(a) Evalute $\int_0^{\pi} \frac{d\theta}{a+\cos\theta}$ ; $a>1$

(b) State and prove Poisson's formula.

— / —

PART-A

(1) Is every ball is an open set? Justify your answer.

(2) Define a homothetic transformation

(3) When will you say arc is rectifiable?

(4) State the Cauchy's theorem in a disk.

(5) Define removable singularity with an example

(6) What is meant by meromorphic function?

(7) Find the residue of $\cot z$ at its pole.

(8) State the argument principle

(9) State the Schwarz's theorem.

(10) Obtain the series expansion for arc $\sin z$.

PART-B

11. a) Prove that on a compact set every continuous function is uniformly continuous

b) State and prove the symmetry principle

12. a) With the usual notations, prove that

$$\left|\int_a^b f(t)\,dt\right| \le \int_a^b |f(t)|\,dt$$

b) Derive Cauchy's representation formula.

13. a) Prove that an analytic function comes arbitrarily close to any complex value in every neighbourhood of an essential singularity.

b) State and prove the local mapping theorem

14. a) State and prove the general Cauchy's theorem

b) Evalute $\int_0^{\infty} \frac{\cos x\,dx}{x^2+a^2}$ (a real).

15. a) If $u_1$ and $u_2$ are harmonic in a region $\Omega$, then prove that $\int_\gamma u_1{}^* du_2 - u_2{}^* du_1 = 0$ for every cycle $\gamma$ which is homologous

Question Paper

Complex Analysis

1) State Heine Borel property
2) Define $C(\alpha)$ and prove that it is open
3) What do you mean by indirectly conformal mapping?
4) What do you say about index of a point w.r.t $\gamma$?
5) Explain essential singular point
6) State Schwartz lemma.
7) Define cross ratio.
8) Find the linear transformation which carries $(0, i, -i)$ into $(1, -1, 0)$
9) Define residue of the function $f(z)$
10) State mean value property.

Section-B.

11) a) Prove that every set has a unique decomposition into components.

b) P.T the cross ratio is real iff the four points lie on a circle or on a straight line.

12) a) State and prove Cauchy integral formula.

b) State and prove Cauchy theorem in a disk

13) a) State and prove Taylor's theorem.

b) State and prove Weierstrass theorem.

14) a) Find poles and residues of $\frac{1}{(z^2-1)^2}$.

b) State and prove argument principle.

15) a) State and prove Schwartz theorem.

b) State and prove Hurwitz theorem.

Section-C.

16) prove that a nonempty open set in the plane is connected iff any two of its points can be joined by a polygon which lies in the set

17) State and prove Cauchy theorem for rectangle.

18) State and prove Schwartz lemma.

When do you say a point $z=b$ is a pole of $f(z)$?

Define a harmonic function?

Given the reflection principle for a harmonic function.

PART-B

Show that a set is compact if it is complete and totally bounded in a metric space.

State and prove the ~~cho~~ cauchy-Reimann equations for an analytic function $f(x,y)=u(x,y)+iv(x,y)$

Show that the line integral $\int_\gamma p\,dx+q\,dy$ defined in $\Omega$ depends only on the end points if and only if there is a function $u(x,y)$ such that $\frac{\partial u}{\partial x}=p$ and $\frac{\partial u}{\partial y}=q$.

State and prove cauchy's integral formula.

Let $z_j$ be the zeros of a function $f(z)$ which is analytic in a circular disc $\Delta$ and does not vanish identically each zero being counted as many times as its order indicates. If $\gamma$ is closed curve not passing through $z_j$ in $\Delta$ that Show that $\frac{1}{2\pi i}\int_\gamma \frac{f'(z)}{f(z)}dz=\sum_j n(\gamma,z_j)$

State and prove the Taylor's theorem.

State and prove the Residue theorem

State and prove the Roche's Theorem.

If $u_1$ and $u_2$ are harmonic in a region $\Omega$ show that $\int_\gamma u_1{*}du_2-u_2{*}du_1=0$ for every cycle $\gamma$ which is homologous to zero in $\Omega$

If $u$ is harmonic show that $\frac{1}{2\pi}\int_{|z|=r} u\,d\theta=\alpha\log r+\beta$.

PART-C

16) State and prove the sufficient condition for the analytically of a function $f(x,y)=u(x,y)+iv(x,y)$.

and prove the Taylor series.

17) State and prove cauchy's Theorem for a rectangle

18) Evaluate $\int_0^{\pi} \log \sin\theta \, d\theta$.

19) State and prove the Laurent's series theorem

20) State and prove $\int_0^{\infty} \frac{\sin x}{x} = \frac{\pi}{2}$.

PART - A.

1. What is called as a region?
2. Define the closure of a set A in a space x
3. If $\gamma = e^{i\theta}$, $0 \leq \theta \leq 2\pi$ and $f(z) = \frac{1}{z}$ for $z \neq 0$ find the value of $\int_\gamma f(z)dz$.
4. When do you say an arc $z = z(t)$ is rectifiable?
5. Define a simply connected region.
6. When do you say a cycle $\gamma$ in $\Omega$ is homologous to zero in the region $\Omega$
7. Define the residue of $f(z)$ at an isolated singularity $z=c$ of order 1.
8. When do you say a point $z=b$ is a pole of $f(z)$?
9. Define a harmonic function
10. Given the reflection principle for a harmonic function.

PART - B.

10a) Show that a set is compact.

11. Evalute $\int_0^{2\pi} F(\cos\theta, \sin\theta) d\theta$.
12. State the Argument principleal.
13. prove That the arithmetic Mean of a harmonic functi over concentric circles $|z| = r$ is a linear function of
14. Define the radius of convergence.

ne The residue of $f(z)$ at an isolated singularity $z=\alpha$ of order 1

> prove that cross ratio of four distinct points in the extended complex plane is real iff they lie on a circle or a straight line

a) i) Prove that $n(\gamma, a) = n(\gamma, b)$ if a and b belong to the same region determined by $\gamma$.

ii) prove that $n(\gamma, a) = 0$ ∀ a in the unbounded region determined by $\gamma$

> Evaluate $\int_{|z-1|=a} \frac{z\,dz}{(z-1)^2(z-a)(z+6)}$ by using Cauchy's integral formula

> State and prove Schwarz lemma.

> State and prove local correspondence Theorem.

> Prove that a region $\Omega$ is said to be simply connected if every closed curve $\gamma$ in $\Omega$ is homologous to zero in $\Omega$

> Prove that if $R(x)$ is a rational function having a simple zero at infinity and real poles, then $\int_{-\infty}^{\infty} R(x)e^{ix}\,dx = 2\pi i \times$ sum of the residues of $R(z)e^{iz}$ in the upper half plane

> Derive laplace equations in polar form from cartesian form

If u is continuous at $\theta$, then prove that $\lim_{z \to e^{i\theta}} P_u(z) = U(\theta)$

PART - C.

prove that $f(z)$ is conformal at $z_0$ iff it has non-zero derivative at $z_0$.

let $f(z)$ be analytic on a disc D except at finite number of points $a_j$. If $\lim_{z \to a_j} (z-a_j)f(z) = 0$ ∀ j then $\int_\gamma f(z)\,dz = 0$ for every closed curve in D not passing through any $a_j$

Prove that if $p\,dx + q\,dy$ is locally exact, then $\int_\gamma p\,dx + q\,dy = 0$ for every cycle homologous to zero in $\Omega$

State and prove Taylor's theorem.

If $f(z)$ is an analytic function which does not vanish identically, then its zeros are isolated

If $f(z)$ is analytic in an annulus, then prove that Laurent series expansion of $f(z)$ exists.

If 'a' is an isolated singularity of $f(z)$ then prove that $\operatorname{Res}_{z=a} f(z)$ is the coefficient of $(z-a)^{-1}$ in the laurent series expansion of $f(z)$ around 'a'.

... the residue of $f(z)$ at an isolated singularity ... order 1

## PART-A

When do you say a space $X$ is compact?

Show that the set of all integers $Z$. is disconnected in R

Define the index of a closed curve $C$, with respect to a point a

When do you say a path $\gamma$ in $\Omega$ is homologous to 0 in $\Omega$

Find the pole and its order for $f(z) = \frac{1}{(z-2)^5}$.

Define a conformal mapping $f(z)$.

State the maximum principle for a harmonic functions

What is the value of the integral of an exact differential over any cycle?

When do you say a cycle $\gamma$ bound a region $\Omega$?

Define a meromorphic function in $\Omega$.

## PART-B.

Show that the image of a connected set under a continuous map is connected

Discuss the transformation $w = e^z$

Find the conjugate harmonic of the function $u = e^x(x\cos y - y s$

Evalute $\int_C \frac{1}{z^2+9}\,dz$ where $C$ is the circle $|z-9| = 3$

State and prove the Taylor series theorem

State and prove the Residue theorem.

State and prove the Rouche's theorem

Evalute $\int_C \frac{2dz}{z^4-1}$ where $C: |z| = 4$, using the method of resid

If $u_1$ and $u_2$ are harmonic in $\Omega$, show that $\int_\gamma u_1 * du_2 - u_2 * du_1 = 0$ for every cycle $\gamma$ which is homolog to zero in $\Omega$

State and prove the Morera's theorem.

## PART-C.

State and prove Cauchy's theorem for Rectangle

Fine the residue of $f(z)$ at an isolated sing[illegible]

... is a pole of $f(z)$?

to zero in $\Omega$.

b) State the prove the Weierstrass's theorem of series.

Part-c.

16a) Show that the function $f(z)$ is analytic if $\frac{\partial f}{\partial \bar{z}}=0$

b) Define a linear transformation on the complex plane. Also prove that a linear transformation, carries circles into circles.

17) State and prove Cauchy's theorem for a rectangle.

18) State and prove the Schwarz lemma.

19 a) State and prove the residue theorem.

b) State and prove Rouche's theorem.

20) Establish the poisson formula $u(a)=\frac{1}{2\pi}\int_{|z|=R}\frac{R^2-|a|^2}{|z-a|^2}u(z)d\theta$

PART-A.

1) Define a connected set

2) Define cross ratio

3) Define rectifiable arcs

4) compute $\int_{|z|=1} e^z z^{-n} dz$

5) State Taylor's theorem.

6) Show that a non constant analytic function maps open sets onto open sets.

7) If $z=a$ is a pole of order $m$ of $f(z)$. What is ~~that~~ the residue of $f(z)$ at $z=a$?

8) State Argument Principle

9) Define uniform convergence of Sequence of functions.

10) State Hurwitz theorem.

PART-B

11 a) State prove that the real line $R$ is connected

b) Suppose $f(z)$ is analytic and satisfies the condition $|f(z)^2-1|<1$ in a region $\Omega$. Show that either $Re\, f(z)>0$ or $Re\, f(z)<0$ throughout $\Omega$

Show that the differential $pdx + q\,dy$ which is defined and continuous in a simple connected region $\Omega$ is exact if and only if $\int_{\partial R} pdx + q\,dy = 0$ for every rectangle $R$ contained in $\Omega$

If $f(z)$ is meromorphic in $\Omega$ with the zeros $a_j$ and the poles $b_k$, show that $\frac{1}{2\pi i}\int_{\gamma} \frac{f'(z)}{f(z)} dz = \sum_j n(\gamma, a_j) - \sum_k n(\gamma, a_k)$

If $f(z)$ is analytic on a closed bounded set $E$, show that the maximum of $|f(z)|$ is taken on the boundary of $E$

State and prove Laurent's series theorem

Evaluate $\int_0^{\pi} \log \sin x \, dx$.

PART - A.

Prove that if $S$ is compact subset of $C$, then $S$ is bounded

Define the principle branch of $\log z$

Compute $\int_{\gamma} x\,dz$ where $\gamma$ is a directed line segment joining $(0,0)$ and $(1,1)$

Prove that $\left| \int_a^b f(t)\,dt \right| \le \int_a^b |f(t)|\,dt$, where $f$ is a complex valued continuous function on $[a,b]$.

Define isolated singularity and given an example

If algebraic over of $f(z)$ at 'a' does not exist and $f(z)$ is not analytic at 'a' what type of singularity at 'a' is for $f(z)$?

State Rouche's theorem.

Evalute $\int_{|z|=1} e^{x} z^{-n} dz$, by using residue theorem.

Write poisson integral $P_v(z)$ and prove that $P_v(z) \ge 0$ where $U \ge 0$

obtain that are $\tan z = z - \frac{z^3}{3} + \frac{z^5}{5} + \frac{z^7}{7} \cdots$

PART - B.

Define a component of a set $E \subseteq C$, and prove that each component is connected, $E$ can be written as a di[illegible] union of its components

3) a) If the piece wise differentiable closed curve $\gamma$ does not pass through the point $\alpha$. prove that the value of the Integral $\int_\gamma \frac{dz}{z-a}$ is an integral Multiple of $2\pi i$

b) State and prove the Morera's theorem.

3) a) Suppose that $f(z)$ is analytic in the region $\Omega'$ obtained by omitting a point $a$ from the region $\Omega$. Prove that a necessary and sufficient condition that there exists an analytic function in $\Omega$ which coincides with $f(z)$ in $\Omega'$ is that $\lim_{z \to a} (z-a) f(z) = 0$.

b) State and prove the Schwarz lemma.

4) a) Prove that a region $\Omega$ is simply connected if and only if $n(\gamma, \alpha) = 0$ for all cycles $\gamma$ in $\Omega$ and all points $\alpha$ which do not belong to $\Omega$

b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx$

5) a) If $u_1$ and $u_2$ are harmonic in a region $\Omega$, then prove that $\int_\gamma u_1 * du_2 - u_2 * du_1 = 0$ for every cycle $\gamma$ which is homologous to zero in $\Omega$

b) Prove that the laurent development is unique.

PART - C.

6) a) Prove : In a compact set every continuous function is uniformly continuous.

b) Find the bilinear transformation which maps $(0, i, -i)$ to $(1, -1, 0)$

b) If the function $f(z)$ is analytic on the rectangle $R$ defined by the inequalities $a \le x \le b$, $c \le y \le d$, prove that $\int_{\partial R} f(z) dz = 0$ Where $\partial R$ denote the boundary of $R$.

3) State and prove Weierstrass theorem regarding essential singularity

b) State and prove the Residue theorem.

State and prove the Taylor series.

lie on a circle (or) on a straight line.

17) State and prove cauchy's Theorem for a rectangle.

18) Define removable Singularity. State and prove the local Mapping Theorem

19 a) Show that a region $\Omega$ is simply connected if and only if $n(\gamma, a) = 0$ for all cycle $\gamma$ in $\Omega$ and all points a which do not belong to $\Omega$

b) State and cauchy's residue theorem.

20) State and prove Schwarz's Theorem.

## PART-A

1) Define a Metric Space. Given an example.

2) Define : Analytic function

3) Compute $\int_\gamma x\,dz$ where $\gamma$ is the directed line segment from 0 to $1+i$

4) State Cauchy's Integral formula

5) Define : Removable Singularity.

6) State the local mapping theorem

7) State Rouche's theorem

8) Find the residue of $f(z) = \cot z$ at each poles

9) If $u(z)$ is harmonic in $\Omega$ then prove that $\frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}$ is analytic in $\Omega$

10) Define poisson integral for u and prove that it is linear function of U.

## PART-B.

11 a) Show that every set has a unique decomposition into components.

b) If $z_1, z_2, z_3, z_4$ are distinct points in the extended plane and T any linear transformation then prove that $(Tz_1, Tz_2, Tz_3, Tz_4) = (z_1, z_2, z_3, z_4)$

Define the residue of $f(z)$ at an isolated singularity $z=\alpha$ of order 1

19) If the ... three ... by ...

17) Show that the differential $p\,dx + q\,dy$ which is defined and continuous in a simple connected region $\Omega$ is exact if and only if $\int_{\partial R} p\,dx + q\,dy = 0$ for every rectangle $R$ contained in $\Omega$

If $f(z)$ is meromorphic in $\Omega$ with the zeros $a_j$ and the poles $b_k$, show that $\frac{1}{2\pi i}\int_{\gamma} \frac{f'(z)}{f(z)} dz = \sum_j n(\gamma, a_j) - \sum_k n(\gamma, b_k)$

If $f(z)$ is analytic on a closed bounded set $E$, show that the maximum of $|f(z)|$ is taken on the boundary of $E$

State and prove Laurent's series theorem

Evaluate $\int_0^{\pi} \log \sin x \, dx$.

— x — x — x —

PART - A.

Prove that if $S$ is compact subset of $C$, then $S$ is bounded

Define the principle branch of $\log z$

Compute $\int_{\gamma} x\,dz$ where $\gamma$ is a directed line segment joining $(0,0)$ and $(1,1)$

Prove that $\left|\int_a^b f(t)\,dt\right| \leq \int_a^b |f(t)|\,dt$, where $f$ is a complex valued continuous function on $[a,b]$.

5) Define isolated singularity and given an example

6) If algebraic over of $f(z)$ at 'a' does not exist and $f(z)$ is not analytic at 'a' what type of singularity at 'a' is for $f(z)$

7) State Rouche's theorem.

8) Evalute $\int_{|z|=1} e^{x} z^{-n}\,dz$, by using residue theorem.

9) write poisson integral $P_v(z)$ and prove that $P_v(z) \geq 0$ when $U \geq 0$

10) obtain that are $\tan z = z - \frac{z^3}{3} + \frac{z^5}{5} + \frac{z^7}{7} \ldots$

PART - B.

11a) Define a component of a set $E \subseteq C$, and prove that ea[ch] component is connected, $E$ can be written as a d[isjoint] union of its components

12) a) State and prove Cauchy's integral formula.

b) State and prove Cauchy's estimate.

13) a) State and prove maximum principle

b) State and prove local mapping theorem.

14) a) State and prove Rouche's theorem.

b) Prove that an analytic function comes arbitrarily close to any complex value in every neighbourhood of an essential singularity

15) a) Discuss geometrical interpretation of Poisson's formula

b) What is the coefficient of $z^7$ in the Taylor development of $\tan z$?

PART - C

16) P.T an analytic function $f(x)$ is a constant in a region $\Omega$. If.

a) $f'(z)=0$ b) $Re f(z) =$ constant c) $Im f(z) =$ constant

d) $|f(z)| =$ constant e) $Arg f(z) =$ constant.

17) State and prove Cauchy's theorem for a rectangle.

18) If $f(z)$ is analytic in the region $\Omega$ and if $a \in \Omega$ such that $f(a)=0$ and $f^{(n)}(a)=0$ $\forall n$ than prove that $f(z)$ is identically zero.

19) Evaluate $\int_0^{\pi} \log \sin x \, dx$.

20) State and prove mean value property.

PART- A

1) Define Jordan arc.

2) Define cross ratio

3) Define winding number

4) Evalute $\int_{|z|=1} \frac{\cos z}{z(z-4)} dz$

5) Define zeros and poles of $f(z)$ at $z=\infty$

6) State Schwartz lemma

7) Define the residue of $f(z)$ at an isolated singularity $z=\alpha$ order

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... curve $\gamma$ does not pass

PART - B.

a) Prove that the nonempty connected subsets of the real line are the intervals

b) Derive the cauchy - Riemann equations in the complex form.

12) a) Prove that the line integral $\int_\gamma p\,dx + q\,dy$. defined in $\Omega$, depends only on the end points of $\gamma$. If and only if there exists a function $U(x,y)$ in $\Omega$ with the partial derivatives

$$\frac{\partial u}{\partial x} = p, \quad \frac{\partial u}{\partial y} = q.$$

b) State and prove Cauchy's estimate

13) a) Prove an analytic function comes arbitrarily close to any complex value in every neighbourhood of an essential singularity

b) If $f(z)$ is defined and continuous on a closed bounded set $E$ and analytic on the interior of $E$, prove that the maximum of $|f(z)|$ on $E$ is assumed on the boundary of $E$.

14) a) State and prove the Rouche's theorem.

b) compute : $\int_0^\pi \frac{d\theta}{a+\cos\theta}$ ; $a>1$

15) a) If $f(z)$ is analytic in the whole plane and real on the real axis purely imaginary on the imaginary axis S.T $f(z)$ is odd

b) State and prove weierstrass's Theorem.

PART - C.

16) a) Define a linear transformation as the complex plane prove $z_1, z_2, z_3, z_4$ are distinct points in the extended complex plane then for any linear transformation $T$, $(Tz_1, Tz_2, Tz_3, Tz_4)$ = $(z_1, z_2, z_3, z_4)$

b) S.T the cross ratio is real iff the four points $z_1, z_2, z_3, z_4$