III B.sc (Mathematics)

Abstract Algebra

Part - A

1) Give an example of a Cyclic group
2) P.T any Cyclic group is Abelian.
3) Define a group Given an example.
4) Define the right coset with an example.
5) Define Permutation.
6) Define Symmetric
7) Define Disjoint.
8) Define Subgroup. Given an example
9) Define Centre of the group.
10) Define normalizer of a group.
11) Define Generator.
12) Define order of an element
13) In a finite group every elements is of finite order.
14) Define left coset.
15) Let H be a subgroup of G. We define a relation $a \sim b \Rightarrow a^{-1}b \in H$ then the relation is an equivalence relation.
16) Define index.
17) Write the statement of Lagranges theorem

18) The order of any element of a finite group G divides the order of G.

19) Let G be a group of order n. Let $a \in G$ then $a^n = e$.

20) Let A and B be subgroups of a finite group G. Such that A is a subgroup of B. S.T $[G:A] = [G:B][B:A]$

21) Define Normal subgroup

22) Define Quotient groups of G modulo n

23) The Centre H of a group G is a normal subgroup of G.

24) Define Isomorphism of a group

25) $(Z, +) \cong (2Z, +)$

26) State Cayley's theorem.

27) Define Automorphism.

28) Define Inner automorphism

29) Define Homomorphism.

30) Define Kernel K.

31) ~~Define~~ ~~fun~~ State fundamental theorem of homomorphism.

32) Any homomorphic unit of cyclic group is cyclic

33) S.T $R^* / \{1, -1\} \cong R^+$

34) State Euler's theorem.

35) Define field

36) Define Skew field.

37) Define boolean ring with example

38) Write Types of ring.

39) In a ring with identity the identity element is unique.

40) Define unit in R

41) Define a ring homomorphism.

42) If $T: V_2(R) \rightarrow V_2(R)$ is defined by $T((a,b)) = (-b,a)$ find the matrix of T with respect to the standard basis $\{e_1, e_2\}$

43) Write a basis for $V_n(R)$

44) State the following statements are true or false

~~45)~~ (a) Z/nz is Cyclic

(b) S_n/A_n is abelian group.

45) Define a linear transformation.

46) Let R be a ring and $a,b \in R$. Prove that $(-a)(-b) = ab$

<u>Part - B</u>

1) P.T a subgroup of Cyclic group is Cyclic

2) State & Prove the Euler's theorem

3) Let N be a normal Subgroup of a group G and G/N is the set of the all right Cosets of N in G P.T G/N is a group under the operators defined by $N_a \cdot N_b = N_{ab}$.

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4) P.T if a group G has exactly one subgroup H of given order, then P.T H is a normal subgroup of G

5) P.T any Centre Z(G) of a group G is a Subgroup of G.

6) Let G be a group and H be a subgroup of G, then prove the following
(i) $a \in H \Leftrightarrow aH = H$
(ii) $aH = bH \Leftrightarrow a^{-1}b \in H$.

7) Let H & K be two finite Subgroups of a group G, then P.T $|HK| = \frac{|H| \cdot |K|}{|H \cap K|}$

8) Let $f: G \to G'$ be a homomorphism. Prove the following (i) $f(e) = e'$
(ii) $f(a^{-1}) = [f(a)^{-1}]$

9) Find the linear transformation $T: V_3(R) \to V_3(R)$ determine by the matrix $\begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$ with respect to the Standard basis. $\{e_1, e_2, e_3\}$.

10) In a group the left and right Cancellation laws hold $ab = ac \Rightarrow b = c$ and $ba = ca \Rightarrow b = c$.

11) ~~Let~~ Any permutation Can be expressed as a product of disjoint Cycles.

12) Let A_n an the ~~set~~ of all even permutation in S_n. Then A_n is a group Containing $n!/2$ Permutation.

13). Let H be a subgroup of G then
(i) The identity element of H is the same as that of G.
(ii) For each $a \in H$ the inverse of a in H is same as the inverse of a in G.

14) If H and K are subgroup of a group G then $H \cap K$ is also a subgroup of G

15) Let G be a group and a be an element of order n in G then $a^m = e$ iff n divides m

16) The order of a permutation P is the L.C.M of the length its disjoint Cycle.

17) Let H be a subgroup of index two in a group G. Then H is a normal subgroup of G.

18) P.T the intersection of two normal subgroups of a groups G is a normal subgroup of G.

19) Let H be a subgroup of G. Let $a \in G$ then aHa^{-1} is a subgroup of G.

20) If H is a subgroup of G and N is a normal subgroup of G then HN is a subgroup of G.

21) Let $f: G \to G'$ be an isomorphism if G is abelian then G' is also abelian.

22) Any infinite cyclic group G is isomorphic to $(Z, +)$

23) Any finite cyclic group of order n is isomorphic to $(Z_n, \oplus)$.

24) Let $f: G \to G'$ be a homomorphism then the Kernal K of f is a normal subgroup of G.

25) Consider G be any group and H be the Centre of G then $G/_H \cong I(G)$

26) Consider $f: G \to G'$ be a homomorphism then f is one to one ~~iff~~ if and only if $Ker(f) = e$

Part - C

1) Let G be a group and $a, b \in G$ then the Equations $ax = b$ and $ya = b$ have unique solution for x and y in G.

2) Let (i) $a^m a^n = a^{m+n}$; $m, n \in Z$
   (ii) $(a^m)^n = a^{mn}$; $m, n \in Z$

3) The union of two Subgroup of Group G is a group if and only if one is contained other.

4) Let A & B be two Subgroups of a group G then AB is a subgroup of G if and only if $AB = BA$.

5) Any Subgroup of Cyclic group is Cyclic

6) Let G be a group and $a, b \in G$ then
   (i) order of a = order of a^{-1}
   (ii) Order of a = order of $b^{-1}ab$
   (iii) Order of ab = order of ba.

7) Let H be a Subgroup of G. The number of left Cosets of H is the same as the number of right cosets of H.

8) A Group G has no proper subgroup if it is a Cyclic group of Prime order.

9) Let H and K be two Subgroups of G of finite index in G. Prove that $H \cap K$ is a subgroup of finite index in G.

10) S.T If a group G has exactly one subgroup H of given order then H is a normal subgroup of G.

11) If G is a group and G' is a set of the binary Operation & there exist a one to one mapping f from G onto G' Such that $f(ab) = f(a) f(b)$ $\forall$ $a, b \in G$ then S.T G' is also a group.

12) ~~State~~ Explain Cayley's Theorem.

13) Explain Fermat's Theorem.

14) S.T $|H|$ divides $|G|$

15) Let G be a group. Then the order of a is Same as the order of Cyclic group generated by a.

16) Let G be a group and H be a Subgroup of G then (i) $a \in H \Rightarrow aH = H$

(ii) $aH = bH \Rightarrow a^{-1}b \in H$

(iii) $a \in bH \Rightarrow a^{-1} \in Hb^{-1}$

(iv) $a \in bH \Rightarrow aH = bH$

17) Let H and K be two subgroups of G of finite index in G. P.T $H \cap K$ is a subgroup of finite index in G.

18) Let N be a subgroup of G. Then the following are equivalent

(i) N is a normal subgroup of G

(ii) $aNa^{-1} = N \ \forall \ a \in G$

(iii) $aNa^{-1} \subseteq N \ \forall \ a \in G$

(iv) $ana^{-1} \in N \ \forall \ n \in N \ \& \ a \in G$.

19) Let N be a normal subgroup of G. Then G/N is a group under the operation defined by $Na \cdot Nb = Nab$

20) Isomorphism is an equivalence relation among groups.