4.2 SPHERE

SECTION A

1. **Find the equation of the sphere having centre at (a, b, c) and radius "r" units?**

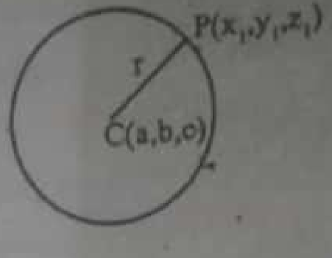
Let C(a, b, c) be the centre and $P(x_1, y_1, z_1)$ be any point on the sphere, then

CP = radius

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} = r$$

$$\sqrt{(x_1 - a)^2 + (y_1 - b)^2 + (z_1 - c)^2} = r$$

$$(x_1 - a)^2 + (y_1 - b)^2 + (z_1 - c)^2 = r^2$$



∴ The equation of the sphere is the locus of P

$$(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2$$

2. **Find the equation of the sphere having centre at (-1, 2, 3) and radius 3?**

The centre and radius form of the sphere is

$$(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2$$

$$(x + 1)^2 + (y - 2)^2 + (z - 3)^2 = 3^2$$

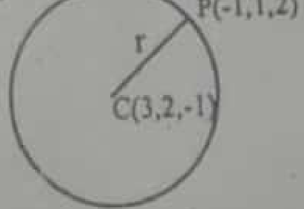
$$x^2 + y^2 + z^2 + 2x - 4y - 6z + 5 = 0$$

3. **Find the equation of the sphere with centre at (3,2,-1) and passing through the point (-1, 1, 2)?**

Here radius r = CP

$$r = \sqrt{(3+1)^2 + (2-1)^2 + (-1-2)^2}$$

$$= \sqrt{26}$$



∴ The equation of the sphere is

$$(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2$$

$(x-3)^2+(y-2)^2+(z+1)^2=\sqrt{26}^2$

$x^2+y^2+z^2-6x-4y+2z-12=0$

4. Write down the general equation and centre, radius of the sphere?

General equation is $x^2+y^2+z^2+2ux+2vy+2wz+d=0$

Centre = $(-u, -v, -w)$

Radius = $\sqrt{u^2+v^2+w^2-d}$

5. Find the centres and radius of the sphere $x^2+y^2+z^2-6x+8y-10z+1=0$?.

Given $x^2+y^2+z^2-6x+8y-10z+1=0$ ------------(1)

comparing (1) with

$x^2+y^2+z^2+2ux+2vy+2wz+d=0$.

$2u=-6 \Rightarrow u=-3$

$2v=8 \Rightarrow v=4$

$2w=-10 \Rightarrow w=-5$ and $d=1$

$\therefore$ Centre $(-u, -v, -w)=(3, -4, 5)$

Radius = $\sqrt{u^2+v^2+w^2-d}=\sqrt{(-3)^2+4^2+(-5)^2-1}=7$ units

6. Find the centre and radius of the sphere $2x^2+2y^2+2z^2+6x-6y+8z+9=0$?.

Dividing the given equation by 2

$x^2+y^2+z^2+3x-3y+4z+9/2=0$ ------------(1)

comparing (1) with

$x^2+y^2+z^2+2ux+2vy+2wz+d=0$.

$2u=3 \Rightarrow u=3/2$

$2v=-3 \Rightarrow v=-3/2$

$2w=4 \Rightarrow w=2$ and $d=9/2$

$\therefore$ Centre $(-u, -v, -w) = (-3/2, 3/2, -2)$

Radius $= \sqrt{u^2 + v^2 + w^2 - d}$

$= \sqrt{(3/2)^2 + (-3/2)^2 + (2)^2 - 9/2} = 2$ units

7. Write down the diameter form of the sphere whose ends of the diameter are (x_1, y_1, z_1) and (x_2, y_2, z_2)?.

$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) + (z-z_1)(z-z_2) = 0$

8. Find the equation of the sphere having ends of the diameters are (2, -3, 1) and (1, -2, -1)?.

The diameter form of the sphere is

$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) + (z-z_1)(z-z_2) = 0$

$(x-2)(x-1) + (y+3)(y+2) + (z-1)(z+1) = 0$

$x^2 + y^2 + z^2 - 3x + 5y + 7 = 0$

9. Find the equation of the sphere which passes through the points (0,0,0), (1,0,0), (0,1,0) and (0,0,1)?.

Let the equation of the sphere is

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ ------------(1)

since (1) is passes through (0,0,0) then (1) becomes

$0^2 + 0^2 + 0^2 + 2u(0) + 2v(0) + 2w(0) + d = 0 \Rightarrow d=0$

since (1) is passes through (1, 0, 0) then

$1^2 + 0^2 + 0^2 + 2u(1) + 0 + 0 + d = 0$

$1 + 2u + d = 0$

$1 + 2u + 0 = 0 \Rightarrow u = -\frac{1}{2}$

Similarly $v = -\frac{1}{2}$

Similarly $w = -\frac{1}{2}$

$\therefore$ The equation of the sphere

10. Write down the equation of a tangent plane to a sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ at the point (x_1, y_1, z_1).

$xx_1 + yy_1 + zz_1 + u(x+x_1) + v(y+y_1) + w(z+z_1) + d = 0$

11. Find the equation of the tangent plane to the sphere $x^2 + y^2 + z^2 - 4x + 6y - 2z + 5 = 0$ at the point (2, 0, 1)?

The equation of the tangent plane at (x_1, y_1, z_1) is

$xx_1 + yy_1 + zz_1 + u(x+x_1) + v(y+y_1) + w(z+z_1) + d = 0$ --(1)

Here $(x_1, y_1, z_1) = (2, 0, 1)$ $u = -2$, $v = 3$, $w = -1$ and $d = 5$

$\therefore$ (1) => $2x + 0y + z - 2(x+2)\ 3(y+0) - 1\ (z+1) + 5 = 0$

=> $y = 0$

That is XZ plane.

12. Write down the equation of the circle obtained by the intersection of the sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ with the plane $ax+by+cz+d=0$?

The equation of the circle is the equation of the sphere together with the equation of the plane.

That is $\left.\begin{array}{r} x^2+y^2+z^2+2ux+2vy+2wz+d = 0 \\ ax+by+cz+d = 0 \end{array}\right\}$

is a circle equation.

13. Show that the spheres $x^2+y^2+z^2+6y+2z+18 = 0$ and $x^2+y^2+z^2+6x+8y+4z+10 = 0$ are cut orthogonally.

$x^2+y^2+z^2+6y+2z+18=0$	$x^2+y^2+z^2+6x+8y+4z+10=0$
Here $u=0$	$u_1=3$
$v=3$	$v_1=4$
$w=1$	$w_1=2$
$d=18$	$d_1=10$

Substituting in

$2uu_1+2vv_1+2ww_1=c+c_1$

$2(0)(3)+2(3)(4)+2(1)(2)=18+10 \Rightarrow 28=28$ satisfied.

$\therefore$ The given spheres are cut orthogonally.

14. Write down the equation of the sphere having the circle $x^2+y^2+z^2+2ux+2vy+2wz+d=0,\ ax+by+cz+d=0$?

$(x^2+y^2+z^2+2ux+2vy+2wz+d)+k(ax+by+cz+d)=0$

EXERCISE 4.2(A) :

1. Find the equation of the spheres of the following
   (i). Centre(3,-4,5) and radius7 (ii). Centre(2,-3,4) and radius3
   (iii). Centre (1/2, -1, 3/2) and radius 2
2. Find the equation of the sphere whose centre at (1, 1, 1) and passes through the point (2, 0, 3)
3. Find the centre and radius of the following spheres.
   (i). $x^2+y^2+z^2-4x+6y-2z+5=0$ (ii). $x^2+y^2+z^2+2y-4z-4=0$
   (iii). $3x^2+3y^2+3z^2-4x+6y-9z+1=0$
4. Find the equation of the sphere on the join of (1, -1, 1) and (-3, 4, 5) as diameter.
5. Find the equations of the sphere through the following points
   (i). (0,0,0),(a,0,0),(0,b,0),(0,0,c) (ii). (0,0,0), (2,0,0), (0,2,0), (0,0,2)
6. Show that the sphere is $x^2+y^2+z^2+6y+2z+8=0$ and $x^2+y^2+z^2+6x+8y+4z+20=0$ are cut orthogonally.
7. Find the equation of the tangent plane to the sphere $x^2+y^2+z^2-4x-6y-8z-9=0$ at (1,2,-2)?

1. **Find the equation of the sphere passing through the points (2,0,1), (1,-5,-1), (0,-2,3) and (4,-1,2).**

Let the equation of the sphere is

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0$$

Since it is passing through the points (2,0,1), (1,-5,-1), (0,-2,3) and (4,-1,2) then

$2^2+0^2+1^2+2u(2)+2v(0)+2w(1)+d=0$

$\Rightarrow 4u+2w+d=-5 \quad$ ----(1)

$1^2+(-5)^2+(-1^2)+2u(1)+2v(-5)+2w(-1)+d=0$

$\Rightarrow 2u-10v-2w+d=-27$ ----(2)

$0^2+(-2)^2+3^2+2u(0)+2v(-2)+2w(3)+d=0$

$\Rightarrow -4v+6w+d=-13 \quad$ ----(3)

$4^2+(-1)^2+(2^2)+2u(4)+2v(-1)+2w(2)+d=0$

$\Rightarrow 8u-2v+4w+d=-21$ ----(4)

(1) - (2) $\quad 2u+10v+4w=22$ (or) $u+5v+2w=11$ ----(5)

(2) - (3) $\quad 2u-6v-8w=-14 \quad u-3v-4w=-7$ ----(6)

(3) - (4) $\quad -8u-2v+2w=8 \quad 4u+v-w=-4$ ----(7)

Solving for u, v, and w by using Cranmer's rule.

$$\Delta = \begin{vmatrix} 1 & 5 & 2 \\ 1 & -3 & -4 \\ 4 & 1 & -1 \end{vmatrix} = -42$$

$$\Delta v = \begin{vmatrix} 1 & 11 & 2 \\ 1 & -7 & -4 \\ 4 & -4 & -1 \end{vmatrix} = -126$$

$$\Delta w = \begin{vmatrix} 1 & 5 & 11 \\ 1 & -3 & -7 \\ 4 & 1 & -4 \end{vmatrix} = 42$$

$$\therefore u, v, w = \frac{\Delta u}{\Delta}, \frac{\Delta v}{\Delta}, \frac{\Delta w}{\Delta} = \frac{84}{-42}, \frac{-126}{-42}, \frac{42}{-42}$$

$\therefore u = -2, v = 3, w = -1$

Substituting u, w, in (1)

$4u+2w+d = -5$

$4(-2)+2(-1)+d = -5$

$d = -5+10 = 5$

$\therefore$ The equation of the sphere is

$x^2+y^2+z^2+2ux+2vy+2wz+d=0$

$x^2+y^2+z^2+2(-2)x+2(3)y+2(-1)z+5=0$

$x^2+y^2+z^2-4x+6y-2z+5=0$

2. Obtain the condition for the plane $lx+my+nz = p$ to be touches the sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$?

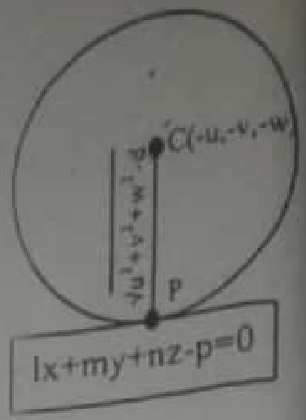
Formula

Let the plane $lx+my+nz-p=0$ touches the sphere at P

The centre of the sphere = $C(-u, -v, -w)$

radius = $\sqrt{u^2+v^2+w^2-d}$

The perpendicular distance from $C(-u, -v, -w)$ to the plane $lx + my + nz - p = 0$ is



$$CP = \pm \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}$$

$$= \pm \frac{l(-u) + m(-v) + n(-w) - p}{\sqrt{l^2 + m^2 + n^2}}$$

$$= \pm \frac{-(lu + mv + nw + p)}{\sqrt{l^2 + m^2 + n^2}}$$

The plane touches the sphere if

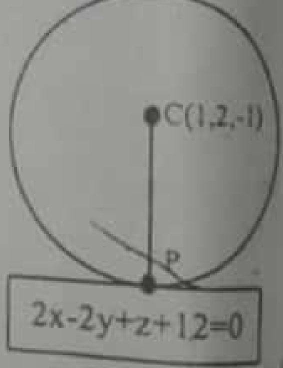
the perpendicular distance from the centre of the sphere to the plane } = radius

$$\pm \frac{-(lu + mv + nw + p)}{\sqrt{l^2 + m^2 + n^2}} = \sqrt{u^2 + v^2 + w^2 - d}$$

squaring on bothsides

$$(lu+mv+nw+p)^2 = (l^2+m^2+n^2)(u^2+v^2+w^2-d)$$

3. **Show that the sphere $x^2+y^2+z^2-2x-4y+2z-3 = 0$ touches the plane $2x-2y+z+12=0$ and find the point of contact ?.**



In the sphere $x^2+y^2+z^2-2x-4y+2z-3 = 0$

$u = -1, v = -2, w = 1, d = -3$

Centre $C(-u, -v, -w) = (1, 2, -1)$

Radius $= \sqrt{u^2 + v^2 + w^2 - d}$

$= \sqrt{(-1)^2 + (-2)^2 + 1^2 + 3} = 3$

The length of the perpendicular distance from the centre (1,2,3) to the plane $2x-2y+z+12 = 0$ is }

$$CP = \pm \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}$$

$$CP = \frac{2(1)-2(2)+1(-1)+12}{\sqrt{2^2 + (-2)^2 + 1^2}} = \frac{9}{3}$$

$CP = 3$

Thus we have CP = Radius

$\therefore$ The given plane touches the sphere

To find the point of contact P :

The equation of the line CP is the line passing through (1,2,-1) and having direction ratios 2, -2, 1.(which is drs of normal to the plane $2x - 2y + z + 12 = 0$).

$$\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$$

$$\frac{x-1}{2} = \frac{y-2}{-2} = \frac{z+1}{1} = r(say)$$

$\therefore P(2r+1, -2r+2, r-1)$

Since P lies on the plane $2x-2y+z+12 = 0$

$2(2r+1) - 2(-2r+2) + (r-1) + 12 = 0$

$4r + 2 + 4r - 4 + r - 1 + 12 = 0$

$\Rightarrow r = -1$

The point of contact $P(2r+1, -2r+2, r-1)$

$= P(2(-1)+1, -2(-1)+2, -1-1)$

$= P(-1, 4, -2)$

4. Find the equation of the sphere passing through the circle $x^2+y^2+z^2-2x-4y = 0$ and $x+2y+3z = 8$ touching the plane $4x+3y = 25$?.

We know that the equation the sphere passing through the circle is

$$(x^2+y^2+z^2-2x-4y) + k\,(x+2y+3z-8) = 0$$

$$x^2+y^2+z^2+(-2+k)x + k\,(-4+2k)y + 3kz - 8k = 0 \quad \text{------(1)}$$

$$u = \frac{-2+k}{2}, \quad v = \frac{-4+2k}{2}, \quad w = \frac{3k}{2}, \quad d = -8k$$

Centre of the sphere (1) is

$$(-u, -v, -w) = \left(\frac{2-k}{2}, \frac{4-2k}{2}, \frac{-3k}{2}\right)$$

$$\text{Radius} = \sqrt{u^2 + v^2 + w^2 - d}$$

$$= \sqrt{\left(\frac{k-2}{2}\right)^2 + \left(\frac{2k-4}{2}\right)^2 + \frac{9k^2}{4} + 8k}$$

$$= \sqrt{\frac{k^2-4k+4}{4} + \frac{4k^2-16k+16}{4} + \frac{9k^2}{4} + 8k}$$

$$= \sqrt{\frac{14k^2-20k+20}{4} + 8k}$$

$$= \sqrt{\frac{14k^2+12k+20}{4}}$$

$$= \sqrt{\frac{7k^2+6k+10}{2}}$$

The length of perpendicular of the plane $4x+3y-0z-25=0$ from the centre

$$CP = \pm \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}$$

$$= \pm \frac{4\left(\frac{2-k}{2}\right) + 3\left(\frac{4-2k}{2}\right) + 0\left(\frac{-3k}{2}\right) - 25}{\sqrt{4^2 + 3^2}}$$

$$= \pm \frac{8-4k+12-6k-50}{10}$$

$$= \pm \frac{-10k-30}{10}$$

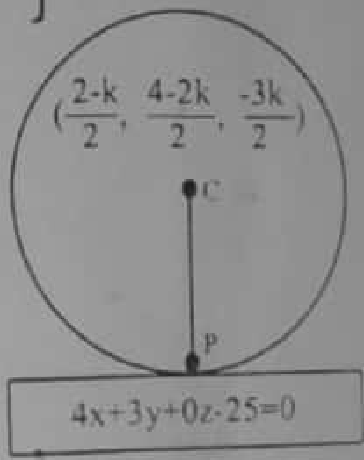
$$= \pm(-k-3)$$

$\therefore CP = \mp(k+3)$

Since the sphere touches the plane 4x+3y=25

$\therefore$ The length of the perpendicular distance from the plane to the centre is CP } = Radius

$$\mp(k+3) = \sqrt{\frac{7k^2+6k+10}{2}}$$



Squaring on both sides

$$(k+3)^2 = \frac{7k^2+6k+10}{2}$$

$$2(k^2+6k+9) = 7k^2+6k+10$$

$$7k^2+6k+10-2k^2-12k-18 = 0$$

$$5k^2-6k-8 = 0$$

$$k = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$k = \frac{6 \pm \sqrt{(-6)^2 - 4.5(-8)}}{2.5}$$

$$= \frac{6 \pm \sqrt{36+160}}{10}$$

$$= \frac{6 \pm 14}{10}$$

$= \frac{20}{10}, \frac{-8}{10}$

$k = 2, -4/5$

If $k = 2$ then the equation of the sphere (1) =>

$x^2+y^2+z^2-2x-4y+2(x+2y+3z-8) = 0$

$x^2+y^2+z^2+6z-16 = 0$

If $k = -4/5$ then the equation of the sphere (1) =>

$x^2+y^2+z^2-2x-4y-\frac{4}{5}(x+2y+3z-8) = 0$

$5x^2+5y^2+5z^2-14x-28y-12z+32 = 0$

5. Find the equation of the sphere with centre at the point (2, -3, 1) and touching the plane $2x+2y-z+12 = 0$?.

The centre and radius form of the sphere is

$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$

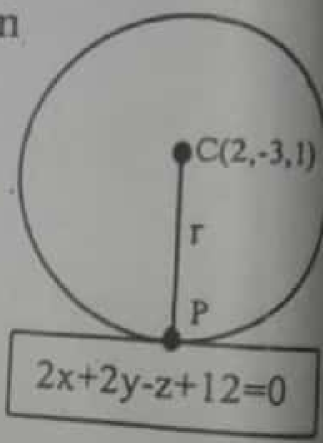
$(x-2)^2 + (y+3)^2 + (z-1)^2 = r^2$ -----(1)

To find radius r :

Radius = The perpendicular distance from (2, -3, 1) to the plane $2x+2y-z+12=0$

$$= \frac{2(2) + 2(-3) - 1 + 12}{\sqrt{2^2 + 2^2 + (-1)^2}}$$

$CP = 3$ that is $r = 3$



$\therefore$ (1) => $(x-2)^2 + (y+3)^2 + (z-1)^2 = 3^2$

$x^2+y^2+z^2-4x+6y-2z+5 = 0$

6. Show that the intersection of the sphere $x^2+y^2+z^2-2x-4y-6z-2 = 0$ and the plane $x+2y+2z = 20$ is a circle whose centre is the point (2, 4, 5) and radius $\sqrt{7}$.

To find the centre and radius of the sphere

$x^2+y^2+z^2-2x-4y-6z-2=0$

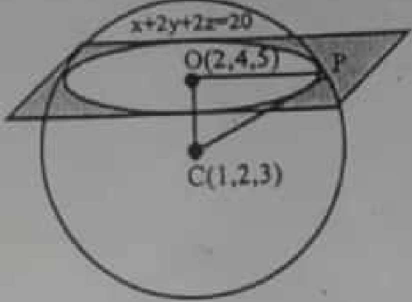
$u=-1,\ v=-2,\ w=-3,\ d=-2$

Centre $C(-u, -v, -w) = (1, 2, 3)$

Radius $= \sqrt{u^2+v^2+w^2-d} = \sqrt{(-1)^2+(-2)^2+(-3)^2+2} = 4$

Let 0 be the centre of the circle

The equation of the line CO is the line passing through (1,2,3) and having drs 1,2,2 (which is drs of normal to the plane x+2y+2z=20)



$$\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$$

$$\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{2} = r(\text{say})$$

Then $O(r+1, 2r+2, 2r+3)$

Since O lies on the plane

$x+2y+2z=20$

$\therefore (r+1)+2(2r+2)+2(2r+3)=20$

$r+1+4r+4+4r+6=20$

$9r = 20-11 \Rightarrow r=1$

$\therefore$ The centre of the circle $O(r+1, 2r+2, 2r+3)$

$O(1+1, 2(1)+2, 2(1)+3)$

$O(2, 4, 5)$

To find radius of circle :

the length CO $= \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$

$= \sqrt{(1-2)^2+(2-4)^2+(3-5)^2}$

$= 3$

From the right angled triangle ΔCOP

$$CO^2 + OP^2 = CP^2$$
$$3^2 + OP^2 = 4^2$$
$$OP^2 = 16-9$$
$$OP^2 = 7$$
$$OP = \sqrt{7}$$

$\therefore$ Centre of the circle is O(2, 4, 5) and radius = $\sqrt{7}$ units.

7. Find the equation of the sphere through the circle $x^2+y^2+z^2=9, 2x+3y+4z=5$ and through the point (1,2,3).

We know that the equation of the sphere having the circle

$$x^2 + y^2 + z^2 - 9 + k(2x + 3y + 4z - 5) = 0 \text{ -----(1)}$$

Since (1) is passes through the point (1,2,3)

$$(1^2 + 2^2 + 3^2 - 9) + k(2(1) + 3(2) + 4(3) - 5) = 0$$
$$5 + 15k = 0 \quad \Rightarrow k = -1/3$$

$\therefore$ (1) =>

$$x^2 + y^2 + z^2 - 9 - 1/3\,(2x + 3y + 4z - 5) = 0$$
$$3x^2 + 3y^2 + 3z^2 - 2x - 3y - 4z - 22 = 0$$

8. Find the equation of the sphere which has its centre on the plane 5x+y-4z+3=0 and passing through the circle $x^2 + y^2 + z^2 - 3x + 4y - 2z + 8 = 0$, 4x-5y+3z-3 = 0?

We know that the equation of the sphere through the circle is

$$x^2 + y^2 + z^2 - 3x + 4y - 2z + 8 + k(4x-5y+3z-3) = 0 \text{ ---(1)}$$
$$x^2 + y^2 + z^2 + (-3 + 4k)x + (4-5k)y + (-2+3k)z + 8 - 3k = 0$$

$$u = \frac{-3+4k}{2}, \quad v = \frac{4-5k}{2}, \quad w = \frac{-2+3k}{2}$$

∴ Centre of the sphere (1) is

$$(-u, -v, -w) = C\left(\frac{3-4k}{2}, \frac{5k-4}{2}, \frac{2-3k}{2}\right)$$

Since the centre lies on the plane $5x + y - 4z + 3 = 0$

$$5\left(\frac{3-4k}{2}\right) + \left(\frac{5k-4}{2}\right) - 4\left(\frac{2-3k}{2}\right) + 3 = 0$$

$$15 - 20k + 5k - 4 - 8 + 12k + 6 = 0$$

$$-3k + 9 = 0$$

$$-3k = -9 \Rightarrow k = 3$$

∴ The equation of the sphere (1) =>

$$(x^2 + y^2 + z^2 - 3x + 4y - 2z + 8) + 3(4x-5y+3z-3) = 0$$

$$x^2 + y^2 + z^2 + 9x - 11y + 7z - 1 = 0$$

9. **Find the equation of the sphere having the circle $x^2+y^2+z^2-6x+3y-z-8=0, 2x+3y-z+6=0$ as the great c**

We know that the equation of the sphere through the

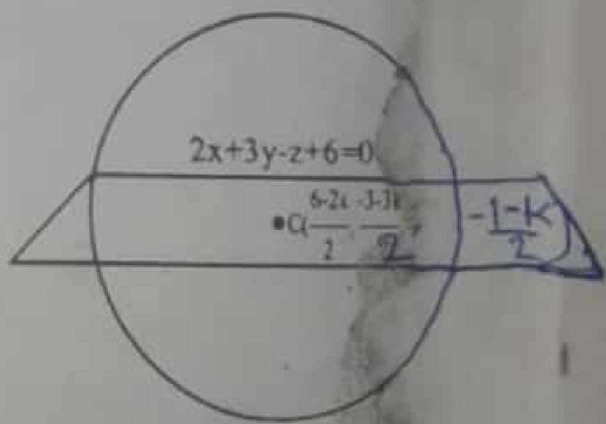
$$(x^2 + y^2 + z^2 - 6x + 3y - z - 8) + k(2x+3y-z+6) = 0 -$$

$$(x^2 + y^2 + z^2 + (-6 + 2k)x + (3+3k)y + (-1-k)z - 8 + 6$$

$$u = \frac{-6+2k}{2}$$

$$v = \frac{3+3k}{2}$$

$$w = \frac{-1-k}{2}$$



Centre of the sphere (1) is

$$C(-u, -v, -w) = C\left(\frac{6-2k}{2}, \frac{-3-3k}{2}, \frac{1+k}{2}\right)$$

Since the circle is a **great circle**, the centre of the sphere lies on the plane $2x + 3y - z + 6 = 0$

$$2\left(\frac{6-2k}{2}\right) + 3\left(\frac{-3-3k}{2}\right) - \left(\frac{1+k}{2}\right) + 6 = 0$$

$$\frac{12-4k-9-9k-1-k+12}{2} = 0$$

$$-14k + 14 = 0$$

$$-14k = -14$$

$$k = 1$$

The equation of the sphere (1) =>

$$(x^2 + y^2 + z^2 - 6x + 3y - z - 8) + 1(2x+3y-z+6) = 0$$

$$x^2 + y^2 + z^2 - 4x + 6y - 2z - 2 = 0$$

10. Prove that the circles

$x^2 + y^2 + z^2 - 2x + 3y - z - 2 = 0 = 2x - 3y + z - 7$ and

$x^2 + y^2 + z^2 - 2x - 4y + 6z + 1 = 0 = x + 2y - 3z - 5$

lies on the same sphere. Also find the equation of the sphere?

The equation of the sphere passing through the first circle is

$$(x^2 + y^2 + z^2 - 2x + 3y - z - 2) + k_1(2x - 3y + z - 7) = 0$$

$$x^2 + y^2 + z^2 + (-2+2k_1)x + (3-3k_1)y + (-1+k_1)z - 2 - 7k_1 = 0 \quad \text{--(1)}$$

The equation of the sphere passing through the second circle is

$$(x^2 + y^2 + z^2 - 2x - 4y + 6z + 1) + k_2(x + 2y - 3z - 5) = 0$$

$$x^2 + y^2 + z^2 + (-2+k_2)x + (-4+2k_2)y + (6-3k_2)z + 1 - 5k_2 = 0 \quad \text{--(2)}$$

If two circles are lies on the same sphere,

then (1) = (2)

$\Rightarrow -2 + 2k_1 = -2 + k_2$ -----(3)

$3 - 3k_1 = -4 + 2k_2$ -----(4)

$-1 + k_1 = 6 - 3k_2$ -----(5)

$-2 - 7k_1 = 1 - 5k_2$ -----(6)

Taking (3) and (4)

$2k_1 - k_2 - 0 = 0$

$-3k_1 - 2k_2 + 7 = 0$

$$\begin{matrix} 2 & -1 & 0 & 2 \\ -3 & -2 & 7 & -3 \end{matrix}$$

$$\frac{k_1}{-7+0} = \frac{k_2}{0-14} = \frac{1}{-4-3}$$

$$\frac{k_1}{1} = \frac{k_2}{2} = \frac{1}{1} \qquad \Rightarrow k_1 = 1, k_2 = 2$$

Substituting the value k_1, k_2 in (5)

$-1 + k_1 = 6 - 3k_2$

$-1 + 1 = 6 - 3(2)$

$0 = 0$ satisfied

Substituting the value k_1, k_2 in (6)

$-2 - 7k_1 = 1 - 5k_2$

$-2 - 7(1) = 1 - 5(2)$

$-9 = -9$ satisfied

$\therefore$ The given two circles lies on the same sphere :

The equation of the sphere is put $k_1 = 1$ in (1)

$(x^2 + y^2 + z^2 - 2x + 3y - z - 2) + 1(2x - 3y + z - 7) = 0$

$x^2 + y^2 + z^2 - 9 = 0$