Dharmapuram Adhinam Arts College

Department of Mathematics

II M.Sc Mathematics

Important Questions

Sub: Functional Analysis

Section-A.

1. Define Banach space ? Give an example.
2. State closed graph theorem.
3. Define Hilbert space ?
4. Define an orthonormal set in Hilbert space ?
5. Define similar matrices ?
6. Define eigen value and eigen vector of T ?
7. Define spectrum of Banach Algebra ?
8. What is topological divisor of zero in a Banach Algebra ?
9. Show that $\|x^* x\| = \|x\|^2$
10. Define self - adjoint in $\mathcal{C}(M)$?

Section-B

11. State and prove the Holder's Inequality
12. State and prove open Mapping theorem
13. State and prove Schwarz Inequality
14. Prove that if M is a proper closed linear subspace of a Hilbert space then there exists a non-zero vector $z_0 \in H$ such that $z_0 \perp M$.
15. Show that if T is normal then x is an eigen vector of T with eigen value λ iff x is an eigen vector of T^* with eigen value $\bar{\lambda}$.

(16) Let B be a basis for H and T an operator whose matrix relative to B in $[\alpha_{ij}]$. Then T is non-singular $\Longleftrightarrow$ $[\alpha_{ij}]$ is non-singular and in this case $[\alpha_{ij}]^{-1} = [T]^{-1}$.

(17) Prove that $\sigma(x)$ is non-empty

(18) prove that $Z \subseteq S$ in a Banach Algebra?

(19) If N is a normed linear space, Prove that the closed Unit sphere S^* in N^* is a compact Hausdroff space in the week * Topology

(20) State and Prove Uniform Boundedness theorem.

Section-C

(21) State and prove Hahn-Banach theorem?

(22) Prove that a closed Convex subset C of a Hilbert space Contains a unique vector of smallest norm?

(23) Let H be an Hilbert Space and let $\{e_i\}$ be an ~~orth~~ orthonormal set in H. Then the following Conditions are equivalent.
(i) $\{e_i\}$ is Complete (ii) $x \perp \{e_i\} \Rightarrow x = 0$
(iii) If x is an arbitrary vector in H, then $x = \sum (x, e_i) e_i$
(iv) If x is an arbitrary vector in H then $\|x\|^2 = \sum |(x, e_i)|^2$.

(24) (i) T is an operator on H for which $(Tx, x) = 0$ for all x, Prove that $T = 0$.
(ii) If P is a projection on H which range M and Nullspace N then $M \perp N \Longleftrightarrow P$ is self-adjoint and in this case $N = M^{\perp}$.

(25) (i) Closed linear subspace M of H is invariant under T an operator $\Longleftrightarrow M^{\perp}$ is invariant under T^*.
(ii) If P is a projection on a Closed linear subspace M of a Hilbert space H then M reduces $T \Rightarrow TP = PT$.

(26) State and prove Spectral theorem.

(27) State and prove Gelfand-Neumark theorem?

(28) show that $(1-rx)$, $(1-xr)$, $(1-xrs)$ are regular?