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Tamil Nadu, India

Programme: M.Sc., Environmental Science and Sustainable Management

Course Title : **Geospatial data Modelling and Informatics**
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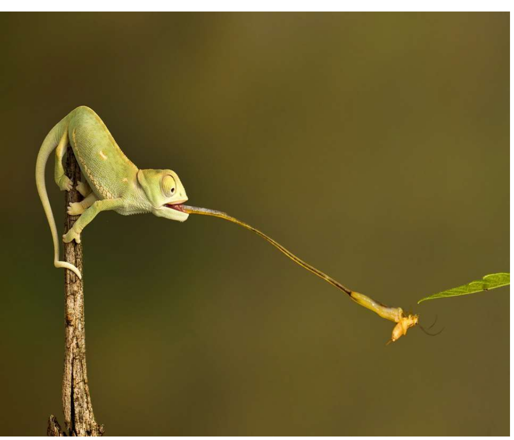
Module-I
Basics of Ecological Models Part 2

Dr. M.Prashanthi Devi
Professor
Department of Environmental Science and Management



LOTKA – VOLTERRA MODEL

Predator

- A **predator** is an organism that hunts, kills and eats other organisms to survive.

Prey

Prey is an organism which gets hunted and taken as food by other organisms.

Lotka – Volterra Model

- Lotka-Volterra model is the simplest model of predator-prey interactions.
- The model was developed independently by Lotka (1925) and Volterra (1926)

Alfred Lotka

Vito Volterra

Alfred James Lotka (March 2, 1880 – December 5, 1949) was a US mathematician, physical chemist, and statistician, famous for his work in population dynamics and energetics.

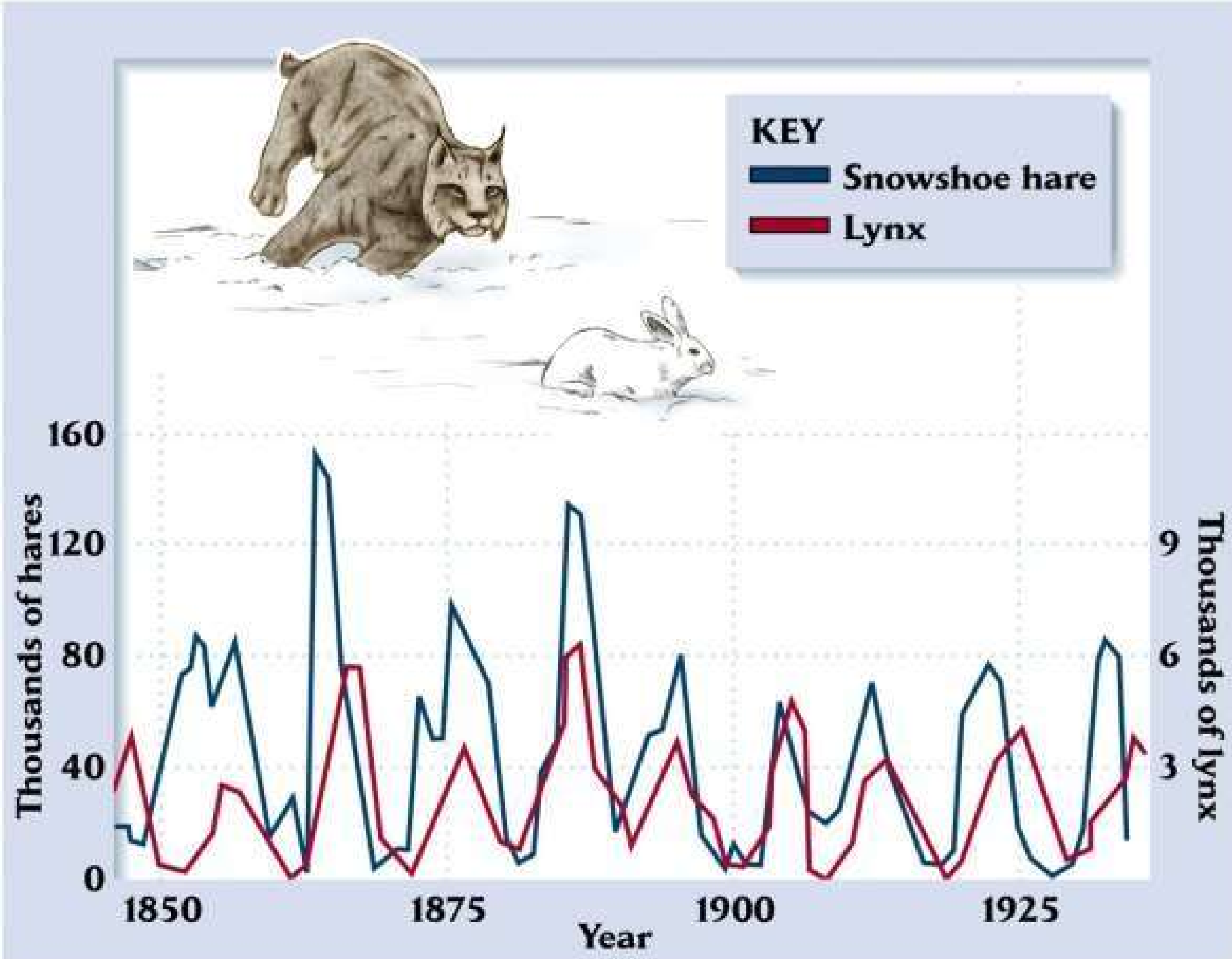
Professor **Vito Volterra** (3 May 1860 – 11 October 1940) was an Italian mathematician and physicist, known for his contributions to mathematical biology.

Lotka – Volterra Model

- Initially proposed by Alfred J. Lotka in the theory of autocatalytic chemical reactions in 1910.
- In 1925 he used the equations to analyse predator–prey interactions in his book on biomathematics.
- The same set of equations was published in 1926 by Vito Volterra, a mathematician and physicist, who had become interested in mathematical biology.
- Volterra's enquiry was inspired through his interactions with the marine biologist Umberto D'Ancona, his son-in-law

Lotka – Volterra Model

- D'Ancona studied the fish catches in the Adriatic Sea and had noticed that the percentage of predatory fish caught had increased during the years of World War I (1914–18).
- This puzzled him, as the fishing effort had been very much reduced during the war years.
- Volterra developed his model independently from Lotka and used it to explain d'Ancona's observation.



Lynx and snowshoe hare data of the Hudson's Bay Company

Lotka–Volterra equations

- The **Lotka–Volterra equations**, also known as the **predator–prey equations**, are a pair of first-order **nonlinear differential equations.**
- Frequently used to describe the dynamics of biological systems in which two species interact, one as a predator and the other as prey.

Assumptions

1.The prey population finds ample food at all times.

1.The food supply of the predator population depends entirely on the size of the prey population.

1.The rate of change of population is proportional to its size.

During the process, the environment does not change in favour of one species, and genetic adaptation is inconsequential.

1.Predators have limitless appetite.

Lotka–Volterra equations

Prey equation

$$\frac{\delta x}{\delta t} = \alpha x - \beta x y$$

Predator equation

$$\frac{\delta y}{\delta t} = \delta x y - \gamma y$$

- x is the number of prey
- y is the number of some predator
- $\frac{\delta x}{\delta t}$ and $\frac{\delta y}{\delta t}$ represent the instantaneous growth rates of the two populations;
- t represents time;
- α, β, γ, δ are positive real parameters describing the interaction of the two species

Prey equation $\frac{\delta x}{\delta t} = \alpha x - \beta x y$

- The prey are assumed to have an unlimited food supply and to reproduce exponentially, unless subject to predation
- This exponential growth is represented in the equation above by the term αx.
- The rate of predation upon the prey is assumed to be proportional to the rate at which the predators and the prey meet, this is represented above by $\beta x y$.
- If either x or y is zero, then there can be no predation.

The rate of change of the prey's population is given by its own growth rate minus the rate at which it is preyed upon.

Predator equation $\frac{\delta y}{\delta t} = \delta x y - \gamma y$

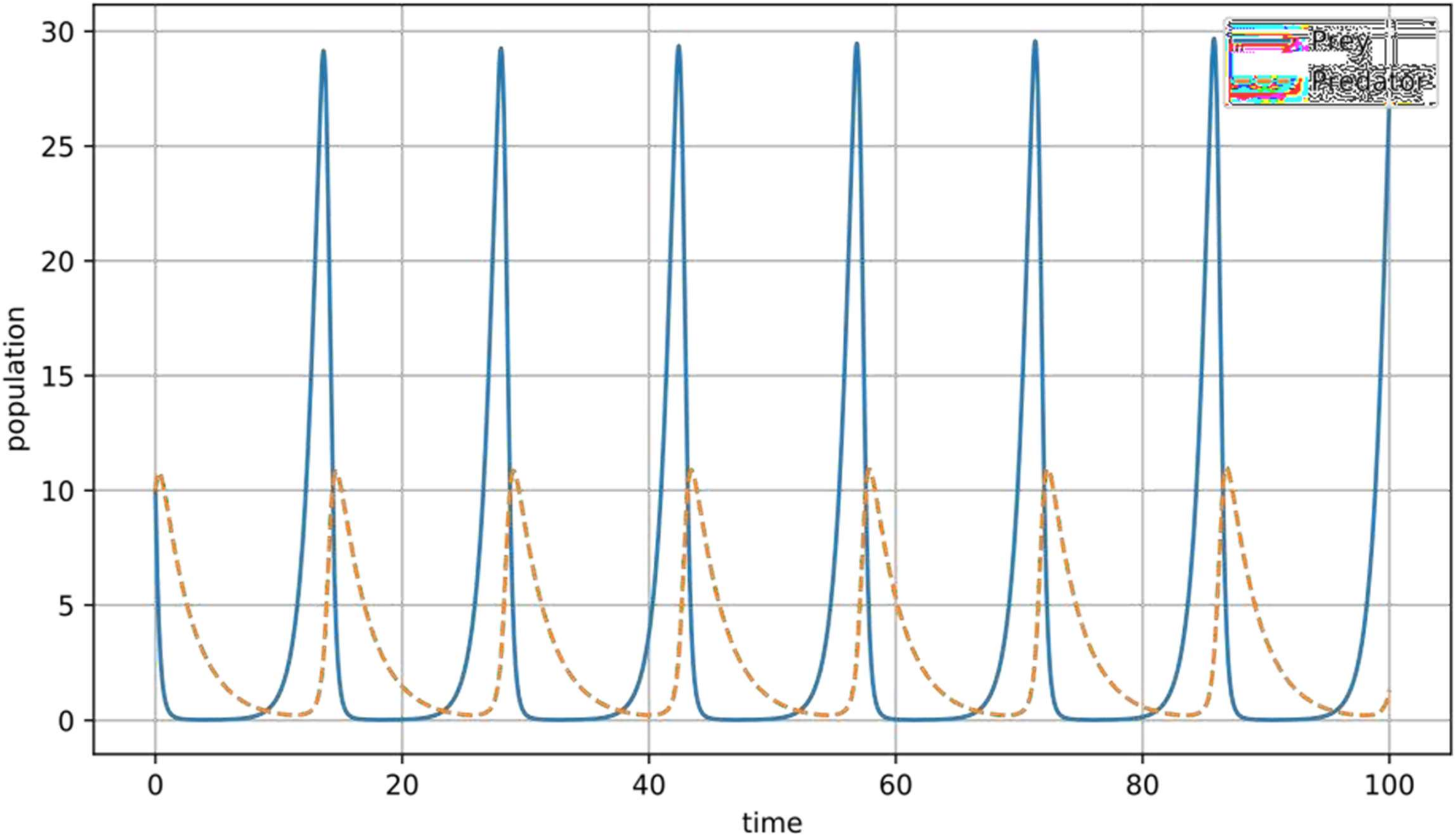
- In this equation, $\delta x y$ represents the growth of the predator population.
- Note the similarity to the predation rate; however, a different constant is used, as the rate at which the predator population grows is not necessarily equal to the rate at which it consumes the prey.
- γy represents the loss rate of the predators due to either natural death or emigration, it leads to an exponential decay in the absence of prey.

The rate of change of the predator's population depends upon the rate at which it consumes prey, minus its intrinsic death rate

Example

- Suppose there are two species of animals, a baboon (prey) and a cheetah (predator).
- If the initial conditions are 10 baboons and 10 cheetahs, one can plot the progression of the two species over time;
- given the parameters that the growth and death rates of baboon are 1.1 and 0.4 while that of cheetahs are 0.1 and 0.4 respectively.
- The choice of time interval is arbitrary.
- One may also plot solutions parametrically as orbits in phase space, without representing time, but with one axis representing the number of prey and the other axis representing the number of predators for all times.

Population of Cheetahs and baboons



- This corresponds to eliminating time from the two differential equations above to produce a single differential equation

$$\frac{\delta x}{\delta t} = \alpha x - \beta x y \qquad \frac{\delta y}{\delta t} = \delta x y - \gamma y$$

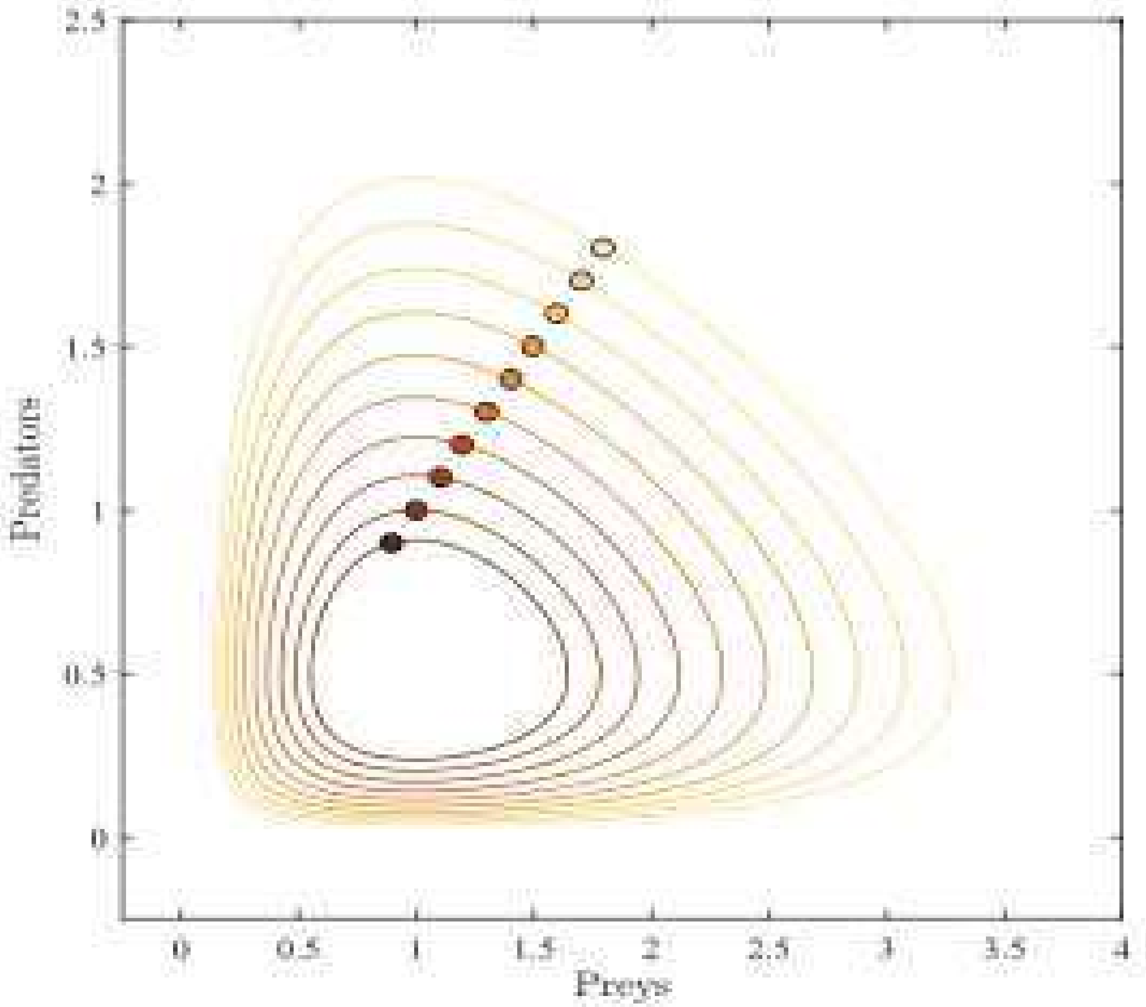
$$\frac{\delta y}{\delta x} = -\frac{y}{x}\frac{\delta x - \gamma}{\beta y - \alpha}$$

$$\frac{\beta y - \alpha}{y}\delta y + \frac{\delta x - \gamma}{x}\delta x = 0$$

$$\int \frac{\beta y - \alpha}{y} \delta y + \frac{\delta x - \gamma}{x} \delta x = 0$$

$$V = \delta x - \gamma\, ln(x) + \beta y - \alpha\, \ln(y)$$

V is a constant quantity depending on the initial conditions and conserved on each curve.



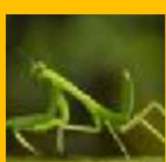
Phase-space plot for the predator prey problem for various initial conditions of the predator population.

$\alpha = 2/3$, $\beta = 4/3$, $\gamma = 1 = \delta$. Assume x, y quantify thousands each. Circles represent prey and predator initial conditions from $x = y = 0.9$ to 1.8, in steps of 0.1. The fixed point is at (1, 1/2).

Equilibrium Solutions to the Equations

Since we know that two populations can be in equilibrium with one another, we can solve the Lotka-Volterra equations to determine when this occurs.

Since equilibrium means that the populations are not changing with respect to time the system of equations can be written as:



$$\mathbf{0 = \alpha x - \beta xy} \qquad \mathbf{0 = \delta xy - \gamma y}$$

Equilibrium Solutions to the Equations

- Which can then be solved for solutions of x and y, giving us the following two solutions:

1. $\boldsymbol{x} = \mathbf{0}$ and $\boldsymbol{y} = \mathbf{0}$
2. $\boldsymbol{x} = \frac{\alpha}{\beta}$ and $\boldsymbol{y} = \frac{\gamma}{\boldsymbol{\delta}}$

- The first solution shows that if both populations are extinct, then they will continue be extinct until an outside factor can change that
- The second solution describes an equilibrium where the population of the prey is equal to the ratio of the constants α and β, and the population of the predator is a ratio of δ and γ
- Since both equations of the second solution depend on the biologically determined parameters, the population at which each equilibrium occurs will depend of the chosen values of the constants.

Summary

- The predator-prey relationship is substantial in maintaining the equilibrium between various animal species.
- Without predators, certain species of prey would force other species to extinction as a result of competition.
- Without prey, there would be no predators
- The key feature of predation thus is the predator's direct impact on the prey population.

Problems with the Lotka-Volterra Equations

- The most significant problem of the LotkaVolterra equations as a biological model is the ability of a prey population to "**bounce back**" even when subjected to extremely low population numbers.
- This is **very rarely seen in real-life scenarios** as the likelihood would be that the prey population would go extinct which would then cause the **extinction** of the predator population very soon after.

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Special adaptations of predators

Birds of prey have keen vision, sharp beaks and talons

Snakes have improved sense of smell and are venomous.

Few have excellent camouflage to stalk and capture prey

Bats have extremely sensitive hearing

Special structures to capture prey

Group hunting to take down animals much larger than themselves

Prey defence

Prey use camouflage to remain undetected by their predators

Some may copy the behaviour of a dangerous animal or have specialised body parts that look dangerous

The skunk sprays the attacker with its signature scent

Few like the porcupine have defense structures

Ground squirrels - alarm calls for warning conspecifics


Thank you