

Class: I M.Sc (Maths)

Sub: Stochastic Processes

Time: 3hrs

Max - 75

PART-A - (10x2=20)

Answer All questions.

1. Give an example of a Stochastic Processes.
2. Define transition Probability matrix for the Markov chain
3. Define accessible and Communicate States.
4. Define a n -Step transition Probability density function.
5. Write down any one of the Postulates for Poisson Process.
6. What is meant by renewal Process.
7. Define a mean recurrent time
8. Define a directly Riemann integrable.
9. If n is the number of Spares in the inventory at $t=0$ then find the expected number of customers waiting for Spares at epoch t .
10. Define an idle Period.

PART-B (5x5=25)

Answer ALL questions.

11a) Explain the Polya's urn model.

(Or)

b) Define Chapman-Kolmogorov equation.

12a) Let $\{X_n\}$, $n \geq 0$ be a Markov chain having State space $S = \{1, 2, 3, 4\}$ and transition Matrix

$$P = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Find the ergodic state.

(Or)

b) If state j is persistent non-null. then Prove that $n \rightarrow \infty$, $P_{jj}^{(n)} \rightarrow t/\mu_{jj}$ when state j is periodic with period L .

13. a) Prove that the difference of two independent Poisson Process is not a Poisson Process.

b) Prove that ^(Or) the Probability of ultimate extinction is 1 when $\mu > \lambda$ and $u < 1$ when $\mu < \lambda$.

14. a) If $P_n \rightarrow \alpha$ then Prove that $V_n \rightarrow \alpha b$ where $b = B(1) = \sum b_n$

b) ^(Or) Derive the integral equation of renewal theory.

15. a) The arrivals at a Counter in bank occur in accordance with a Poisson Process at an average rate of 8 per hour. The duration of service of a customer has exponential distribution with mean of 6 minutes. Find the Probability that on arriving customer.

(i) has to wait on arrival.

(ii) find 4 customers in the system and

(iii) has to spend less than 15 minutes in the bank.

^(Or)

b) Explain the $M/G/\infty$ model.

Part - C ($3 \times 10 = 30$)

Answer any Three questions.

16. Write a note on Markov-Bernoulli chain.

17. Explain the determination of higher transition Probabilities

18. Find the solution of the equations for a finite state process.

19. State and prove elementary renewal theorems.

20. Derive an integral equation satisfied by the differential equation of busy period initiated by a single unit or customer.