**Unit –II**

**Centre of Gravity**

**Definition:** The Centre of gravity of a body is the point at which the resultant of the weights of all the particles of the body acts, whatever may be the orientation of the body. The total weight of the body may be supposed to act at its centre of gravity.

Suppose the particles A, B, C… of a body have masses m1, m2, m3 … Let their coordinates in a rectangular Cartesian coordinate system be(x1,y1,z1),(x2,y2,z2),…(xn,yn,zn).

Z

P dm

**Z**

Y

Y

X

Then, the coordinates of the centre of gravity G of the body are

=Σ mnyn;

Σ mn

y = Σ mnyn; z = mnzn

Σ mn Σ mn

Suppose an element P of the body has a mass dm (Fig. 3.1) and is coordinate are x, y, z. Then,

=∫x dm = 1 ∫x dm; y =1 ∫y dm ; z = 1 ∫z dm

∫dm M M M

Here, the integrals extend over all elements of the body, and M=∫dm=Total mass of the body.

**Centre of Gravity of a rigid solid cone:**

Let ABC represent a solid cone of height h and semi-vertical angle α.The cone may be considered to be made up of a large number of circular discs parallel to the base. The centre of gravity of the each disc lies at its centre. Therefore, the C.G., of the cone should lie along the axis AD of the cone.

A

αy

B1 C1

r

B C

Consider a disc B1C1 of thickness dy at a distance y below the vertex A. If r is the radius of the disc, than

r = y tan α

Volume of the cone = л r2 h where h=dy

Volume of the disc = Area x thickness = л y2 tan2 α dy

Mass of the disc= dm=лy2 ρ tan2 α dy.

Where ρ = density of the cone.

The distance of the C.G., of the cone from the vertex is given by

y =∫ y dm = o∫h лy3 ρ tan2 α dy = o∫h y3 dy = 3/4 h.

∫dm o∫h лy2 ρ tan2 α dy o∫h y2 dy

Therefore, the C.G., of the cone is along it’s at a distance of 3/4h from the vertex.

**Centre of Gravity of a solid hemisphere:**

L et ABC represent a solid hemisphere of radius r, centre 0 and density ρ. Consider an elementary slice of the hemisphere with radius y and thickness dx, at a distance x from 0.

A

dx

y

O C

x

B

Volume of the slice = лy2dx = л (r2-x2)dx.

Mass of the slice = dm = ρл (r2- x2 ) dx.

The distance of the C.G of the hemisphere from O is given by

x = ∫ x dm =0∫r x ρл(r2-x2) dx= 0∫r(r2 x –x3) dx

∫ dm 0∫r ρл(r2-x2) dx0∫r(r2 –x2) dx

x = 3/8 r.

Hence, the C.G., of the solid hemisphere is on its axis at a distance 3/8 r. from the centre.

***Centre of gravity of a hollow hemisphere***

Let ACB be a section of a hemisphere of radius r, centre O and surface density ρ (Fig. 3.5). Imagine the surface of the hemisphere to be divided into slices like PQQ1 P1 by planes parallel to AB. If ےPOC = θ and ے POQ = dθ, then

A

Q

dθ

P

θ

O C

r

P1

Q1

B

Radius of the ring = r sin θ

Width of the ring = r dθ

Area of the ring = 2лr sinθ.rdθ

Therefore, mass of the ring = dm = 2 лr2ρ sin θ dθ.

The C.G., of this ring is at the Centre of the ring at a distance r cos θ from O.

The distance of the C.G., of the hollow hemisphere from O is given by

x = ∫ x dm = 0∫л/2 (r cosθ)2л2ρ sin θ dθ = 0∫л/2 sin θ cos θ dθ

∫ dm 0∫л/2 2лr2 ρ sinθ.dθ 0∫л/2 sinθ.dθ

x = r/2

The C.G., of a hollow hemisphere is on its axis at a distance r/2 from the centre. i.e., the gravity is at the midpoint of the radius OC.

***Centre of gravity of a solid tetrahedron***

Let ABCD be the tetrahedron and G1 the centre of gravity of the base BCD. Let h be the altitude of the tetrahedron and ρ its density. Suppose the tetrahedron is divided into thin slices by planes parallel to the base BCD. Consider one such slice B1C1D1 of thickness dx at a depth x below A. let s be the area of the triangular base BCD. Then we have,

B1C1 = x

BC h

A

B1 D1

C1

B D

• G1

C

If a1 and a are the altitudes of triangles B1C1D1 and BCD respectively.

a1 = x

a h

Now, area of B1C1D1 = ½ B1C1 x a1

Area of BCD = ½ BC x a = S

Hence, Area of B1C1D1  =B1C1 x a1 = x2

S BC a h2

Therefore, Area of B1C1D1 =Sx2/h2

Volume of the slice B1C1D1 =Sx2dx/h2

Mass of the slice = dm = ρ Sx2dx/h2

The distance of the centre of gravity of the tetrahedron from A is given by

x = ∫ x dm = 0∫hxρ Sx2dx/h2= 0∫hx3dx = ¾ h

∫ dm 0∫hρ Sx2dx/h2 0∫hx2dx

Hence, the C.G., of a uniform tetrahedron lies at a point G on the line AH such that AG : GH = 3:1.

**FLOATING BODIES**

* + The state of equilibrium of a solid body partially or fully immersed in a liquid or gas is called floating
  + In a body to float, the weight of the body should be supported by the buoyant force.

i.e., FB = ρg× Displaced volume of liquid = Weight of the floating body

* + Any floating object displaces its own weight of fluid When any boat

displaces a weight of water equal to its own weight, it floats. This is called the "principle of flotation": A floating object displaces a weight of fluid equal to its own weight.

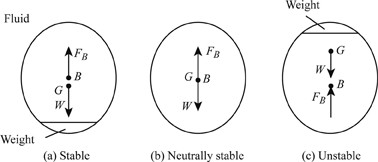
* + Buoyancy is defined as the force applied to the object in an upward direction. Usually, most of the objects experience a force of buoyancy when they are immersed in a fluid and the object also appears to have lost its weight.
  + Archimedes’ principle states that when a body is immersed in a fluid it experiences an upward force of buoyancy equal to the weight of the fluid displaced by the immersed portion of the body.

1. **CONDITIONS OF EQUILIBRIUM OF A FLOATING BODY**

(Material for exam)

The following are the conditions for equilibrium for a floating body

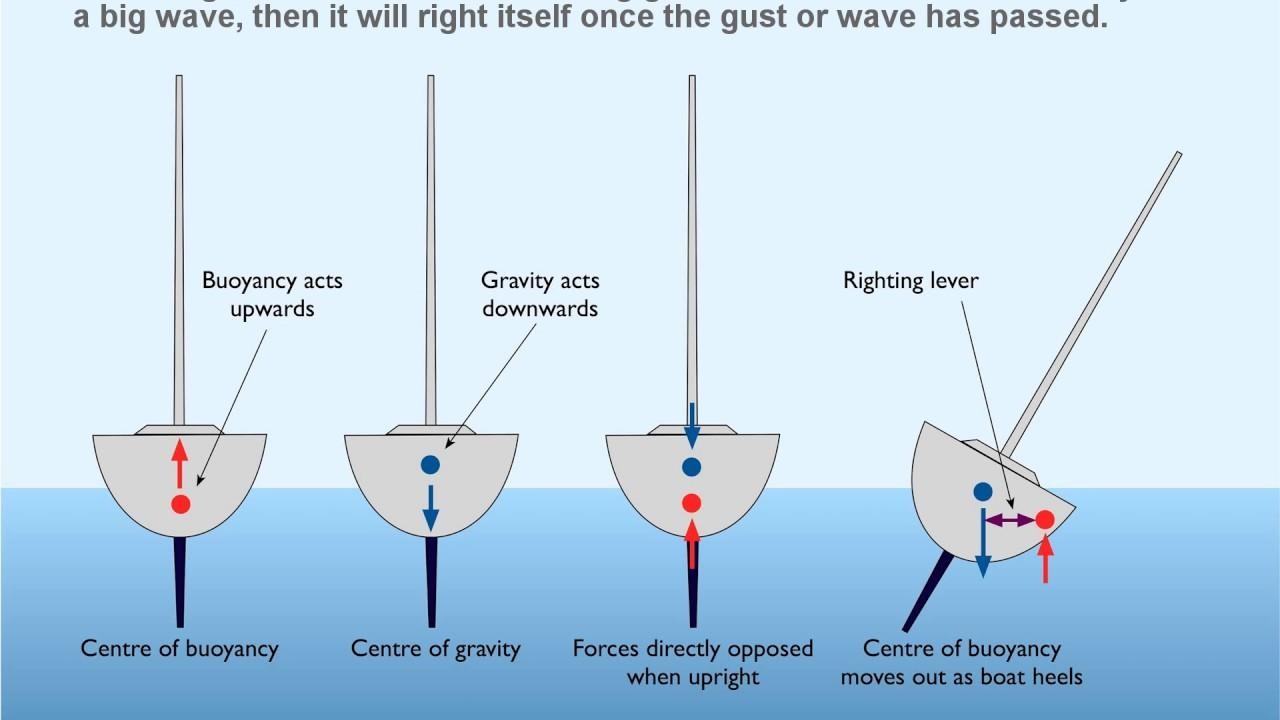
* The weight of the liquid displaced by the immersed part must be equal to the weight of the body.
* The centre of gravity and the centre of buoyancy of the body should be along the same vertical line.
* The centre of gravity lies vertically below the centre of buoyancy (stable equilibrium) and if centre of gravity lies above the centre of buoyancy (unstable equilibrium).



**Stable Equilibrium for Floating Bodies:** If a floating body is given a small angular displacement (disturbing moment) and after the removal of that Force or moment, body comes back to its original position. It is called as Stable Equilibrium.

**Unstable Equilibrium for Floating Bodies:** If a floating body is given a small angular displacement (disturbing moment) and after the removal of that Force or moment, body does not come back to its original position. It is called as UnStable Equilibrium.

**Neutral Equilibrium for floating bodies:** If a body in water or any liquid is given a slight angular displacement, it will neither rotates nor goes to the original position but attains a new position. This type of Equilibrium is known as Neutral Equilibrium.



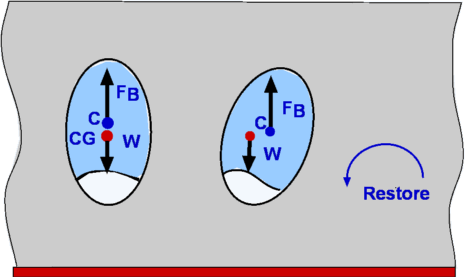
1. **STABILITY OF EQUILIBRIUM OF A FLOATING BODY**

Stability of Immersed and Floating Bodies

Stability becomes an important consideration when floating bodies such as a boat or ferry is designed. It is an obvious requirement that a floating body such as a boat does not topple when slightly disturbed.

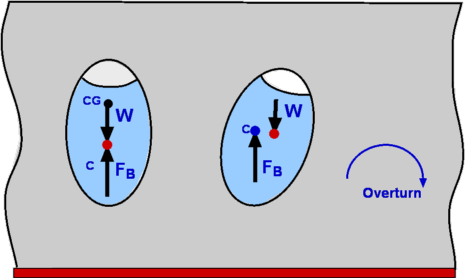
We say that a body is in stable equilibrium if it is able to return to its position when slightly disturbed. Failure to do so denotes unstable equilibrium.

Consider the immersed body shown in fig if the center of gravity of the body lies below the center of buoyancy stable equilibrium prevails



**Stability of immersed bodies**

An overturning couple leading to unstable equilibrium results if the center of gravity is above the center of buoyancy.



Instability of immersed bodies

It becomes more complicated when floating bodies are considered. Now as the body rotates responding to any disturbance the center of buoyancy can shift. This could render the body stable even though the center of gravity is above the center of buoyancy.

This is particularly true of the bodies with a broader base such as a barge (Fig. 3). A slender body as shown in Fig. 4 is very susceptible for instability.

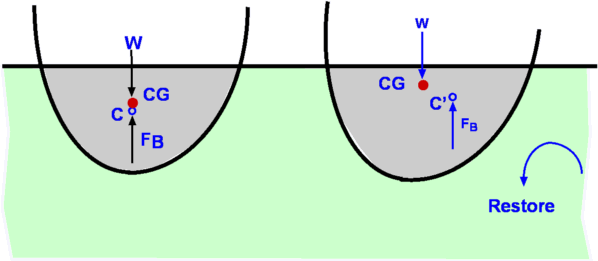
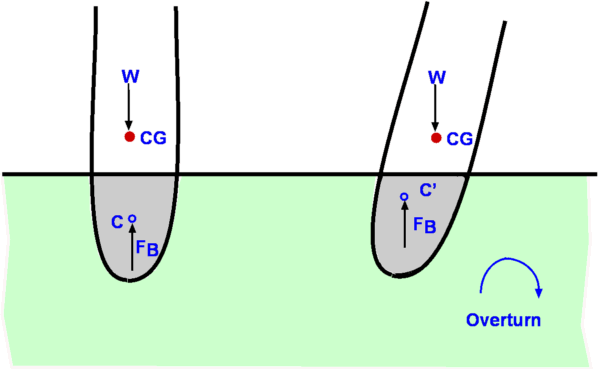


Figure 3: **stability of floating bodies**



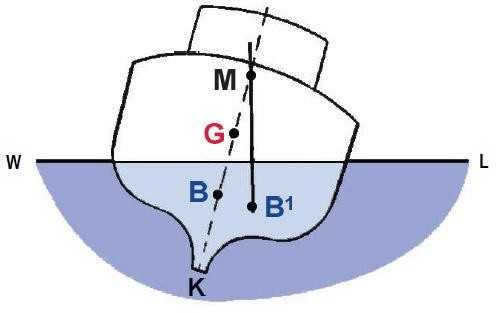
**instability of floating bodies**

**\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\***

**META CENTRE**

When a small angular displacement is given to a body floating in a liquid, it starts oscillation about some point M. This point about which the body starts oscillating is called the metacentre

Metacentre (M) may be defined as the point of intersection of the axis of body passing through centre of gravity (G) and original centre of buoyancy (B) and a vertical line passing through the new centre of buoyancy (B1) of the titled position of the body.



***Metacentric height***: The distance between the centre of gravity of a floating body and the metacentre, i.e. distance GM is called meta-centric height. Relation between centre of gravity and metacentre in different three types of equilibrium:

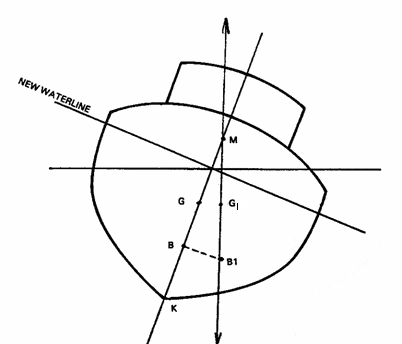
1. ***Stable equilibrium***: In this, position of metacentre (M) remains higher than centre of gravity of body.
2. ***Unstable equilibrium***: In this position of metacentre (M) remains lower than centre of gravity of body.
3. ***Neutral equilibrium***: The position of metacentre (M) coincides with centre of gravity of body.

**EXPERIMENTAL DETERMINATION OF A METACENTRIC HEIGHT OF A SHIP**

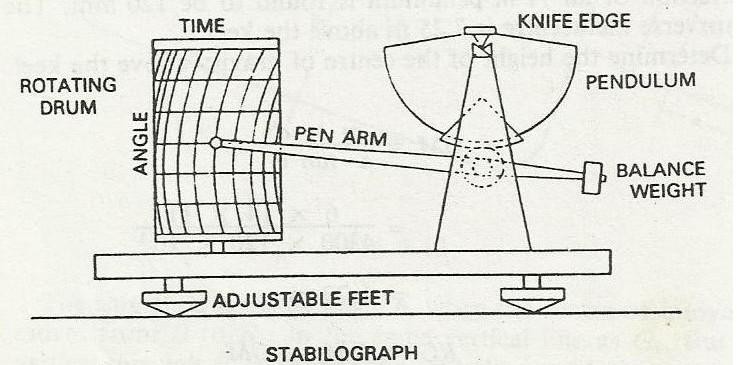
## Inclining Experiment- Determining Metacentric height of the ship

The Metacentric height of the ship plays an important role in setting the loading capacity and stability of the ship. The Initial metacentric height of the ship is determined by an inclining experiment after the ship is completely built.

lets consider a ship which was in eqilibrium is now inclined by an angle θ.



When a ship is heeled by an angle, the center of buoyancy is shifted from B to B1.When a vertical line is drawn from B and B1, they intersect at a point known as **metacentre** of the ship. **The metacentric height** is the distance between the centre of gravity and metacentre of the ship i.e. GM and it is used to calculate the stability of the ship.



## Inclining Experiment Requirements

* The experiment is carried out when the ship is built completely or when major structural changes have been done.
* The experiment is carried out with empty ship or as near to empty ship as possible.
* The ship must be in upright position.
* The ship should be sheltered and in calm waters.
* Mooring ropes should be slackened and gangway lifted.
* Draught and density of water are to be correctly noted.
* All tanks in the ship must be empty or pressed up tight to reduce free surface effect.
* Only those people responsible for conducting the experiment must go onboard.

**The Experiment**

To conduct this experiment, a special tool known as stabilograph is required. The tool consists of a heavy metal pendulum balanced on a knife edge and connected to a pointer to record the heel angle readings.

Normally minimum of two stabilographs are used and are placed at maximum distance from each other i.e. one in forward and one at aft.

Four masses are placed on the ships deck, two on each side of the mid ship, placed away from the centre line.

In the next step, the masses are moved one at a time until all four are on the same side, then all four on the other side, and lastly two on each side.

The deflection on both the stabilographs is recorded for all the movement of mass and an average of these readings are used to determine metacentric height.

Suppose θ is the angle of heel and G1 is the moved position of the centre of gravity after inclination. Then by trigonometry,

GG1= GM tanθ

Also GG1 is = m x d/Δ Where m= mass moved

d= distance by which the mass is moved Δ= displacement of ship in water

Hence GM = m x d /Δtanθ and GM is **metacentric height**

Where tanθ can be determined by the readings of stabilograph.

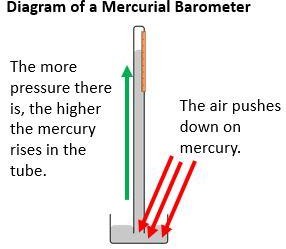
# ATMOSPHERIC PRESSURE

Atmospheric pressure is a force in an area pushed against a surface by the weight of the atmosphere of Earth, a layer of air. This is because high places do not

have as much air above them, pushing down. Barometers can be used to measure atmospheric pressure.

**THE BAROMETER**

# Barometer, device used to measure [atmospheric pressure](https://www.britannica.com/science/atmospheric-pressure). Because atmospheric pressure changes with distance above or below [sea level](https://www.britannica.com/science/sea-level), a barometer can also be used to measure altitude.



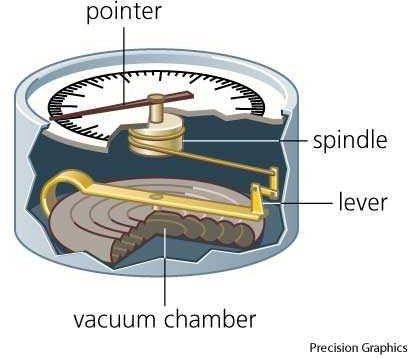
There are two main types of barometers: mercury and aneroid.

In the [mercury barometer](https://www.britannica.com/technology/mercury-barometer), Air pressure pushes down on the surface of the mercury, making some rise up the tube. The greater the air pressure, the higher the mercury rises. You can read the pressure off a scale marked onto the glass.

atmospheric pressure balances a column of mercury, the height of which can be precisely measured. To increase their accuracy, mercury barometers are often corrected for ambient temperature and the local value of gravity.

[Pressure](https://www.britannica.com/science/pressure) : [SI](https://www.britannica.com/science/International-System-of-Units) unit called the Pascal

A nonliquid barometer called the [aneroid barometer](https://www.britannica.com/technology/aneroid-barometer) is used in portable instruments and in aircraft altimeters because of its smaller size and convenience.

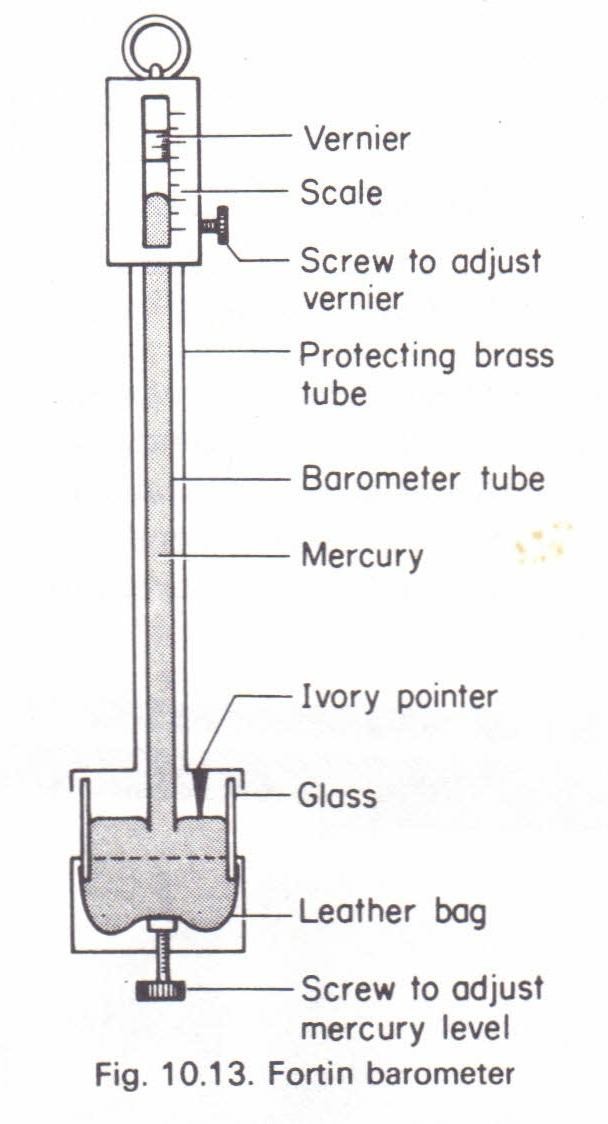
*Aneroid barometer*

# It contains a flexible-walled evacuated capsule, the wall of which deflects with changes in atmospheric pressure. This deflection is coupled to an indicating needle.

A barometer that mechanically records changes in barometric pressure over time is called a barograph.

**FORTIN’S BAROMETER**

The barometer is an instrument used to measure atmospheric pressure. Fortin’s barometer is a modified form of Torricelli’s simple barometer.



## Construction

A Fortin’s barometer consists of a narrow glass tube of length about 90 cm. This tube is closed at one end. The tube is completely filled with mercury and kept inverted in a cistern filled with dry mercury. Usually, the glass tube is protected by enclosing it in a brass tube.

The upper part of the brass tube has a slit that enables the level of the mercury in the glass tube to be seen. A scale graduated in millimetres is attached to the brass tube. This functions as the main scale.

For accurate measurement, a vernier scale that can slide over the main scale is also fixed to the barometer. The vernier scale can be moved up and down using a screw.

The bottom of the cistern is like a bag made of flexible leather. The mercury level can be adjusted by means of a screw provided underneath. There is an ivory pointer in the cistern, placed at the top. The tip of this pointer coincides with the zero of the main scale.

The level of the mercury column in the cistern can be changed with the screw under it. It is so adjusted that the ivory point is exactly at the surface of the mercury in the cistern. The whole apparatus is fixed in a vertical position.

## Working

Any change in the atmospheric pressure is accompanied by an immediate change in the level of the mercury in the glass tube. As the height of the mercury column in the barometer changes, mercury flows between the tube and cistern. As a result, the level of the mercury in the cistern also changes.

To determine the length of the mercury column in the barometer, it is necessary to know the position of the free surface in the cistern as well as in the tube.

The first step in measuring atmospheric pressure using Fortin’s barometer is to set the mercury level in the cistern. Using the adjustment screw, set the level of the mercury in the cistern such that the ivory pointer just touches the mercury.

The reading of the top of the mercury column is then measured using both the main scale and the vernier scale. Before the readings are noted, the vernier scale needs to be positioned properly.

The vernier scale is to be adjusted so that its edge and the corresponding reading in the main scale just set tangentially to the meniscus.

Now, the readings on the main scale and the vernier scale are noted, and the atmospheric pressure is calculated.

## Advantages

The main advantages of Fortin’s barometer are:

* It is portable.
* It allows the mercury level in the cistern to be set to zero. This makes the reading more accurate.

**CORRECTION FOR A BAROMETER**

# Correction of barometer readings:

The mercury barometer’s reading should be corrected to the one and the standard condition.

Standard condition is defined as a temperature of 0 °C, where the density of mercury is13.5951 g/cm3 and a gravity acceleration of 980.665 cm/s2.

During actual observation, the reading should be corrected for the index error, temperature correction, and gravity acceleration as follows:

1. Corrections on index error

Individual mercury barometers include index errors (difference between the value indicated by an individual instrument and that of the standard). The index error is found by comparison with the standard, and the value is stated on a "comparison certificate".

1. Corrections for temperature

The temperature correction means to correct a barometric reading, obtained at a certain temperature, to a value when mercury and graduation temperatures are 0 °C. The temperature of the attached thermometer is used for this purpose.

The height of the mercury column varies with temperature, even the atmospheric pressure is unchanged. The graduation of the barometer is engraved so that the correct pressure is indicated when temperature is 0

°C. In a case that when temperature is above 0 °C, the graduation expands and the measured value will be smaller than the true value.

This effect of temperature must be corrected from these two aspects collectively.

The correction value for temperature *Ct* is expressed as follows:

Ct = - H (μ – λ)t / 1 + μt

where:

*H* hPa is the barometric reading after the correction for index error.

*t* °C is the temperature indicated by the attached thermometer. μ is the volume expansion coefficient of mercury.

λ is the linear expansion coefficient of the tube.

There is a small difference in absolute values for correction between temperatures below and above 0 °C. The values for correction at temperatures above 0 °C are negative and those below 0 °C are positive.

1. Corrections for gravity

Gravity affects the height of the mercury column. After the corrections for index error and temperature, the reading under the local acceleration of gravity has to be reduced to the one under the standard gravity acceleration. This is called corrections for gravity.

The gravity value for correction *Cg* is derived by:

Cg = H0 – H = H ( g – g0 / g0 )

where:

*g0* is the standard gravity acceleration.

*g* is the gravity acceleration at an observing point.

*H* is the barometric reading after the index error and temperature corrections

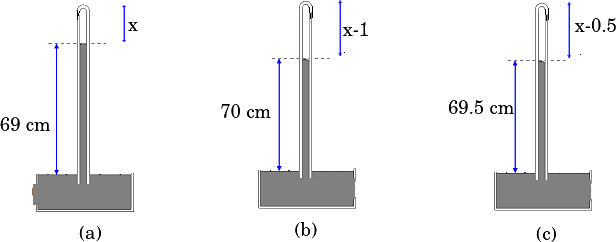
*H0* is the value already corrected for gravitation

When the gravity acceleration at the observing point is larger than the standard gravity acceleration, the gravity value for correction is positive. Otherwise, the value for correction is negative.

**FAULTY BAROMETER**

A faulty barometer contains certains amount of air and saturated water vapour A barometer is faulty. When the true barometer reading are 73 and 75 cm of Hg, the

faulty barometer reads 69 cm and 70 cm respectively. What is the true .reading when the faulty barometer reads 69.5 cm?



ANSWER

Let the density of mercury and cross section area of barometer be *ρ* and *A* respectively. For faulty barometer, *Po*=*ρgh*+*Pair*

Case (a): *ρg*(73)=*ρg*(69)+*P*1 ⟹*P*1=*ρg*(4) Case (b): *ρg*(75)=*ρg*(70)+*P*2 ⟹*P*2=*ρg*(5) Also *P*1*V*1=*P*2*V*2

∴ *ρg*(4)×*Ax*=*ρg*(5)×*A*(*x*−1) ⟹*x*=5*cm*

Case (c): Let the true pressure be shown by barometer reading *h*

Using *P*1*V*1=*P*3*V*3 where *V*3=*A* (*x*−0.5) = *A* (4.5)

∴ *ρg*(4)×*A*(5)=*P*3×*A*(4.5) ⟹*P*3=*ρg*(4.44)

∴ *ρgh* = *ρg*(69.5)+*ρg*(4.44) ⟹*h*=73.94*cm*

The true reading of the barometer will be 73.94 cm.

**VARIATION OF ATMOSPHERIC PRESURE WITH ALTITUDE**

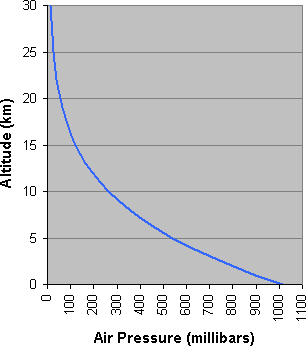
Atmospheric pressure, also known as barometric pressure is the [pressure](https://en.wikipedia.org/wiki/Pressure) within the [atmosphere of Earth](https://en.wikipedia.org/wiki/Atmosphere_of_Earth).

The [standard atmosphere](https://en.wikipedia.org/wiki/Atmosphere_(unit)) (symbol: atm) is a unit of pressure defined as

101,325 [Pa](https://en.wikipedia.org/wiki/Pascal_(unit)) , which is equivalent to 760 [mm Hg](https://en.wikipedia.org/wiki/Millimeter_of_mercury). The Earth's atmospheric pressure at sea level is approximately 1 atm.

The pressure at any level in the atmosphere may be interpreted as the total weight of the air above a unit area at any elevation. At higher elevations, there are

fewer air molecules above a given surface than a similar surface at lower levels. So that the pressure decreases with increasing altitude.



Pressure on Earth varies with the altitude of the surface. So air pressure on mountains is usually lower than air pressure at sea level.

Pressure varies smoothly from the Earth's surface to the top of the [mesosphere](https://en.wikipedia.org/wiki/Mesosphere). Although the pressure changes with the weather that is Temperature and humidity also affect the atmospheric pressure.

