\vec{r}}{r} = 0$

36. If $\phi = (x^2 + y + z - 1)$ find $\text{grad } \phi$ at $(1, 0, 0)$

37. $\vec{F} = 2xy\vec{i} + (x^2 + 2yz)\vec{j} + (y^2 + 1)\vec{k}$ is solenoidal (or) non-solenoidal

38. If $\phi = x^3 + y^3 + z^3 - 3xyz$ find $\text{div}(\text{grad } \phi)$

39. Show that $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ is irrotational

40. If $\vec{f} = (x^2 - y^2)\vec{i} + 2xy\vec{j} + (y^2 - 2xy)\vec{k}$

find $\text{div } \vec{f}$

41. Prove that $\text{Curl}(r^n \vec{r}) = 0$

42. Define solenoidal vector

5 mark

1. solve $p^3 - 7p - 6 = 0$

2. solve $(D^2 - 2D + 2)y = e^x \sin x$

3. solve $(D^3 - D^2 - D + 1)y = 1 + x^2$

4. solve $(D^2 - 4D - 5)y = e^{-3x} + 3 \cos 4x$

5. solve $x^2 p^2 + 3xy p + 2y^2 = 0$

6. solve $xy p^2 + p(3x^2 - 2y^2) - 6xy = 0$

7. solve $y^2 p - xy q = x(z - 2y)$

8. Eliminate the arbitrary constant a and b

$(x+a)^2 + (y-b)^2 = z^2 \cot^2 \alpha$

9. solve $p \cot x + q - \cot y = z$

10. solve $p \tan x + q \tan y = z \tan z$

11. find $p x + q y = z$

12. find (a) $L[e^{-at} f(t)] = F(s+a)$ where

$F(s) = L\{f(t)\}$

(b) Evaluate $\int_0^\infty t \cdot e^{-3t} \sin t dt$

13. solve $L(\sin^3 2t)$

14. If $L\{f(t)\} = f(s)$

(a) $\lim_{t \rightarrow 0} f(t) = \lim_{s \rightarrow \infty} s f(s)$

(b)

⑥ $\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} s \cdot F(s)$

15. find $L^{-1}(\sin t \cdot e^{-t} \cdot t)$

16. Evaluate $L(t \sin at)$

17. Evaluate $\int_0^{\infty} \frac{e^{-t} \sin t}{t} dt$

18. find $L(t^n) = \frac{n!}{s^{n+1}}$

19. find (a) $L(e^{at})$ (b) $L(\cos(at))$

20. find $L^{-1}\left(\frac{s}{(s^2+a^2)^2}\right)$

21. find $L^{-1}\left(\frac{1}{s(s+1)(s+2)}\right)$

22. find $L^{-1}\left(\frac{s+2}{(s^2+4s+5)^2}\right)$

23. find $L^{-1}\left(\frac{s}{(s^2+4)(s^2+1)}\right)$

24. find $L^{-1}\left(\frac{s}{(s-1)^4}\right)$

25. Evaluate $L^{-1}\left(\frac{1}{(s+1)(s^2+1)}\right)$

26. Evaluate $L^{-1}\left(\frac{s^2-s+2}{s(s-3)(s+2)}\right)$

27. find $L^{-1}\left(\frac{s}{s^2+2s+5}\right)$

28. Find the directional derivative of $\phi = x^2 y z + 4 x z^2$ at $(1, 1, 1)$ direction of $\vec{i} + \vec{j} + \vec{k}$
29. Find the unit normal vector to the surface $x^2 + y^2 + z^2 = 2$ at the point $(1, -1, 2)$
30. Find $f = x^2 z \vec{i} - 2y^3 z^2 \vec{j} + xy^2 z \vec{k}$ divergences and curl f at the point $(1, -1, 1)$
31. If \vec{A}, \vec{B} are irrotational then prove that $\vec{A} \times \vec{B}$ is solenoidal.
32. If $\vec{F} = 3xy \vec{i} - y^2 \vec{j}$ and C is a straight line joining $(0, 0, 0)$ to $(1, 1, 1)$ find $\int_C \vec{F} \cdot d\vec{r}$
33. Using stoke theorem prove that $\int_C \vec{r} \cdot d\vec{r} = 0$ where $\vec{r} = x \vec{i} + y \vec{j} + z \vec{k}$
34. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ if $f = x^2 \vec{i} - xy \vec{j}$ where C is the curve $y = x^2$ from $(0, 0)$ to $(1, 1)$
35. Find the unit normal vector to the surface $x^3 - xyz + z^2 = 7$ which ϕ at the point $(1, -1, 2)$

1. Find the ~~show~~ particular integral of the differential equation of $(D^2 + 9)y = 4 \sin 3x$
2. Solve $(D^2 + 2D + 5)y = x e^x + e^{2x}$
3. solve $y - 2px = x^2 p^4$
4. solve $y = px + \sqrt{p^2 + 1}$
5. solve $y^2 \log y = xyp + p^2$
6. solve $(D^2 - 8D + 9)y = 8 \cos 5x$
7. solve $(D^2 + 2D + 5)y = x e^x$
8. solve $(\frac{d^2 y}{dx^2} + 3\frac{dy}{dx} + 2y) = \sin 3x$
9. Evaluate $P(1+q)^2 = q(z-a)$
10. Evaluate $Pq = y$
11. find $P + q = Pq$
12. solve $(pyz + qzx = xy) (2p + 3q) = 1$
13. Find, $p - q = \log(x+y), P\sqrt{x} + q\sqrt{y} = \sqrt{z}$
14. solve $(y^2 + z^2)p - xyq + xzr = 0$
15. solve $z = px + qy + \sqrt{1 + p^2 + q^2}$
16. find (i) $L(t)$ (v) $L(\cos 3t)$
 ii) $L(t^2) = \frac{2}{s^3} + \frac{2t}{s^2} + \frac{t^2}{s}$
 iii) $L(t^{\frac{1}{2}}) = \frac{\sqrt{\pi}}{2s^{3/2}} + \frac{1}{2s^{1/2}}$
 iv) $L(t^{-\frac{1}{2}})$

17. (a) find $L\{f(t)\}$, $f(t) = (t-1)^2$ when $t > 1$, $f(t) = 0$ when $t < 1$

(b) find $L(t^n)$

18. (a) Evaluate $\int_0^{\infty} t e^{-3t} \cos t dt$

(b) Evaluate $L\left(\frac{1 - \cos t}{t}\right)$

19. (a) find $L\left(\frac{\sin 2t}{t}\right)$

(b) find $L(\sinh at) = \frac{1}{a} f\left(\frac{s}{a}\right)$ where $L(\sinh t) = \frac{1}{s^2 - 1} = F(s)$

20. Evaluate (a) $L\{f(t)\} = F(s)$ then $L\{t \cdot f(t)\} = -\frac{d}{ds} f(s)$

(b) solve $L(\cos bt)$ and $L(\sin bt)$

21. Solve the equation using Laplace transformation

$$\frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} - 3y = \sin t \text{ given that}$$

$$y = \frac{dy}{dt} = 0 \text{ when } t = 0$$

22. using Laplace transform to solve: $t^2 y'' - (2+t)y' + 3y = (t-1)$, when $y(0) = 0$

23. solve $\frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} + 13y = 2e^{-x}$, $y(0) = 0$, $y'(0) = -1$ by using Laplace transform.

24. Solve the equation $\frac{d^2y}{dt^2} + \frac{2dy}{dt} - 3y = \sin t$ given that $y = \frac{dy}{dt} = 0$ when $t=0$
25. Solve $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} - 5y = 5$ given that $y=0$, $\frac{dy}{dt} = 2$ when $t=0$
26. solve $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 4e^{-t}$ given that $y = \frac{dy}{dt} = 0$ when $t=0$.
27. solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \sin 3x$
28. Determine y which satisfies the equation $\frac{dy}{dt} + 3y + 2 = \int_0^t y dt = t$ for which $y(0) = 0$
29. Find the unit normal vector to the surface $\phi = x^2yz + 4xz^2$ at $(1, -2, 1)$ and also find directional derivative $2\vec{i} - \vec{j} - \vec{k}$
30. If $\vec{F} = (y^2 - z^2 + 3yz - 2x)\vec{i} + (3xz + 2xy)\vec{j} + (3xy - 2xz + 2z)\vec{k}$ is solenoidal irrotational