# UNIT - 2

Obtain all the basic solutions to the following system of linear equations.

$$x_1 + 2x_2 + x_3 = 4 \rightarrow (1)$$

$$2x_1 + x_2 + 5x_3 = 5 \rightarrow (2).$$

Solution :-

$$x_1 + 2x_2 + x_3 = 4 \rightarrow (1)$$

$$2x_1 + x_2 + 5x_3 = 5 \rightarrow (2)$$

$n \rightarrow$ no. of variables $= 3$

$m \rightarrow$ no. of equations $= 2$

Now $n-m = 3-2 = 1$ (non-basic solution)

The matrix form of the given equation is

$$AX = b$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

where $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 5 \end{bmatrix}$, $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $b = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

case 1:

Let $x_1 = 0$ be a non-basic solution.

$(1) \Rightarrow \quad 2x_2 + x_3 = 4 \rightarrow (3)$

$(2) \Rightarrow \quad x_2 + 5x_3 = 5 \rightarrow (4)$

$$2x_2 + x_3 = 4$$

$(4) \times 2 \Rightarrow \quad 2x_2 + 10x_3 = 10$

$$(-) \quad (-) \quad (-)$$

$$-9x_3 = -6$$

$$x_3 = 2/3.$$

$(4) \Rightarrow \quad x_2 + 5x_3 = 5$

$$x_2 = 5 - 5x_3$$

$$x_2 = 5 - \frac{10}{3}$$

$$x_2 = 5/3.$$

$$\therefore \boxed{x_1 = 0 \,;\, x_2 = 5/3 \,;\, x_3 = 2/3}$$

It is feasible and also non-degenerate solution,

Case 2:

Let $x_2 = 0$ be a non-basic solution

$$① \Rightarrow x_1 + x_3 = 4 \rightarrow ③$$

$$② \Rightarrow 2x_1 + 5x_3 = 5 \rightarrow ④$$

$$③ \times 2 \Rightarrow 2x_1 + 2x_3 = 8$$

$$2x_1 + 5x_3 = 5$$

$$(-) \quad (-) \quad (-)$$

$$-3x_3 = 3$$

$$x_3 = -1$$

$$① \Rightarrow x_1 - 1 = 4$$

$$x_1 = 4 + 1$$

$$x_1 = 5$$

$$\therefore \boxed{x_1 = 5;\ x_2 = 0;\ x_3 = -1}$$

It is infeasible solution and also non-degenerate solution.

Case 3:

Let $x_3 = 0$

$$① \Rightarrow x_1 + 2x_2 = 4 \rightarrow ③$$

$$② \Rightarrow 2x_1 + x_2 = 5 \rightarrow ④$$

$$③ \times 2 \Rightarrow 2x_1 + 4x_2 = 8$$

$$2x_1 + x_2 = 5$$

$$(-) \quad (-) \quad (-)$$

$$3x_2 = 3$$

$$x_2 = 1$$

$$③ \Rightarrow x_1 + 2(1) = 4$$

$$x_1 = 2$$

$$\boxed{x_1 = 2;\ x_2 = 1;\ x_3 = 0}$$

It is feasible and non-degenerate solution.

| Basic | Non-basic |
|---|---|
| $x_2 = 5/3$<br>$x_3 = 2/3$ | $x_1 = 0$ |
| $x_1 = 5$<br>$x_3 = -1$ | $x_2 = 0$ |
| $x_1 = 2$<br>$x_2 = 1$ | $x_3 = 0$ |

2. Obtain all the basic solutions to the following system of linear equations

$$x_1 + 2x_2 + 3x_3 = 4$$
$$x_1 - 2x_2 + x_3 = 5$$

A. $x_1 + 2x_2 + 3x_3 = 4$

$x_1 - 2x_2 + x_3 = 5$

no. of variables = 3

no. of equations = 2

Now $n - m = 3 - 2 = 1$ (non-basic)

The matrix form of given equation is

$$Ax = b$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

Case 1:

Let $x_1 = 0$;

① $\Rightarrow 2x_2 + 3x_3 = 4 \rightarrow$ ③

② $\Rightarrow -2x_2 + x_3 = 5 \rightarrow$ ④

$$\begin{array}{r} 2x_2 + 3x_3 = 4 \\ -2x_2 + x_3 = 5 \\ \hline 4x_3 = 9 \end{array}$$

$x_3 = 9/4$

① $\Rightarrow 2x_2 + 3(9/4) = 4$

$2x_2 + 27/4 = 4$

$2x_2 = 4 - \frac{27}{4}$

$2x_2 = -11/4$

$x_2 = -11/8$.

$\boxed{x_1 = 0 \; ; \; x_2 = -11/8 \; ; \; x_3 = 9/4}$

It is an infeasible and also non-degenerate solution.

Case 2:

Let $x_2 = 0$;

① ⇒ $x_1 + 3x_3 = 4$ → ③
② ⇒ $x_1 + x_3 = 5$ → ④

$$x_1 + 3x_3 = 4$$
$$\underline{\overset{(-)}{x_1} + \overset{(-)}{x_3} = \overset{(-)}{5}}$$
$$2x_3 = -1$$
$$x_3 = -1/2$$

① ⇒ $x_1 + 3(-1/2) = 4$

$$x_1 - 3/2 = 4$$
$$x_1 = 11/2$$

$$\boxed{x_1 = 11/2 ;\ x_2 = 0 ;\ x_3 = -1/2}$$

It is infeasible and non-degenerate solution.

Case 3 :-

Let $x_3 = 0$

① ⇒ $x_1 + 2x_2 = 4$ → ③
② ⇒ $x_1 - 2x_2 = 5$ → ④

$$x_1 + 2x_2 = 4$$
$$\underline{x_1 - 2x_2 = 5}$$
$$2x_1 = 9$$
$$x_1 = 9/2$$

① ⇒ $9/2 + 2x_2 = 4$

$$2x_2 = 4 - 9/2$$
$$2x_2 = -1/2.$$
$$x_2 = -1/4$$

$$\boxed{x_1 = 9/2 ;\ x_2 = -1/4 ;\ x_3 = 0.}$$

It is infeasible and also non-degenerate solution.

| Basic | Non-basic |
|---|---|
| $x_2 = -11/8$<br>$x_3 = 9/4$ | $x_1 = 0$ |
| $x_1 = 11/2$<br>$x_3 = -1/2$ | $x_2 = 0$ |
| $x_1 = 9/2$<br>$x_2 = -1/4$ | $x_3 = 0$ |