1st B.Sc (Maths)

Analytical Geometry 3D

Part - A

1) If the direction cosines of a line are $1/c, 1/c, 1/c$ then find the value of c

2) If α, β, γ be the angles which a line makes with the +ve direction of the axes, P.T $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

3) Find the distance between the points $(4, -3, 6)$ and $(-2, -1, -3)$

4) Write down the formula for normal form of the equation of a plane.

5) Find the angle between the two lines whose direction ratios are $1, 2, \sqrt{3}-1, -\sqrt{3}-1, 4$

6) What is mean by Co-planar lines.

7) What is meant by unsegmmetric form of the Equation of a line.

8) Define Skew lines.

9) Find the value of k so that the lines $\frac{x-1}{3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{1} = \frac{y-5}{1} = \frac{z-6}{-5}$ are Perpendicular to each other.

10) Write down the Conditions for a line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ in the plane $ax + by + cz + d = 0$

11) When do you say that the two lines $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$ are Coplanar

12) Write the Condition for two lines are Co-planar.

13) Determine the equation of the sphere with Centre (a,b,c) & radius r units

14) When two sphere with radius r_1 and r_2 and Centre C_1 & C_2 touch each externally.

15) Define great circle in a sphere.

16) Find the Centre & radius of the sphere $2x^2+2y^2+2z^2-2x+4y+2z+3=0$.

17) Define sphere.

18) S.T the sphere $2x-2y+z+12=0$ touches the sphere $x^2+y^2+z^2=2x-4y+12z-3=0$.

19) $ax^2+by^2+cz^2+2fy+2gzx+2hxy=0$ Write down the Condition for the second order homogeneous equation.

20) Find the equation of the cone of the second degree which passes through the axes.

21) Define right Circular Cone.

22) Write down the equation of the tangent plane at (x_1,y_1,z_1) to the Cone.

23) Write down the conditions $ax^2+by^2+cz^2+2fyz+2gzx+2hxy=0$ represents (i) a Cone (ii) a pair of Planes.

24) Write the Condition for the plane $lx + my + nz = p$ to touches the Conicoid $ax^2 + by^2 + cz^2 = 1$

25) Give the general Equation to a Cone which touches the Co-ordinate planes.

26) Define Central quadric.

27) Write down the Eqn of the normal to the Cone at (x_1, y_1, z_1)

28) Define Cone.

29) Write the condition for the plane $lx + my + nz = 0$ to touches the quadric Cone $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$.

30) Write down the two tangent planes to be a Conicoid parallel to the plane $lx + my + nz = 0$.

Part - B

1) The direction Cosines of the lines are connected by the relations $l + m + n = 0$

2) S.T the lines whose direction Cosines are given by $al + bm + cn = 0$, $fmn + gnl + hlm = 0$ are Perpendicular is $f/a + g/b + h/c = 0$.

3) Find the direction Cosines of two lines which are determined by the relations $l + m + n = 0$, $mn + 6ln - 12lm = 0$.

4) Find the distance of the points $(2, 3, 4)$ & $(1, 1, 4)$ from the Planes $3x - 6y + 2z + 11 = 0$.

5) Find the equation of the plane passing through $(-1,1,1)$, $(1,-1,1)$ & Perpendicular to the plane $x+2y+2z=5$

6) Find the equation of the plane through the point $(1,1,1)$ & the intersection of the plane $x+y+z=6$ & $2x+3y+4z+5=0$.

7) Determine the image of a plane $P(1,3,4)$ in the Plane $2x-y+z+3=0$.

8) P.T the line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ is parallel to the plane $2x+y-2z=3$

9) S.T the lines $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$, $\frac{x-1}{2}=\frac{y+1}{1}=\frac{z+10}{8}$ intersection and find the co-ordinates of the point of intersection.

10) A Square ABCD of diagonal $2a$ is folded along the diagonal AC, so that the planes DAC, BAC are at right angles, S.T the Shortest distance b/w DC & AB is then $2a/\sqrt{3}$

11) S.T the 2 lines $\frac{x+5}{3}=\frac{y+4}{1}=\frac{z-7}{-2}$ and $3x+2y+z-2=0=x-3y+2z-3$ are Coplanar & find the Eqn of the plane.

12) Find the length of the perpendicular from the point $P(5,4,-1)$ upon the line $\frac{x-1}{2}=\frac{1}{4}y=\frac{1}{5}z$.

13) If r be the radius of the circle $x^2+y^2+z^2+2ux+2vy+2wz+d=0$, $lx+my+nz=0$ P.T $(r^2+d)(l^2+m^2+n^2)=(mw-nv)^2+(nu-lw)^2+(lv-mu)^2$.

14) S.T the plane $lx+my+nz=p$ touches the sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ if $(ul+vm+wn+p)^2=(l^2+m^2+n^2)(u^2+v^2+w^2-d)$

15) Find the centre & radius of the circle $x+2y+2z=15$, $x^2+y^2+z^2-2y-4z=11$.

16) Find the eqn of two tangent planes to the sphere $x^2+y^2+z^2+9$ which passing through the line $x+y=6$, $x-2y=3$.

17) Find the eqn of the sphere through 4 pts $(4,-1,2)$, $(0,-2,3)$, $(4,-5,-1)$ & $(2,0,1)$

18) S.T the plane $2x+2y+z+12=0$ & touches the sphere $x^2+y^2+z^2-2x-2y-2z-3=0$ also find the point of contact.

19) S.T the eqn of a right circular cone whose vertex is 0, axis OZ & semi vertical angle α is $x^2+y^2+z^2+\tan^2\alpha$

20) Find the equation of the tangent planes to the cone $9x^2-4y^2+16z^2=0$ which contains the line $x/32=y/72=z/27$.

21) Find the Eqn of Cone whose Vertex is $(1,2,3)$ and which passes through the circle $x^2+y^2+z^2=4$, $x+y+z=1$.

22) P.T the Equation $2x^2-7y^2+2z^2-10yz-8zx$ ~~[illegible]~~ $-10xy+6y+12y-6z-3=0$ represents a Cone whose Vertex is $(1/4, 1/2, 1/4)$.

23) S.T $33x^2+13y-95z^2-144yz-96zx-48xy=0$ represents a Right Circular Cone whose axis is the line $3x=y=z$ find the vertical angle.

24) S.T the Cone whose vertex is at the Origin & which passes through the circle of intersection of the sphere $x^2+y^2+z^2=3r^2$ & any Plane at a distance r from the origin has 3 mutually Perpendicular generators.

25) P.T the Cones $ax^2+by^2+cz^2=0$ and $x^2/a+y^2/b+z^2/c=0$ are reciprocal.

26) Find the angles the lines are given by $x+y+z=0$, $yz/b-c+zx/c-a+xy/a-b=0$.

Part - C

1) A line makes angle $\alpha, \beta, \gamma, \delta$ with 4 diagonals of a Cube, P.T $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = 4/3$

2) S.T the straight line whose d.c are given by the eqn $al + bm + cn = 0$, $ul^2 + vm^2 + wn^2 = 0$ are Perpendicular (or) Parallel awarding as
$a^2(v+w) + b^2(w+u) + c^2(u+v) = 0$ (or)
$a^2/u + b^2/v + c^2/w = 0$.

3) Find the length & eqn of the line of the shortest distance b/w the lines
$\frac{x+3}{-4} = \frac{y-6}{3} = \frac{z}{2}$, $\frac{x+2}{4} = \frac{y}{1} = \frac{z-7}{1}$

4) Find the magnitude & the eqn of the line of shortest distance b/w the lines
$\frac{x-3}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$, $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

5) P.T the line $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z-10}{8}$, $\frac{x-4}{1} = \frac{y+3}{1} = \frac{z-1}{7}$ intersect find also their point of intersection & the plane through them.

6) Obtain the eqn of sphere which passes through the circle $x^2+y^2+z^2-2x+2y+4z-3=0$.

7) S.T the sphere $x^2+y^2+z^2 = 64$ & $x^2+y^2+z^2-12x+4y-6z+48=0$ touches internally & find this Point of Contact.

8) Find the eqn of the tangent plane to the sphere $x^2+y^2+z^2+2x-2y+6z-7=0$ which intersect the line $6x-3y-2z=0=3z+2$

9) Derive the eqn of the sphere drawn on the line joining (x,y,z) & (x_2,y_2,z_2)

10) Find the Condition for the eqn $ax^2+by^2+cz^2+2fyz+2gzx+2hxy=0$ to represent a right circular cone

11) S.T the eqn of right cone which passes through $(2,1,3)$ & has its vertex at the point $(1,1,2)$ & axis the line $\frac{x-1}{2}=\frac{y-1}{-4}=\frac{z-2}{3}$ is $17x^2-14y^2+7z^2+24yz+16xy-127x-18x-114y-28z+70=0$

12) S.T the 2 cones $ayz+bzx+cxy=0$, $fyz+gzx+hxy=0$ intersection in the Co-ordinate axes & in the line $x(bh-eg)=y(cf-af)=z(ag-bf)$ planes through ox and oy include an angle 60°. S.T their line of intersection lies on the cone $z^2(x^2+y^2+z^2)=3x^2y^2$

13) Find the eqn of the tangent plane to the cone $9x^2-4y^2+16z^2=0$ which contain the line $x/32=y/72=z/72$

14.) Find the Eqn of the two tangent planes of the ellipsoid $2x^2+2y^2+z^2=2$ which pass through the line $x=0,\ x+y=10$.

15) S.T the plane $3x+2y+z=k$ touches the ellipsoid $3x^2+4y^2+z^2=20$ if $k=\pm 10\sqrt{2}$ Find the length of the Chord of Contact b/w the two tangent plane.