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SUBJECT CODE : MCA24201

UNIT-I

Operation Research

Introduction:

The term operation research was first coined by Mccloskey and Trefthen in 1940. This new science came into existence as a result of research on military operations during world war II.

O.R:

New approach to systematic and scientific study of the operations of the system was called the operations research (or) operational research.

O.R has been variously described as the "science of use" "quantitative common sense", "scientific approach to decision-making problems". etc.

Nature and features of O.R:

i) O.R is the application of scientific methods, techniques and tools to problems involving the operation of a system so as to provide those in control of the system with optimum solution to the problem.

— Churchman, Ackoff and Arnoff

ii) O.R is a scientif knowledge through interdisciplinary team effort for the purpose of determining the best utilization of limited resources"

— H.A. Taha

ADVANTAGES AND LIMITATIONS OF MODELS:

i) Through a model, the problem under considera becomes controllable

ii) It provides some logical & systematic approach to the problem.

iii) It indicates the limitations and scope of an activity.

iv) It helps in incorporating useful tools that dis eliminate duplication of method applied to solve any specific problems.

v) It helps in finding avenues for new research & improvements in a system.

vi) It provides economic description and explanations of the operations of the system they represent.

Limitation :-

i) Models are only an attempt in understanding operations and should never be considered as absolute in any sense.

ii) validity of any model with regard to corresponding operation can only be verified by carring out the experiment and ~~observation~~ observing relevant data characteristics.

iii) construction of models require experts from various disciplines.

Objectives of O.R:

i) OR aims to decision making and improve the quality of each and every operations of the business.

ii) It aims to maximize the profit and reduce cost of each and every operation, by optimization of total output.

iii) OR aims to increase the productivity in the business by optimization of full output in the business.

iv) To develop more effective approach to complete the particular task.

v) To learn all about administration and management in socio culture for the purpose of effective implementation at every stage.

vi) OR also aims to ~~indu~~ introduced many new digital concepts in operational Management.

Scope:

1) In agriculture:-

* Increase population result in many pr

* optimum allocation of land to a variety of crops as per the climatic conditions.

* optimum distribution of water from numerous resources like canal for irrigation purpose.

Hence there is a requirement of determining best policies under the given restrictions. Therefore a good quantity of work can be done in this direction

2) In industry:

* Mostly Industry make decisions on past basis and hence chances of serious loss happens. This loss can be compensated through OR techniques.

* Thus OR is helpful for the industry director in deciding optimum distribution of several limited resources like men, machines, material, etc to reach at the optimum decision.

3) In production management:-

* A production manager can utilize OR techniques to calculate the number and size of the items to be produced.

* In scheduling and sequencing the production machines

* In computing the optimum product mix

* To choose, locate and design the sites for the production plans

4) Finance, Budgeting and Investment:

*) cash flow analysis, long range capital requirement, dividend policies, investment portifolious.

* Credit policies, credit risks and delinquent account procedures.

* claim and complaint procedure.

5) Marketing:

* product selection, timing, competitive actions
* Advertising mean with respect to cost and time
* Number of salesmen, frequency of calling of account, etc.
* Effectiveness of market research.

6) personel:

* Forecasting the manpower requirement, recruitment policies and job assignments.
* selection of suitable personal with the consideration for age and silks, etc.
* Determination of optimum number of persons for each service centre.

phases:-

1) Pre modeling phase:
   1) Identification of problem.
   2) Quantify the problem.

2) Modeling phase:
   3) Data collection.
   4) Formulation a mathematical model of problem
   5) Identification of possible alternative solutions.

3) Implementation phase:
   6) Interpretation of solution.
   7) model validation
   8) Monitor and control

Models:-

A model is an ideal representation of a real system. System can be a problem, process, operation, obeject or event.

Types of Models:

1) classifications based on Functions:

i) Normative Models ii) predictive models
iii) Descriptive Models.

Normative Models:

These models provide the best solution to problems subject to certain limitations. These models are also called optimization models or prescriptive models because they precribe what have to be done.

Exp: Linear programming, x-ray of healthy man, CPM & PERT planning model.

ii) predictive model:-

These models predict the outcomes regarding certain event due to a given set of alternatives for the problem. They can answer, "what is type of questions"

Ex: Television network predict the election outcome before counting the votes based on the survey results.

iii) Descriptive Models:-

These models describe the system under study based on observation, survey, questionnaire results.

Ex: Organization chart, plant layout diagram, scale models etc.

2) classification based on structure:-

i) Iconic Models:
Iconic Models is a physical or pictorial or visual repres./ of the real system. They are scaled up or scaled down versions of the particular system they represent.
Ex: Model or Blue prints of proposed building, models of sun & planets are scaled down & model of atom, models of cells in human body are scaled up.

ii) Analogue Models:-
These models repre./ a system by a set of properties which is different from the original system & the does not resemble it physically.
EX: A barometer that indicates change in atomospheric pressure through movement of a needle, graphs, flow diagrams, charts etc.

3) classification bassed on Nature of an environment
i) Deterministic Models:
In these models all parameters and functional relationship are assumed to be known with certainty when decision is to be made.
EX: Linear programming, Transportation, Assignment Models.

ii) probabilistic Models or stochastic Models:

These type of models usually such situation in which outcome of managerial action can not be predicted with certainty.

Ex: Insurance companies are willing to ensure against risk of fire, accidents, sickness.

Linear Programming Problem

Max (or) Min $z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$ — (1)

where c_j's are real constants.

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &\le \text{ or } \ge \text{ or } = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &\le \text{ or } \ge \text{ or } = b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &\le \text{ or } \ge \text{ or } = b_m \end{aligned}\right\} \text{ — (2)}$$

where a_{ij}'s b_j's are real constants.

$$\& \ x_j \ge 0 \quad j = 1, 2 \cdots n \text{ — (3)}$$

Linear programming problem deals with the optimization (Max (or) Min) of a function of decision variables known as obejective function, subject to a set of simultanous linear equations known as constraints and non-negative constrains is called LPP.

Here (1) is called as the objective function
(2) is called the subject constrains
(3) are called the non-negative restrictions.

procedure for Mathematical formulation of LPP:

1) Identify the unknown decision variable to be determined and assign symbols to them.
2) Identify all the restrictions or constraints in the problem and express them as linear equations or inequalities of decision variables.
3) Identify the objective or aim & represent it also as a linear function of decision variables.
4) Express the complete formulation of LPP as a general mathematical model.

problems:

1) A firm manufactures two types of products A & B and sells them at a profit of Rs. 2 on type A and Rs. 3 on types B. Each product is processing time on M_1 and 2 minutes on M_2. Type B requires 1 minute on M_1 & 1 minute on M_2. Machine M_1 is avaliable for not more than 6 hours 40 minutes while machine M_2 is avaliable for 10 hours during any working day. Formulate the problem as a LPP so as to maximize the profit.

Soln:

let the firm decide to produce x_1 units of product A and x_2 units of products B to maximize its profit.

To produce these unit of type A & type B products, it requires.

x_1+x_2 processing minutes on M_1
$2x_1+x_2$ processing minutes on M_2

Since machine M_1 is avalible for not more than 6 hours & 40 minutes & machine M_2 is available for 10 hours doing any working day, the constraints are

1hr = 60min
60x6 = 360 min
+40
400

$$x_1+x_2 \le 400$$

$$2x_1+x_2 \le 600$$

Since the profit from type A is Rs. 2 & profit from type B is Rs. 3, the total profit is $2x_1+3x_2$. As the objective is to maximize the profit, the objective function is maximize $Z=2x_1+3x_2$.

∴ The complete formulation of the LPP is

Maximize $Z = 2x_1+3x_2$

subject to the constraints

$$x_1+x_2 \le 400$$

$$2x_1+x_2 \le 600$$

$$\& \; x_1, x_2 \ge 0$$

2) A firm produces three products. These products are processed on three different machines. The time required to manufacture one unit of each of the three products and the daily capacity of the three machines are given in the table below.

Machine	Time per unit (min)			Machine capacity (Min/day)
	pro 1	pro 2	pro 3	
M_1	2	3	2	440
M_2	4	–	3	470
M_3	2	5	–	430

It is required to determine the no. of units to be manufactured for each product daily. The profit per unit for product 1, 2 & 3 is Rs. 4, Rs. 3 & Rs. 6 respectively. It is assumed that all the amounts produced are consumed in the market. Formulate ~~and~~ the mathematical model for the problem.

Soln:

Let x_1, x_2 & x_3 be the number units of products 1, 2 & 3 produced respectively.

To produce these amount of products 1, 2 & 3 it requires.

$2x_1 + 3x_2 + 2x_3$ min on M_1

$4x_1 + 3x_3$ min on M_2

$2x_1 + 5x_2$ min on M_3

But the capacity of the Machines M_1, M_2 & M_3 are 440, 470 & 430 (min/day).

∴ The constraints are

$$2x_1 + 3x_2 + 2x_3 \leq 440$$

$$4x_1 + 3x_3 \leq 470$$

$$2x_1 + 5x_2 \leq 430$$

$$\& \; x_1, x_2, x_3 \geq 0$$

since the profit per unit for product 1,2 & 3 is Rs. 4, Rs. 3 & Rs. 6 respectively, the total profit is $4x_1 + 3x_2 + 6x_3$. As the objective is to maximize the profit, the objective function is maximize $Z = 4x_1 + 3x_2 + 6x_3$

$\therefore$ The complete formulation of the LPP is

Maximize $Z = 4x_1 + 3x_2 + 6x_3$

Subject to the constraints

$$2x_1 + 3x_2 + 2x_3 \leq 440$$

$$4x_1 + 3x_3 \leq 470$$

$$2x_1 + 5x_2 \leq 430$$

$$\& \ x_1, x_2, x_3 \geq 0.$$

3) A person wants to decide the constituents of a diet which will fulfil his daily requirments of proteins, fats and carbohydrates at the minimum cost. The choice is to be made from four different types of foods. The yields per unit of these foods are given in the following table.

Food type	yield/unit			cost/unit (Rs)
	proteins	Fats	carbohydrates	
1	3	2	6	45
2	4	2	4	40
3	8	7	7	85
4	6	5	4	65
Minimum	800	200	700	

Solu:

let x_1, x_2, x_3 & x_4 be the unit of food of type 1, 2, 3 & 4 used respectively.

From these units of food of type 1, 2, 3 & 4

$3x_1 + 4x_2 + 8x_3 + 6x_4$ proteins/day

$2x_1 + 2x_2 + 7x_3 + 5x_4$ Fats/day

$6x_1 + 4x_2 + 7x_3 + 4x_4$ Carbohy/day.

Since the minimum requirement of these proteins, fats and carbohydrates are 800, 200 and 700 respectively, the constraints are

$$3x_1 + 4x_2 + 8x_3 + 6x_4 \geq 800$$
$$2x_1 + 2x_2 + 7x_3 + 5x_4 \geq 200$$
$$6x_1 + 4x_2 + 7x_3 + 4x_4 \geq 700$$
$$\& \; x_1, x_2, x_3, x_4 \geq 0.$$

$\because$ the costs of these food of type 1, 2, 3 & 4 are Rs. 45, Rs. 40, Rs. 85, & Rs. 65 per unit, the total cost is Rs. $45x_1 + 40x_2 + 85x_3 + 65x_4$. As the obejective is to minimize the total cost, the objective function is

$$\text{Minimize } z = 45x_1 + 40x_2 + 85x_3 + 65x_4$$

$\therefore$ The complete formulation of the LPP is

$$\text{Minimize } z = 45x_1 + 40x_2 + 85x_3 + 65x_4$$

subject to
$$3x_1 + 4x_2 + 8x_3 + 6x_4 \geq 800$$
$$2x_1 + 2x_2 + 7x_3 + 5x_4 \geq 200$$
$$6x_1 + 4x_2 + 7x_3 + 4x_4 \geq 700$$
$$\& \; x_1, x_2, x_3, x_4 \geq 0$$

Formulation of LPP (Graphical solution of LPP)

The major steps in the solu/ of a LPP by graphical method ~~are~~

1) Identify the problem - the decision variable, the objective & the restrictions.
2) Set up the mathematical formulation of the problem.
3) Plot a graph representing all the constraints of the problem and identify the feasible region (solution space). The feasible region is the intersection of all the regions repre/ by the constraints of the problem and is restricted to the first quadrant only.
4) The feasible region obtained in step 3 may be bounded (or) unbounded. Compute the coordinates of all the corner points of the feasible region.
5) Find out the value of the objective function at each corner (solution) point determined in step 4.
6) Select the corner point the optimizes (maxi (or) mini) the values of the objective function. It gives the optimum feasible solution.

Problem

1) Solve the following LPP Method using graphical method

$$\text{Max } Z = 3x_1 + 2x_2$$

subject to $-2x_1 + x_2 \le 1$

$x_1 \le 2$

$x_1 + x_2 \le 3$ and $x_1, x_2 \ge 0$

Soln:

First consider the inequality constraints as equalities.

$-2x_1 + x_2 = 1$ —— (1)

$x_1 = 2$ —— (2)

$x_1 + x_2 = 3$ —— (3)

and $x_1 = 0$ —— (4)

$x_2 = 0$ —— (5)

equ (1) ⇒ put in $x_1 = 0$

$-2x_1 + x_2 = 1$

$-2(0) + x_2 = 1$ $\quad$ (0,1)

$\boxed{x_2 = 1}$

(1) ⇒ put in $x_2 = 0$ $\quad -2x_1 + (0) = 1$

$-2x_1 = 1 \Rightarrow \boxed{x_1 = -1/2}$

$(-1/2, 0)$

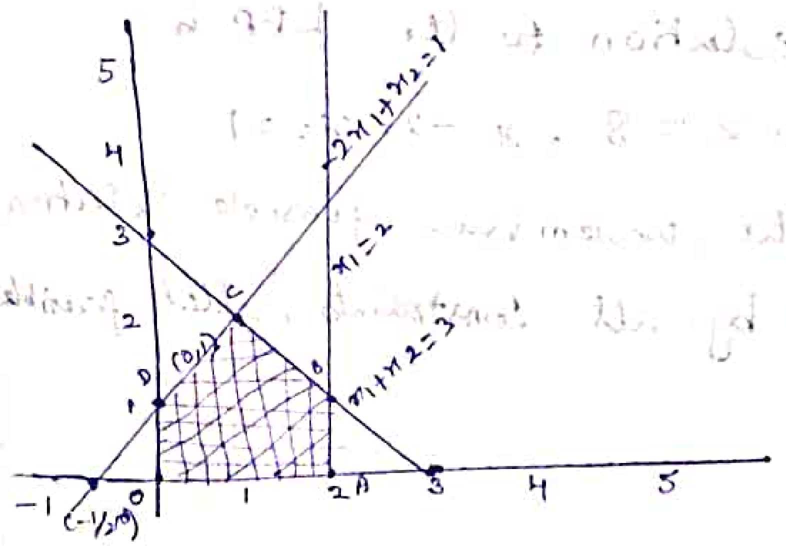
equ (3) ⇒ put in $x_1 = 0$

$x_1 + x_2 = 3$

$0 + x_2 = 3$ $\quad \boxed{x_2 = 3}$ $\quad$ (0,3)

put in $x_2 = 0$ $\quad x_1 + (0) = 3$ $\quad$ (3,0)

$\boxed{x_1 = 3}$



B point $x_1+x_2=3$

$-x_1=2$

$x_2=1$

$x_1+x_2=3$

$x_1+(1)=3$

$x_1=3-1$

$\boxed{x_1=2}$

C point $-2x_1+x_2=1$

$-x_1+x_2=3$

$+3x_1=+2$

$x_1=2/3$

$x_1+x_2=3$

$2/3+x_2=3$

$x_2=3-\frac{2}{3}=\frac{9-2}{3}$

$=7/3$

$\boxed{x_2=7/3}$

∴ The vertices of the solution space are O(0,0) A(2,0) B(2,1) C(2/3, 7/3) & D(0,1)

The values of z at these vertices are given by

vertex	value of z ($z=3x_1+2x_2$)
O(0,0)	$3(0)+2(0)=0$
A(2,0)	$3(2)+2(0)=6$
B(2,1)	$3(2)+2(1)=\boxed{8}$
C($\frac{2}{3}$, $\frac{7}{3}$)	$3(2/3)+2(7/3)=\frac{20}{3}$
D(0,1)	$3(0)+2(1)=2$

Since the problem is of Maximization type the optimum solution to the LPP is,

Maximum $z=8$, $x_1=2$; $x_2=1$

Hence the problem have feasible solution

The area bounded by all constraints, called feasible region.

2) Use the graphical method to solve the following LPP.

Minimize $z = -x_1 + 2x_2$

subject constraints $-x_1 + 3x_2 \leq 0$

$x_1 + x_2 \leq 6$

$x_1 - x_2 \leq 2$ and

$x_1, x_2 \geq 0$.

Ans: Min: -2

3) use graphical method for LPP

Maximize $z = 6x_1 + x_2$

S.con $2x_1 + x_2 \geq 3$

$x_2 - x_1 \geq 0$ & $x_1, x_2 \geq 0$

Solu:

First consider the inequality constraints as equations.

$2x_1 + x_2 = 3$ — (1)

$x_2 - x_1 = 0$ — (2)

and $x_1 = 0$ — (3)

$x_2 = 0$ — (4)

equ (1) $\Rightarrow$ put $x_1 = 0$

$2x_1 + x_2 = 3$

$2(0) + x_2 = 3$

$(0, 3)$

$x_2 = 0 \Rightarrow 2x_1 + 0 = 3$

$x_1 = 3/2$

$(3/2, 0)$

equ (2) $\Rightarrow x_2 - x_1 = 0$

put $x_1 = 0 \Rightarrow x_2 - 0 = 0$

$x_2 = 0$

$(0, 0)$

$x_2 = 0 \Rightarrow 0 - x_1 = 0$

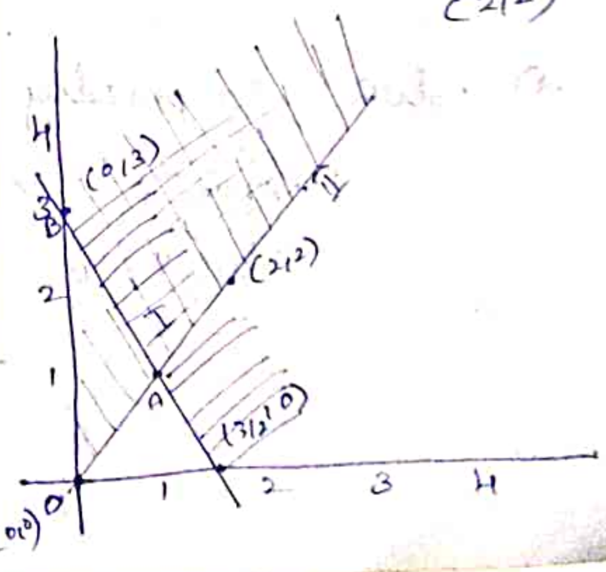
$x_1 = 0$

$(0, 0)$

$x_1 = x_2$

we take any value

$(2, 2)$



Two extreme point of the feasible region are A & B

The feasible region is unbounded.

A point

$$2x_1 + x_2 = 3$$
$$\overset{+}{-}x_1 \overset{-}{+} x_2 = 0$$

$$3x_1 = 3$$
$$x_1 = \frac{3}{3}$$
$$\boxed{x_1 = 1}$$

put x_1 value in any equation

$$2(1) + x_2 = 3$$
$$x_2 = 3 - 2$$
$$\boxed{x_2 = 1}$$

A(1, 1)

B point

(0, 3)

∴ The vertices of solution are A(1,1), B(0,3)

vertex	value of z $z = 6x_1 + x_2$
A(1,1)	$6(1) + 1 = 7$
B(0,3)	$6(0) + 3 = 3$

$z(A) = 7$, $z(B) = 3$

But there are points in this convex region for which z will have much higher values. In fact, the maximum value of z occurs at infinity. Hence the problem has unbounded solution.

4) solve graphically the following LPP.

Maximize $\cancel{Z = 4x_1 + 3x_2}$ $z = x_1 + x_2$

subject to $x_1 + x_2 \leq +1$

$$-3x_1 + x_2 \geq 3$$
$$x_1, x_2 \geq 0$$

Soln:

$x_1+x_2=1$ — ①

$-3x_1+x_2=3$ — ②

equ ① $\Rightarrow$ $x_1+x_2=1$

put $x_1=0$ $\quad 0+x_2=1$

$\boxed{x_2=1}$

$(0,1)$

put $x_2=0 \Rightarrow x_1+0=1$

$x_1=1$

$(1,0)$

equ ② $\quad -3x_1+x_2=3$

put $x_1=0$ $\quad -3(0)+x_2=3$

$\boxed{x_2=3}$

$(0,3)$

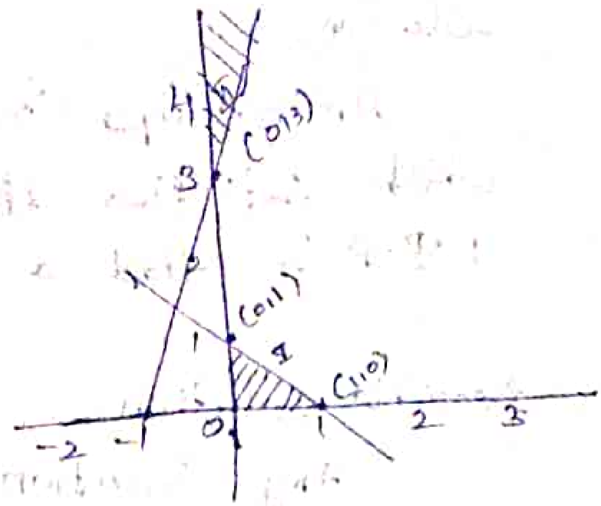
put $x_2=0$ $\quad -3x_1+x_2=3$

$-3x_1+0=3$

$x_1=\frac{-\cancel{3}}{\cancel{3}}$

$\boxed{x_1=-1}$

$(-1,0)$



Here the constraints are not satisfied simultaneously.

$\therefore$ The given LPP has no feasible region and hence we get an infeasible solution

5) Find graphical method Maximum $z=10x_1+6x_2$

Subject to $5x_1+3x_2\le 30$

$x_1+2x_2\le 18$ &

$x_1, x_2\ge 0$

6) find graphical Maximum $z=3x_1+2x_1$

Subject to $2x_1+x_2\le 2$

$3x_1+4x_2\ge 12$ &

x_1 & $x_2\ge 0$

Solution:

An n-tuple $(x_1, x_2 \dots x_n)$ of real numbers which satisfies the constraints of a General L.P.P is called a Solution to the General LPP

Feasible solution:-

Any solution to a General LPP which also satisfies the non-negative restrictions of the problem. is called a feasible solution to the General LPP.

optimum Solution:-

Any feasible solu.) which optimizes (min (or) max) the objective function of a General LPP is called a optimum Solution.

Slack and surplus variables:-

Slack:

let the constraints of a general LPP be

$\sum_{j=1}^{n} a_{ij} x_j \le b_i \quad i=1,2 \dots$

Then the non-negative variables x_{n+i} which satisfy $\sum_{j=1}^{n} a_{ij} x_j + x_{n+i} = b_i$ are called slack variable.

surplus:-

let the Constraints of a general LPP be $\sum_{j=1}^{a} a_{ij} x_j \ge b_i \quad i=k+1, k+2 \dots$

Then the non-negative variable x_{n+i} which satisfy $\sum_{j=1} a_{ij} x_j - x_{n+i} = b_i$ are called surplus variable

Simplex Method :-

1) Use simplex method to solve the LPP

$$\text{Maxi } z = 4x_1 + 10x_2$$

subject to $2x_1 + x_2 \le 50$

$2x_1 + 5x_2 \le 100$

$2x_1 + 3x_2 \le 90$ and $x_1, x_2 \ge 0$

Soln.

Introducing the slack variables s_1, s_2 & s_3 the problem in standard form becomes

$$\text{Max } z = 4x_1 + 10x_2$$

sub to $2x_1 + x_2 + s_1 + 0s_2 + 0s_3 = 50$

$2x_1 + 5x_2 + 0s_1 + s_2 + 0s_3 = 100$

$2x_1 + 3x_2 + 0s_1 + 0s_2 + s_3 = 90$

and $x_1, x_2, s_1, s_2, s_3 \ge 0$

∴ there are 3 equations with 5 variables, the initial basic feasible solution is obtained by equation $(5-3) = 2$ variable to zero.

∴ The initial basic feasible solution is $s_1 = 50$, $s_2 = 100$, $s_3 = 90$

The initial simplex table :

	c_j		4	10	0	0	0	
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	s_3	$\theta = \min \frac{X_{Bi}}{a_{ir}}$
0	s_1	50	2	1	1	0	0	$\frac{50}{1} = 50$
0	s_2	100	2	(5) pivot	0	1	0	$\frac{100}{5} = 20$ *
0	s_3	90	2	3	0	0	1	$\frac{90}{3} = 30$
$z_j - c_j$		0	-4	-10 ↑	0	0	0	

Here the net evaluation are calculated as.

$z_j - c_j = c_B a_j - c_j$

$z_1 - c_1 = c_B a_1 - c_1 = (0\ 0\ 0)[2\ 2\ 2]^T - 4 = -4$

$z_2 - c_2 = c_B a_2 - c_2 = (0\ 0\ 0)[1\ 5\ 3]^T - 10 = -10$

$z_3 - c_3 = c_B a_3 - c_3 = (0\ 0\ 0)[1\ 0\ 0]^T - 0 = 0$

$z_4 - c_4 = c_B a_4 - c_4 = [0\ 0\ 0][0\ 1\ 0]^T - 0 = 0$

$z_5 - c_5 = c_B a_5 - c_5 = [0\ 0\ 0][0\ 0\ 1]^T - 0 = 0$

Since there are some $(z_j - c_j) < 0$, the current basic feasible solution is not optimal.

To find the entering variable:

Since $(z_2 - c_2) = -10$ is the most negative, the corresponding non-basic variable x_2 enters the basis. The column corresponding to this x_2 is called the key column or pivot column.

Find the leaving variable.

Find the ratio $\theta = \min\left\{ \frac{x_{Bi}}{a_{ir}}, a_{ir} > 0 \right\}$

$= \min\left\{ \frac{50}{1}, \frac{100}{5}, \frac{90}{3} \right\}$

$= \min\{50, 20, 30\}$

$= 20$

New pivot equ = old pivot equ ÷ pivot element

$= (100 \quad 2 \quad 5 \quad 0 \quad 1 \quad 0) \div 5$

$= (20 \quad \frac{2}{5} \quad 1 \quad 0 \quad \frac{1}{5} \quad 0)$

New S_1 equ = old S_1 equ $-$ (corresponding column co-eff) $\times$ (New pivot equation)

$$
\begin{array}{rcccccc}
= & 50 & 2 & 1 & 1 & 0 & 0 \\
(-) & (20 & 2/5 & 1 & 0 & \frac{1}{5} & 0) \times 1 \\
\hline
 & 30 & 8/5 & 0 & 1 & -\frac{1}{5} & 0 \\
\hline
\end{array}
$$

New S_3 equ $= 90 \quad 2 \quad 3 \quad 0 \quad 0 \quad 1$

$(-) \; (20 \quad 2/5 \quad 1 \quad 0 \quad 1/5 \quad 0) \times 3$

$$
\begin{array}{rcccccc}
= & 90 & 2 & 3 & 0 & 0 & 1 \\
(-) & 60 & 6/5 & 3 & 0 & 3/5 & 0 \\
\hline
 & 30 & 4/5 & 0 & 0 & -3/5 & 1 \\
\end{array}
$$

$(Z_j - C_j) = 0 \quad -4 \quad -10 \quad 0 \quad 0 \quad 0 - (20 \quad \frac{2}{5} \quad 1 \quad 0 \quad \frac{1}{5} \quad 0) \times -10$

$$
\begin{array}{rcccccc}
= & 0 & -4 & -10 & 0 & 0 & 0 \\
(-) & -200 & -\frac{20}{5} & -10 & 0 & -\frac{10}{5} & 0 \\
\hline
 & 200 & 0 & 0 & 0 & \frac{10}{5} & 0 \\
\hline
\end{array}
$$

First Iteration:

	C_j		4	10	0	0	0
C_B	Y_B	X_B	x_1	x_2	S_1	S_2	S_3
0	S_1	30	$8/5$	0	1	$-1/5$	0
10	x_2	20	$2/5$	1	0	$1/5$	0
0	S_3	30	$4/5$	0	0	$-\frac{3}{5}$	1
$Z_j - C_j$		200	0	0	0	2	0

Since all $Z_j - C_j \geq 0$ the current basic feasible solution is optimal.

$\therefore$ The optimal solution is:

$$\text{Max } Z = 200 \quad x_1 = 0 \quad x_2 = 20$$

2) Find the non-negative values of x_1, x_2 & x_3 which Maximize $Z = 3x_1 + 2x_2 + 5x_3$

sub to
$$x_1 + 4x_2 \leq 420$$
$$3x_1 + 2x_3 \leq 460$$
$$x_1 + 2x_2 + x_3 \leq 430$$
$$x_1, x_2, x_3 \geq 0$$

Soln:

Given the LPP, By intro. slack variable,

$$\text{Max } Z = 3x_1 + 2x_2 + 5x_3$$

sub to
$$x_1 + 4x_2 + 0x_3 + S_1 + 0S_2 + 0S_3 = 420$$
$$3x_1 + 0x_2 + 2x_3 + 0S_1 + S_2 + 0S_3 = 460$$
$$x_1 + 2x_2 + x_3 + 0S_1 + 0S_2 + S_3 = 430$$

and $x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$

Since there are 3 equation with 6 variable, the initial basic feasible solu/ is obtained by equating $(6-3)=3$ variable to zero.

$\therefore$ The initial basic feasible solu/ is

$S_1 = 420,\ S_2 = 460,\ S_3 = 430 \quad (x_1 = x_2 = x_3 = 0$ non-basic$)$

The initial simplex table

$c_j \ (3 \quad 2 \quad 5 \quad 0 \quad 0 \quad 0)$

C_B	y_B	x_B	x_1	x_2	x_3	S_1	S_2	S_3	θ
0	S_1	420	1	4	0	1	0	0	
0	S_2	460	3	0	(2)	0	1	0	$\frac{460}{2} = 230^*$
0	S_3	430	1	2	1	0	0	1	$\frac{430}{1} = 430$
$z_j - c_j$		0	-3	-2	-5	0	0	0	

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\uparrow$

$z_1 - c_1 = C_B a_1 - c_1 = (0 \ 0 \ 0)\ [1 \ 3 \ 1]^T - 3 = -3$

$z_2 - c_2 = C_B a_2 - c_2 = (0 \ 0 \ 0)\ [4 \ 0 \ 2]^T - 2 = -2$

$z_3 - c_3 = C_B a_3 - c_3 = (0 \ 0 \ 0)\ (0 \ 2 \ 1]^T - 5 = -5$

$z_4 - c_4 = C_B a_4 - c_4 = [0 \ 0 \ 0]\ [1 \ 0 \ 0]^T - 0 = 0$

$z_5 - c_5 = C_B a_5 - c_5 = [0 \ 0 \ 0]\ [0 \ 1 \ 0]^T - 0 = 0$

$z_6 - c_6 = C_B a_6 - c_6 = [0 \ 0 \ 0]\ [0 \ 0 \ 1]^T - 0 = 0$

$(z_j - c_j) < 0$, the curren basic feasible solution is not optimal.

$\therefore$ $(z_3 - c_3) = -5$ is the most negative, the corresponding non-basic variable x_3 enters into the basis.

The column corres/ to this x_3 is called the key column or pivot column.

To find leaving variable

$$\theta = \min\left\{ \frac{x_{Bi}}{a_{ir}}, a_{ir} > 0 \right\}$$

$$= \min\left\{ \frac{420}{0}, \frac{460}{2}, \frac{430}{1} \right\}$$

$$= \min\{ \cancel{-}\ 230, 430 \} = 230$$

New pivot equation = old pivot equation ÷ pivot element

$= (460\ 3\ 0\ 2\ 0\ 1\ 0) \div 2$

$= (230\ 3/2\ 0\ 1\ 0\ \frac{1}{2}\ 0)$

New s_1 equ = old s_1 equ $-$ (its entering column coeffi) $\times$ (New pivot equ)

$= (420\ 1\ 4\ 0\ 1\ 0\ 0) - (230, \frac{3}{2}\ 0\ 1\ 0\ \frac{1}{2}\ 0) \times 0$

= 420	1	4	0	1	0	0
− 0	0	0	0	0	0	0
420	1	4	0	1	0	0

New s_3 equ =

430	1	2	1	0	0	1
− (230	3/2	0	1	0	$\frac{1}{2}$	0) × 1
200	$-\frac{1}{2}$	2	0	0	$-\frac{1}{2}$	1

New $(z_j - c_j)$ equ. = 0   -3   -2   -5   0   0   0

$(-)\left(230 \quad 3/2 \quad 0 \quad 1 \quad 0 \quad \frac{1}{2} \quad 0\right) \times (-5)$

$$
\begin{array}{ccccccc}
= 0 & -3 & -2 & -5 & 0 & 0 & 0 \\
-1150 & -\frac{15}{2} & 0 & -5 & 0 & -\frac{5}{2} & 0 \\
\hline
1150 & 9/2 & -2 & 0 & 0 & 5/2 & 0 \\
\hline
\end{array}
$$

First Iteration:

	C_j		3	2	5	0	0	0	
C_B	y_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	θ
0	S_1	420	1	4	0	1	0	0	$\frac{420}{4} = 105$
5	x_3	230	3/2	0	1	0	$\frac{1}{2}$	0	$\frac{230}{0} = -$
0	S_3	200	-1/2	2	0	0	-1/2	1	$\frac{200}{2} = 100^*$
$z_j - c_j$		1150	9/2	-2	0	0	5/2	0	

$(z_2 - c_2) = -2$, the basic feasible solu. is not optimal.

∴ Here the non-basic variable x_2 enters into the basis and the basic variable S_3

New pivot equ = old pivot equ ÷ pivot element

$= (200 \quad -1/2 \quad 2 \quad 0 \quad 0 \quad -\frac{1}{2} \quad 1) \div 2$

$= (100 \quad -\frac{1}{4} \quad 1 \quad 0 \quad 0 \quad -\frac{1}{4} \quad \frac{1}{2})$

New s_1 equ $= (420 \ 1 \ 4 \ 0 \ 1 \ 0 \ 0) - (100 \ -\frac{1}{4} \ 1 \ 0 \ 0 \ -\frac{1}{4} \ 1/2) \times 4$

$= 420 \ 1 \ 4 \ 0 \ 1 \ 0 \ 0$

$- (400 \ -1 \ 4 \ 0 \ 0 \ -1 \ 2)$

$20 \ 0 \ 0 \ 0 \ 1 \ 1 \ -2$

Second Iteration.

	C_j		3	2	5	0	0	0
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3
0	s_1	20	0	0	0	1	1	-2
5	x_3	230	3/2	0	1	0	1/2	0
2	x_2	100	$-1/4$	1	0	0	$-1/4$	1/2
$z_j - c_j$		1350	4	0	0	0	2	1

$(z_j - c_j)$ equ $= 1150 \ 9/2 \ -2 \ 0 \ 0 \ 5/2 \ 0$

$- (100 \ -1/4 \ 1 \ 0 \ 0 \ -1/4 \ 1/2) \times -2$

$= 1150 \ 9/2 \ -2 \ 0 \ 0 \ 5/2 \ 0$

$- (-200 \ \cancel{2}/\cancel{4}2 \ -2 \ 0 \ 0 \ \cancel{2}/\cancel{4}2 \ \frac{-2}{2})$

$1350 \ 8/2 \ 0 \ 0 \ 0 \ 2 \ +1$

$(z_j - c_j) \geq 0$, the current basic feasible soln. is optimal.

$\therefore$ The optimal soln. is Max $z = 1350$, $x_1 = 0$

$x_2 = 100$, $x_3 = 230$.

1) using simplex method, Max $z = x_1 + 4x_2 + 5x_3$ subject to $3x_1 + 6x_2 + 3x_3 \leq 22$, $x_1 + 2x_2 + 3x_3 \leq 14$ and $3x_1 + 2x_2 \leq 14$.

Ans. Max $z = \frac{74}{3}$; $x_1 = 0$, $x_2 = 2$, $x_3 = \frac{10}{3}$

2) using simplex method Min $z = 8x_1 - 2x_2$ subject to $-4x_1 + 2x_2 \leq 1$, $5x_1 - 4x_2 \leq 3$ and $x_1, x_2 \geq 0$

[Hint: Min z type, we shall convert it in to a maximization type. so

$\text{Max}(-z) = \text{Max } z^* = -8x_1 + 2x_2$]

Ans: min $z = -1$ $x_1 = 0$, $x_2 = 1/2$

Simplex Algorithm: [for maximization type only]

step:1 check $b_i \geq 0 \ \forall i$. If $b_i < 0$ for some i, then multiply the corresponding constraint by (-1).

step:2 Convert all the inequalities into equations by introducing slack / surplus / artifical variables.

step:3 obtain initial basic feasible solution.

Step:4 Compute the net evaluations $z_j - c_j$ by $z_j - c_j$

i) If $z_j - c_j \geq 0 \ \forall j$ then the (b.f.s) is an {o.b.f.s}

ii) If $z_j - c_j < 0$ for some j then go to next step

step:5 let $z_r - c_r$ be the most negative of $z_j - c_j$

i) If all $a_{ir} \leq 0$ then there is an unbounded Solu.

ii) If $a_{ir} > 0$ for some i, then the corresponding a_r enters the basis.

Step: 6 Compute $\min\left\{ \frac{x_{Bi}}{a_{ir}},\ a_{ir} > 0 \right\}$.

If $\frac{x_{BK}}{a_{kr}}$ is the minimum, then y_k will leave the basis. The element y_{kr} is known as the leading element or pivotal element

Step: 7 Convert the leading element to unity by dividing its row by the leading element itself. and all other elements are calculate by

$$y_{ij}^1 = y_{ij} - \frac{y_{kj}}{y_{kr}} \cdot y_{ir} \qquad i = 1, 2 \dots m+1, \ i \neq k$$

and $$y_{kj}^1 = \frac{y_{kj}}{y_{kr}}$$

Step: 8 Go to step 4. Repeat the procedure until the optimum solution (or) unbounded solution is obtained.

Remark: ① For minimization problem the objective fun. can be connected as

min z = − max (−z)

② In simplex method the pivotal element always positive

Artifical variables:

There are two methods to solve the LPP by using artifical variables.

i) Two phase method

ii) Big-M method [Penalty method (or) charnes method]

Two-phase method:

phase-I

convert the given LPP into standard form put the coefficient of artificial variables as -1 for maximation (or) 1 for minimization and all other variables as 0 in the objective function.

Apply simplex algorithm:

i) If max $z^* < 0$ and atleast one artifical variable is present in the basis with positive values then the LPP does not posses any feasible solution.

ii) If max $z^* = 0$ and atleast one artificial variable is present in the basis with zero value then go to phase II

iii) If max $z^* = 0$ & no artifical variable is present then go to phase II

phase: II

Assign actual coefficients to the decision variables in the objective function and a value '0' to the artificial variable that appear at zero value in phase - II

Apply simplex algorithm to the modified table to get the optimum solution.

Big-M method:

The Big-M method is an alternative method of solving a linear programming problem involving artificial variables. In this method assign a very high penalty (say M) to the artifical variables in the objective function

Step:1
Write the given LPP into its standard form and check whether there exists a starting basic feasible solution.

(a) If there is a ready starting basic feasible solution; move onto step 3

b) If there does not exist a ready starting basic feasible solution, move on to step 2

Step:2
Add artificial variable to the left side of each equation that has no obvious starting basic variables. Assign a very high penalty (say M) to these variables in the objective function

Step: 3 Apply simplex method to the modified LPP following cases may arise at the last ~~state~~ iteration.

a) At least one artificial variable is present in the basis with zero value. In such a case the current optimum basic feasible solution is degenerate

b) At least one artificial variable is present in the basis with a positive value. In such a case, the given LPP does not posses an optimum basic feasible solution. The given problem is said to have a ~~peso~~ pseudo-optimum basic feasible solution

Note:

= → Add artificial variable only

≥ → subtract surplus variable + Add arti-l variable

≤ → Add slack variable only.

Big-M method:

1) using penalty method (or) Big method. solve the LPP by simplex method.

$$\text{Max } z = 3x_1 + 2x_2$$

$$\text{sub to } \quad 2x_1 + x_2 \le 2$$

$$3x_1 + 4x_2 \ge 12$$

$$\text{and } \quad x_1, x_2 \ge 0.$$

Solu: By introducing non-negative slack variable s_1 & surplus variable s_2.

$$\text{Max } z = 3x_1 + 2x_2 + 0S_1 + 0S_2 - MR_1$$

$$2x_1 + x_2 + S_1 + 0S_2 = 2$$

$$3x_1 + 4x_2 + 0S_1 - S_2 + R_1 = 12$$

$$x_1, x_2, S_1, S_2, R_1 \geq 0$$

The Initial basic feasible solution is given by $S_1 = 2$, $R_1 = 12$ $(x_1 = x_2 = S_2 = 0)$

		c_j	3	2	0	0	$-M$	
C_B	Y_B	X_B	x_1	x_2	S_1	S_2	R_1	θ
0	S_1	2	2	1	1	0	0	$2/1 = 2 \leftarrow$
$-M$	R_1	12	3	4	0	-1	1	$12/4 = 3$
$z_j - c_j$		$-12M$	$-3M-3$	$-4M-2$	0	M	0	

$\uparrow$

$z_1 - c_1 = C_B a_1 - c_1 = (0 \ -M)(2 \ 3) - 3 = -3M - 3$

$z_2 - c_2 = -4M - 2$

$z_3 - c_3 = 0$

$z_4 - c_4 = M$

$z_5 - c_5 = -M - (-M) = 0$

Most -ve

$-3M - 3 = -3(2) - 3$
$= -6 - 3$
$= -9$

$-4(2) - 2$
$= -8 - 2 = -10$

$z_j - c_j < 0$ the current basic feasible solution is not optimal

$\because (z_2 - c_2) = -4M - 2$, is the most negative the corresponding non-basic variable x_2 enters into the basic

New pivot equation = old pivot equ ÷ pivot elt.

= (2 2 1 1 0 0) ÷ 1

= 2 2 1 1 0 0

New R_1 equ = old R_1 equ − (Corr. Column coeff) × (New pivot equ)

= (12 3 4 0 −1 1) − (4)(2 2 1 1 0 0)

= 12 3 4 0 −1 1

(−) 8 8 4 4 0 0

4 −5 0 −4 −1 1

First Iteration:

		c_j	3	2	0	0	−M
C_B	y_B	x_B	x_1	x_2	S_1	S_2	R_1
2	x_2	2	2	1	1	0	0
−M	R_1	4	−5	0	−4	−1	1
$z_j - c_j$		−4M+4	5M+1	0	4M+2	M	0

$(z_j - c_j)$ = −12M −3M−3 −4M−2 0 M 0 − (−4M−2)

(2 2 1 1 0 0)

= −12M0 −3M−3 −4M−2 0 M 0

(−) (−8M−4) (−8M−4) (−4M−2) (−4M−2) 0 0

−4M+4 5M+1 0 4M+2 M 0

since all $(z_j - c_j) \geq 0$ and an artificial variable R_1 appears in the basis at non-zero level, the given LPP does not possess any feasible solution. But the possess a pseudo optimal solution.

2) Min $z = 4x_1 + x_2$ using penality method.

sub to $3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 4$ & $x_1, x_2 \geq 0$

Solu:

Max $z = -4x_1 - x_2 + 0S_1 + 0S_2 - MR_1 - MR_2$

sub to $3x_1 + x_2 + R_1 = 3$

$4x_1 + 3x_2 - S_1 + R_2 = 6$

$x_1 + 2x_2 + S_2 = 4$

Initial simplex method.

		c_j	-4	-1	0	0	-M	-M	
C_B	Y_B	X_B	x_1	x_2	S_1	S_2	R_1	R_2	θ
-M	R_1	3	3	1	0	0	1	0	$3/3 = 1$ *
-M	R_2	6	4	3	-1	0	0	1	$6/4 = 3/2$
0	S_2	4	1	2	0	1	0	0	$4/1 = 4$
$z_j - c_j$		-9M	-7M+4	-4M+1	M	0	0	0	

↑

New pivot equ = old piv equ ÷ pivot eld.

$= (3 \quad 3 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0) \div 3$

$= (1 \quad 1 \quad 1/3 \quad 0 \quad 0 \quad 1/3 \quad 0)$

R_2 equ $= (6 \quad 4 \quad 3 \quad -1 \quad 0 \quad 0 \quad 1) - (4)(1 \quad 1 \quad \frac{1}{3} \quad 0 \quad 0 \quad 1/3 \quad 0)$

=	6	4	3	-1	0	0	1	
(-)	4	4	4/3	0	0	4/3	0	
	2	0	5/3	-1	0	-4/3	1	

$3 - \frac{4}{3} =$

$9 - \frac{4}{3} = 5/3$

S_2 equ

=	4	1	2	0	0	0	0	
(-)	1	1	1/3	0	0	1/3	0	$1 - \frac{1}{3}$
	3	0	5/3	0	0	-1/3	0	$2 - \frac{1}{3} = \frac{5}{3}$

$z_j - c_j = (9M \quad -7M+4 \quad -4M+1 \quad M \quad 0 \quad 0 \quad 0) -$

$(-7M+4)(1 \quad 1 \quad 1/3 \quad 0 \quad 0 \quad 1/3 \quad 0)$

Unit - II

Transportation Model

Mathematical formulation of TP

$$\text{Min } z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Sub to $\sum_{j=1}^{n} x_{ij} = a_i \quad i = 1, 2, \dots m$

$$\sum_{i=1}^{m} x_{ij} = b_j \quad j = 1, 2, \dots n$$

& $x_{ij} \geq 0$ for all i & j

Note: 1

The two sets of constraints will be consistent of

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

(total supply) = total demand

which is the necessary and sufficient condition for a transportation problem to have a feasible solution ~~for a~~ are called problems satisfying this condition balanced transportation problems

Note: 2

i) If $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$ then the transportation problem is called to be unbalanced.

ii) The unbalanced problem can be balanced by adding dummy supply (row) or a dummy demand (column) as the need arises.

Note : 3

If the no. of positive allocation at any stage of feasible solution is less than the required no (m+n-1) than the solution is said to be degenerate. otherwise than the solution is non-degenerate.

note : 4

The transportation table having positive allocation in a cell is called occupied cell. otherwise called empty or unoccupied or non-occupied cells

Def 1:

A set of non-negative values x_{ij}, $i=1,2,\cdots n$ $j=1,2,\cdots n$ that satisfies the constraints (r)m condition and also the non-negativity restrictions is called feasible solution to the transportation problem

def 2:

A feasible solution to a $(m \times n)$ transportation problem that contains no more than $m+n-1$ non-negative allocation is called a basic feasible solution (BFS) to the transportation problem.

def 3:

A feasible solution (not necessarily basic) is said to be an <u>optimal solution</u> if it minimizes that the total transportation cost

note: The no. of basic variables in an $m \times n$ balanced transportation problem is atmost $m+n-1$

Note: The no. of non-basic variables is an $m \times n$ balanced transportation problem is atleast $mn-(m+n-1)$

Initial basic feasible solution:

There are three different methods to obtain the IBFS

1) North-West corner rule
2) Least-cost method
3) Vogel's Approximation method [VAM]

optimal test

1) Stepping stone method
2) Modified Distribution method [MODI]

1) Determine basic feasible solution to the following transportation problem using north-west corner rule

	A	B	C	supply
p	9	8	5	25
q	6	8	4	35
r	7	6	9	40
Demand	30	25	45	

soln:

9	8	5	25
6	8	4	35
7	6	9	40
30	25	45	

Since $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = 100$. the given problem is balanced

$\therefore$ There exists a feasible solution to TP

$\boxed{25}$ 9	8	5	~~25~~ 0
$\boxed{5}$ 6	$\boxed{25}$ 8	$\boxed{5}$ 4	~~35~~ ~~30~~ 5
7	6	$\boxed{40}$ 9	40
~~30~~ ~~5~~ 0	~~25~~ 0	~~45~~ 40	

The no. of positive independent allocation is equal to $m+n-1 = 3+3-1 = 5$

This solution is non-degenerate basic feasible

The initial transportation cost $= 25\times 9 + 5\times 6 + 25\times 8 + 5\times 4 + 40\times 9$

$= 225 + 30 + 200 + 20 + 360$

$= 835$

2) using NWCR

5	7	8	65
4	4	6	42
6	7	7	43
70	30	50	

Soln:

since $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = 150$

The given problem is balanced

$\therefore$ There exists a feasible solution to TP

$\boxed{65}$ 5	7	8	~~65~~ 0
$\boxed{5}$ 4	$\boxed{30}$ 4	$\boxed{7}$ 6	~~42~~ ~~37~~ 7
6	7	$\boxed{43}$ 7	43
~~70~~ ~~5~~ 0	~~30~~ 0	~~50~~ 43	

The no of positive independent allocati...

to $m+n-1 = 3+3-1 = 5$

This solution is non-degenerate basic feasible

the initial transportation problem cost =

$65 \times 5 + 5 \times 4 + 30 \times 4 + 7 \times 6 + 43 \times 7$

3) Least cost method

				supply
1	2	1	4	30
3	3	2	1	50
4	2	5	4	20
20	40	30	10	

demand 20 40 30 10

Soln:

since $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = 100$ the given problem is balanced

∴ There exists a feasible solution to the TP

$\boxed{20}$ 1	2	$\boxed{10}$ 1	4	~~30~~ ~~10~~ 0
3	$\boxed{20}$ 3	$\boxed{20}$ 2	$\boxed{10}$ 1	~~50~~ ~~40~~ 20
4	$\boxed{20}$ 2	5	4	~~20~~
~~20~~ 0	~~40~~ 20	~~30~~ ~~20~~ 0	~~10~~ 0	

The no. of positive independent allocation is equal to $m+n-1 = 3+4-1 = 6$

This solution is non-degenerate basic feasible

The initial transportation problem cost =

$20\times1 + 10\times1 + 20\times3 + 20\times2 + 10\times1 + 20\times2$

$= 20+10+60+40+10+40$

$= 180$

3) VAM

						Row penalties
S_1	$\boxed{200}$ 11	13	17	14	~~250~~ 50	(2)
S_2	16	18	14	10	300	(4)
S_3	21	24	13	10	400	(3)
	~~200~~ 0	225	275	250		
column penalties	↑(5)	(5)	(1)	(4)		

					Row penalties
	$\boxed{50}$ 13	17	14	0	(1)
	18	14	10	300	(4)
	24	13	10	400	(3)
	175	275	250		
column penalties	(5)↑	(1)	(0)		

				Row penalties
$\boxed{175}$ 18	14	10	125	(4)
24	13	10	400	(3)
0	275	250		
Column penalties				
(6)	(1)	(0)		

			Row penalties
14	$\boxed{125}$ 10	0	(4)
13	10	400	(3)
275	125		
Column penalties			
(1)	(0)		

			Row penalties
$\boxed{275}$ 13	10	125	(3)
0	125		
Column penalties			
(13)	(10)		

$\boxed{125}$ 10

$\boxed{200}$ 11	$\boxed{50}$ 13	17	14
16	$\boxed{175}$ 18	14	$\boxed{125}$ 10
21	24	$\boxed{275}$ 13	$\boxed{125}$ 10

$m+n-1 = 3+4-1$

$= 6$

non - degenerate

The initial TP cost $= 200\times11 + 50\times13 + 175\times18 + 125\times10 + 275\times13 + 125\times10$

$= 12075.$

4) Solve by MODI method :-

21	16	25	13	11
17	18	14	23	13
32	27	18	41	19
6	10	12	15	

soln:

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = 43$$

The given problem is balanced

					Row Penalties
21	16	25	$\boxed{11}$ 13	0	(3)
17	18	14	23	13	(3)
32	27	18	41	19	(9)
6	10	12	4		
Column Penalties (4)	(2)	(4)	(10)↑		

					Row Penalties
17	18	14	$\boxed{4}$ 23	9	(3)
32	27	18	41	19	(9)
6	10	12	0		
Column Penalties (15)	(9)	(4)	(18)↑		

				Row Penalties
$\boxed{6}$ 17	18	23	3	(1)
32	27	41	19	(5)
0	10	12		
Column Penalties (15)↑	(9)	(4)		

				Row penalties
$\boxed{6}$ 17	18	14	3	(3)
32	27	18	19	(9)
0	10	12		
(15) ↑	(9)	(4)		

column penalties

			Row penalties
18	14	3	(4)
27	$\boxed{12}$ 18	7	← (9)
10	0		
(9)	(4)		

column penalties

		Row penalties
18	3	(18)
$\boxed{7}$ 27	0	← (27)
3		
(9)		

column penalties

$\boxed{3}$ 18

21	16	25	[11] 13
[6] 17	[3] 18	14	[4] 23
32	[7] 27	[12] 18	41

$\therefore$ The no. of positive independent allocation is equal to

$m+n-1 = 3+4-1 = 6$

$\therefore$ The soln is non-degenerate basic feasible

$$\left.\begin{array}{r}\text{The initial transportation} \\ \text{cost}\end{array}\right\} = \begin{array}{l} 11 \times 13 + 17 \times 6 + 3 \times 18 + 4 \times 23 \\ + 27 \times 7 + 18 \times 12 \end{array}$$

$$= 796.$$

To find the optimal solution, we apply MODI method

u_i and v_j for each occupied cell (i,j) using the relation

$$\boxed{c_{ij} = u_i + v_j}$$

As the 2nd row contains maximum no. of allocation, we

choose $u_2 = 0$

	$V_1 = 17$	$V_2 = 18$	$V_3 = 9$	$V_4 = 23$
$u_1 = -10$	21	16	25	$\boxed{11}$ 13
$u_2 = 0$	$\boxed{6}$ 17	$\boxed{3}$ 18	14	$\boxed{4}$ 23
$u_3 = 9$	32	$\boxed{7}$ 27	$\boxed{12}$ 18	41

occupied cells

$$\boxed{C_{ij} = u_i + V_j}$$

$C_{14} = u_1 + V_4 \Rightarrow u_1 + 23 = 13$

$\boxed{u_1 = -10}$

$C_{21} = u_2 + V_1 \Rightarrow 0 + V_1 = 17$

$\boxed{V_1 = 17}$

$C_{22} = u_2 + V_2 \Rightarrow 0 + V_2 = 18$

$\boxed{V_2 = 18}$

$C_{24} = u_2 + V_4 \Rightarrow 0 + V_4 = 23$

$\boxed{V_4 = 23}$

$C_{32} = u_3 + V_2 \Rightarrow u_3 + 18 = 27$

$\boxed{u_3 = 9}$

$C_{33} = u_3 + V_3 \Rightarrow 9 + V_3 = 18$

$\boxed{V_3 = 9}$

unoccupied cell

$$\boxed{d_{ij} = C_{ij} - (u_i + V_j)}$$

$d_{11} = 21 - (u_1 + V_1) \Rightarrow 21 - 7$

$\boxed{d_{11} = 14}$

$d_{12} = 16 - (u_1 + V_1) \Rightarrow 16 - 8$

$\boxed{d_{12} = 8}$

$d_{13} = 25 + 1$

$\boxed{d_{13} = 26}$

$d_{23} = 14 - (9)$

$\boxed{d_{23} = 5}$

$d_{31} = 32 - 26$

$\boxed{d_{31} = 6}$

$d_{34} = 41 - 32$

$\boxed{d_{34} = 9}$

14 21	8 16	26 25	$\boxed{11}$ 13
$\boxed{6}$ 17	$\boxed{3}$ 18	5 14	$\boxed{4}$ 23
6 32	$\boxed{7}$ 27	$\boxed{12}$ 18	9 41

since all $d_{ij} \geq 0$ the solution under the test is optimum and unique.

$\therefore$ the optimum allocation cells is given

$$x_{14} = 11,\ x_{21} = 6,\ x_{23} = 3,\ x_{24} = 4,\ x_{32} = 7,\ x_{33} = 12$$

The transportation cost $= 13 \times 11 + 17 \times 6 + 18 \times 3 + 23 \times 4 + 27 \times 7 + 18 \times 12$

$= 796$

cost = Rs. 160

unbalanced TP :-

				Supply
11	20	7	8	50
21	16	20	12	40
8	12	18	9	70
30	25	35	40	

demand 30 25 35 40

soln:

$\Sigma a_i = 50+40+70 = 160$

$\Sigma b_j = 30+25+35+40 = 130$

$$\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$$

The given TP is unbalanced. so we change the problem Balanced

The amount of shortage is 30. its occur on demand.

The amount of shortage is 30 so we add dummy column in matrine with zero cost

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

The given TP is balanced

							i)	ii)
	11	20	$\boxed{35}$ 7	8	0	15 ~~50~~	(7)	(1
	21	16	20	12	$\boxed{30}$ 0	10 ~~40~~	(12)	(4
	8	12	18	9	0	70	(8)	(1
	30	25	~~35~~ 0	40	~~30~~ 0			
i)	(3)	(4)	(11)	(1)	(0)			
ii)	(3)	(4)	(11)	(1)	–			

					iii)	iv)	v)
	11	20	$\boxed{15}$ 8	15	(3)	(3)	(3)
	21	16	$\boxed{10}$ 12	0 ~~10~~	(4)	–	–
	$\boxed{30}$ 8	$\boxed{25}$ 12	$\boxed{15}$ 9	45 ~~70~~ ~~15~~ 0	(1)	(1)	(1)
	~~30~~ 0	~~25~~ 0	~~40~~ 30 15				
iii)	(3)	(4)	(1)				
iv)	(3)	(8)	(1)				
v)	(3)	–	(1)				
vi)	–	–	(1)				

11	20	$\boxed{35}$ 7	$\boxed{15}$ 8	0
21	16	20	$\boxed{10}$ 12	$\boxed{30}$ 0
$\boxed{30}$ 8	$\boxed{25}$ 12	18	$\boxed{15}$ 9	0

The no. of positive independent allocation is equal to

$m+n-1 = 3+5-1$

$= 7$

$\therefore$ The solution is non-degenerate basic feasible

The initial transportation cost $\Big\}$ $= 7 \times 35 + 8 \times 15 + 10 \times 12 + 0 \times 30$

$+ 8 \times 30 + 12 \times 25 + 9 \times 15$

$= 245 + 120 + 120 + 0 + 300 + \overset{240}{\cancel{180}} + 135$

~~= Rs. 1160~~

= Rs. 1160

	$v_1=8$	$v_2=12$	$v_3=8$	$v_4=9$	$v_5=-3$
$u_1=-1$	11	20	$\boxed{35}$ 7	$\boxed{15}$ 8	0
$u_2=3$	21	16	20	$\boxed{10}$ 12	$\boxed{30}$ 0
$u_3=0$	$\boxed{30}$ 8	$\boxed{25}$ 12	18	$\boxed{15}$ 9	0

occupied cells

$c_{ij}=u_i+v_j$

$C_{13}=u_1+v_3=7 \Rightarrow -1+v_3=7$

$\boxed{v_3=8}$

$C_{31}=u_3+v_1=8 \Rightarrow 0+v_1=8$

$\boxed{v_1=8}$

$C_{14}=u_1+v_4=8 \Rightarrow u_1+9=8$

$\boxed{u_1=-1}$

$C_{32}=u_3+v_2=12 \Rightarrow 0+v_2=12$

$\boxed{v_2=12}$

$C_{24}=u_2+v_4=12 \Rightarrow u_2+9=12$

$\boxed{u_2=3}$

$C_{34}=u_3+v_4=9 \Rightarrow 0+v_4=9$

$\boxed{v_4=9}$

$C_{25}=u_2+v_5=0 \Rightarrow 3+v_5=0$

$\boxed{v_5=-3}$

unoccupied cells

$d_{ij} = c_{ij} - (u_i + v_j)$

$d_{11} = 11 - (-1+8)$

$\boxed{d_{11} = 4}$

$d_{12} = 20 - (-1+12)$

$\boxed{d_{12} = 9}$

$d_{15} = 0 - (-1-3)$

$\boxed{d_{15} = 4}$

$d_{21} = 21 - (3+8)$

$\boxed{d_{21} = 10}$

$d_{22} = 16 - (3+12)$

$\boxed{d_{22} = 1}$

$d_{23} = 20 - (3+8)$

$\boxed{d_{23} = 9}$

$d_{33} = 18 - (0+8)$

$\boxed{d_{33} = 10}$

$d_{35} = 0 - (0-3)$

$\boxed{d_{35} = 3}$

11 (4)	20 (9)	7 [35]	8 [15]	0 (4)
21 (10)	16 (1)	20 (9)	12 [10]	0 [30]
8 [30]	12 [25]	18 (10)	9 [15]	0 (3)

since all $d_{ij} > 0$ the solution under the test is optimal and unique

∴ The optimum allocation cells is given

$x_{13} = 35,\ x_{14} = 15,\ x_{24} = 10,\ x_{25} = 30,\ x_{31} = 30$

Assignment Problem

mathematical formula of an assignment problem:

2m (consider of an assignment problem of assigning

n jobs to n machine (1 job to 1 machine)

Let c_{ij} be the unit cost of assign i^{th} machine to

the j^{th} job

let $x_{ij} = \begin{cases} 1, & \text{if } j^{th} \text{ job is assigned to } i^{th} \text{ machine} \\ 0, & \text{if } j^{th} \text{ job is not assigned to } i^{th} \text{ machine} \end{cases}$

(model)
The assignment model is given by the LPP

$$\text{minimize } z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to the constraints

$$\sum_{i=1}^{n} x_{ij} = 1, \; j = 1, 2, \ldots n$$

$$\sum_{j=1}^{n} x_{ij} = 1, \; i = 1, 2, \ldots n \quad \text{and } x_{ij} = 0 \text{ (or) } 1$$

difference between Transportation & Assignment Problem

Transportation problem	Assignment Problem
supply at any shows maybe any positive quantity a_i	~~demand at any~~ (machine) supply at any shows ~~maybe~~ will be 1 that is $a_i = 1$

demand at any destination maybe any Positive quantity b_j	demand at any destination (job) will be 1 that is $b_j = 1$
one or more shows to any number of destination	one shows (machine) to only one destination (job)

1) consider the problem of assigning 5 jobs to 5 person the assignment cost are

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline A & 8 & 4 & 2 & 6 & 1 \\ B & 0 & 9 & 5 & 5 & 4 \\ C & 3 & 8 & 9 & 2 & 6 \\ D & 4 & 3 & 1 & 0 & 3 \\ E & 9 & 5 & 8 & 9 & 5 \end{array}$$

determine the optimum assignment schedule.-

soln:-

$$\begin{pmatrix} 8 & 4 & 2 & 6 & 1 \\ 0 & 9 & 5 & 5 & 4 \\ 3 & 8 & 9 & 2 & 6 \\ 4 & 3 & 1 & 0 & 3 \\ 9 & 5 & 8 & 9 & 5 \end{pmatrix}$$

Since the no.of rows is equal to the no.of columns in the cost matrix. so the given assignment problem is balanced step 1: reducing row

select the smallest cost element in each row and subtract this from all the elements of the corresponding row

$$\begin{pmatrix} 7 & 3 & 1 & 5 & 0 \\ 0 & 9 & 5 & 5 & 4 \\ 1 & 6 & 7 & 0 & 4 \\ 4 & 3 & 1 & 0 & 3 \\ 4 & 0 & 3 & 4 & 0 \end{pmatrix}$$

step 2: reducing column

select the smallest cost element in each column and sub this from all the element of the corresponding column

$$\begin{pmatrix} 7 & 3 & 0 & 5 & 0 \\ 0 & 9 & 4 & 5 & 4 \\ 1 & 6 & 6 & 0 & 4 \\ 4 & 3 & 0 & 0 & 3 \\ 4 & 0 & 2 & 4 & 0 \end{pmatrix}$$

step 3:

$$\begin{pmatrix} 7 & 3 & \cancel{0} & 5 & \boxed{0} \\ \boxed{0} & 9 & 4 & 5 & 4 \\ 1 & 6 & 6 & \boxed{0} & 4 \\ 4 & 3 & \boxed{0} & \cancel{0} & 3 \\ 4 & \boxed{0} & 2 & 4 & \cancel{0} \end{pmatrix}$$

No. of matrix = No. of allocation

Step 4:

Since each row and each column contains exactly one assignment

i.e) exactly one encircled zero

The current assignment is optimal

∴ The optimum assignment schedule is given by :

$A \to 5,\ B \to 1,\ C \to 4,\ D \to 3,\ E \to 2$

The optimum (minimum) assingment cost $= (1+0+2+1+5$

$= 9$ units

2) The processing time in hours of the jobs when allocated to the different machines are indicated below assign the machine for the job show that the total processing time is minimum.

$$\begin{array}{c c} & \begin{array}{ccccc} M_1 & M_2 & M_3 & M_4 & M_5 \end{array} \\ \begin{array}{c} J_1 \\ J_2 \\ J_3 \\ J_4 \\ J_5 \end{array} & \begin{pmatrix} 9 & 22 & 58 & 11 & 19 \\ 43 & 78 & 72 & 50 & 63 \\ 41 & 28 & 91 & 37 & 45 \\ 74 & 42 & 27 & 49 & 39 \\ 36 & 11 & 57 & 22 & 25 \end{pmatrix} \end{array}$$

soln:

Step 1:

$$\begin{pmatrix} 20 & 13 & 49 & 2 & 10 \\ 0 & 35 & 29 & 7 & 20 \\ 13 & 0 & 63 & 9 & 17 \\ 47 & 15 & 0 & 22 & 12 \\ 25 & 0 & 46 & 11 & 14 \end{pmatrix}$$

Step 2:

$$\begin{pmatrix} 0 & 13 & 49 & 0 & 0 \\ 0 & 35 & 29 & 5 & 10 \\ 13 & 0 & 63 & 7 & 7 \\ 47 & 15 & 0 & 20 & 2 \\ 25 & 0 & 46 & 9 & 4 \end{pmatrix}$$

Step 3:

$$\begin{pmatrix} \cancel{0} & 13 & 49 & \boxed{0} & \cancel{0} \\ \boxed{0} & 35 & 29 & 5 & 10 \\ 13 & \boxed{0} & 63 & 7 & 7 \\ 47 & 15 & \boxed{0} & 20 & 2 \\ 25 & \cancel{0} & 46 & 9 & 4 \end{pmatrix}$$

No. of matrix ≠ No. of allocation

Step 4:

$$\begin{pmatrix} \cancel{0} & 13 & 49 & \boxed{0} & \cancel{0} \\ \boxed{0} & 35 & 29 & 5 & 10 \\ 13 & \boxed{0} & 63 & 7 & 7 \\ 47 & 15 & \boxed{0} & 20 & 2 \\ 25 & \cancel{0} & 46 & 9 & 4 \end{pmatrix} \begin{matrix} \\ \\ \checkmark \\ \\ \checkmark \end{matrix}$$

Step 5:

$$\begin{pmatrix} \cancel{0} & 17 & 49 & \boxed{0} & \cancel{0} \\ \boxed{0} & 39 & 29 & 5 & 10 \\ 9 & \boxed{0} & 59 & 3 & 3 \\ 47 & 19 & \boxed{0} & 20 & 2 \\ 21 & \cancel{0} & 42 & 5 & \boxed{0} \end{pmatrix}$$

Step 6:

The optimum assignment schedule is given by

$J_1 \to M_4$, $J_2 \to M_1$, $J_3 \to M_2$, $J_4 \to M_3$, $J_5 \to M_5$

The optimum (minimum) assignment cost $= (11 + 43 + 28 + 2$

$+ 25$

$= 134$ ~~units~~ hrs

unbalance Assignment Problem:-

If the no. of rows is ~~equal~~ not equal to the no. of column in the cost matrise of the given assignment problem ~~is~~ then the given assignment problem is unbalanced.

A company has 4 machine to 3 jobs what are the job assignment which will minimize the cost

$$\begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{cccc} 1 & 2 & 3 & 4 \\ \left(\begin{array}{cccc} 18 & 24 & 28 & 32 \\ 8 & 13 & 17 & 19 \\ 10 & 15 & 19 & 22 \end{array} \right. \end{array}$$

Since the no. of rows is less than the no. of columns in the cost matrix. The given assignment prblm is unbalanced.

P1:

$$\begin{array}{c} A \\ B \\ C \\ D \end{array} \begin{pmatrix} 18 & 24 & 28 & 32 \\ 8 & 13 & 17 & 19 \\ 10 & 15 & 19 & 22 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

To make it a balanced one add a dummy job D (row) with zero cost element

Step 2:

$$\begin{pmatrix} 0 & 6 & 10 & 14 \\ 0 & 5 & 9 & 11 \\ 0 & 5 & 9 & 12 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Step 3: column

$$\begin{pmatrix} 0 & 6 & 10 & 14 \\ 0 & 5 & 9 & 11 \\ 0 & 5 & 9 & 12 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Step 4:

$$\begin{pmatrix} \boxed{0} & 6 & 10 & 14 \\ \cancel{0} & 5 & 9 & 11 \\ \cancel{0} & 5 & 9 & 12 \\ \cancel{0} & \boxed{0} & \cancel{0} & \cancel{0} \end{pmatrix}$$

No. of matrix ~~rows~~ ≠ no. of allocation

Step 5: ✓

$$\begin{pmatrix} \boxed{0} & 6 & 10 & 14 \\ \cancel{0} & 5 & 9 & 11 \\ \cancel{0} & 5 & 9 & 12 \\ \cancel{0} & \boxed{0} & \cancel{0} & \cancel{0} \end{pmatrix} \begin{array}{c} \\ ✓ \\ ✓ \\ \\ \end{array}$$

Step 5:

$$\begin{pmatrix} \boxed{0} & 6 & 10 & 14 \\ \cancel{0} & 5 & 9 & 11 \\ \cancel{0} & 5 & 9 & 12 \\ \cancel{0} & \boxed{0} & \cancel{0} & \cancel{0} \end{pmatrix}$$

Step 6:

$$\begin{pmatrix} \boxed{0} & 1 & 5 & 9 \\ \cancel{0} & \boxed{0} & 4 & 6 \\ \cancel{0} & \cancel{0} & 4 & 7 \\ 5 & \cancel{0} & \boxed{0} & \cancel{0} \end{pmatrix}$$

No. of matrix $\neq$ No. of allocation

Step 7:

$$\begin{pmatrix} \boxed{0} & 1 & 5 & 9 \\ \cancel{0} & \boxed{0} & 4 & 6 \\ \cancel{0} & \cancel{0} & 4 & 7 \\ 5 & \cancel{0} & \boxed{0} & \cancel{0} \end{pmatrix}$$

Step 8:

$$\begin{pmatrix} \boxed{0} & 1 & 1 & 5 \\ \cancel{0} & \boxed{0} & \cancel{0} & 2 \\ \cancel{0} & \cancel{0} & \boxed{0} & 3 \\ 9 & 4 & \cancel{0} & \boxed{0} \end{pmatrix}$$

No. of matrix = No. of allocation

The optimum assignment schedule is given by

A → 1 , B → 2 , C → 3 , D → 4

the optimum (minimum) assignment cost = (18 + 13 + 19 + 0)

= 50 units

Maximize cost in assignment

The maximization problem has to be converted into an equalant minimization problem and then slove by the usual hungarian method

~~step 1~~

i) since max z = −min(−z) multiple all the cost element c_{ij} of the cost matrix by −1

ii) subtract all the cost element c_{ij} of the cost matrix from the highest cost in the cost matrix

cost matrix

1)

		Districts			
		1	2	3	4
salesman	A	16	10	14	11
	B	14	11	15	15
	C	15	15	13	12
	D	13	12	14	15

Find the assignment of salesman to various district which will yield maximum profit

The given AP is balanced

Since this is a maximization problem it can be co,

Step 1:

$$\begin{pmatrix} 0 & 6 & 2 & 5 \\ 2 & 5 & 1 & 1 \\ 1 & 1 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}$$

Step 2:

$$\begin{pmatrix} 0 & 6 & 2 & 5 \\ 1 & 4 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 2 & 3 & 1 & 0 \end{pmatrix}$$

Step 3: Column

$$\begin{pmatrix} 0 & 6 & 2 & 5 \\ 1 & 4 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 2 & 3 & 1 & 0 \end{pmatrix}$$

Step 4:

$$\begin{pmatrix} \boxed{0} & 6 & 2 & 5 \\ 1 & 4 & \boxed{0} & \cancel{0} \\ \cancel{0} & \boxed{0} & 2 & 3 \\ 2 & 3 & 1 & \boxed{0} \end{pmatrix}$$

No. of matrix = No. of allocation

Step 5:

~~the~~ The optimum assignment schedule is given by

A → 1 , B → 3 , C → 2 , D → 4

The optimum (minimum) assignment

cost = 16 + 15 + 15 + 15

= 61 units

Travelling salesmaproblem :-

1) solve the following tranvelling salesman problem

		TO			
		A	B	C	D
	A	–	46	16	40
	B	41	–	50	40
From	C	82	32	–	60
	D	40	40	36	–

soln:

step 1:

$$\begin{pmatrix} \infty & 46 & 16 & 40 \\ 41 & \infty & 50 & 40 \\ 82 & 32 & \infty & 60 \\ 40 & 40 & 36 & \infty \end{pmatrix}$$

step 2: row

$$\begin{pmatrix} \infty & 30 & 0 & 24 \\ 1 & \infty & 10 & 0 \\ 50 & 0 & \infty & 28 \\ 4 & 4 & 0 & \infty \end{pmatrix}$$

step 3: column

$$\begin{pmatrix} \infty & 30 & 0 & 24 \\ 0 & \infty & 10 & 0 \\ 49 & 0 & \infty & 28 \\ 3 & 4 & 0 & \infty \end{pmatrix}$$

step4:

$$\begin{pmatrix} \infty & 30 & \boxed{0} & 24 \\ \boxed{0} & \infty & 10 & \cancel{0} \\ 49 & \boxed{0} & \infty & 28 \\ 3 & 4 & \cancel{0} & \infty \end{pmatrix}$$

step 5:

$$\begin{pmatrix} \infty & 30 & \boxed{0} & 24 \\ \boxed{0} & \infty & 10 & \cancel{0} \\ 49 & \boxed{0} & \infty & 28 \\ 3 & 4 & \cancel{0} & \infty \end{pmatrix}$$

No.of matrix ≠ No.of allocation

Step 6:

$$\begin{pmatrix} \infty & 27 & \boxed{0} & 21 \\ \cancel{0} & \infty & 13 & \boxed{0} \\ 49 & \boxed{0} & \infty & 28 \\ \boxed{0} & 1 & \cancel{0} & \infty \end{pmatrix}$$

No. of matrix = No. of allocation

The optimum assignment schedule is given by

~~A → C → B → D → A~~

A → C , B → D , C → B , D → A

i.e) A → C , C → B , B → D , D → A

The route conditions are A → C → B → D → A

∴ The required minimum cost = 16 + 40 + 32 + 40

= 128 units

H.W	A	B	C	D	E
A	-	3	6	2	3
B	3	-	5	2	3
C	6	5	-	6	4
D	2	2	6	-	6
E	3	3	4	6	-

Ans : 15

Queuing Theory

Def:
A flow of customers from infinite (or) finite population towards the service facility forms a queue.

The arrival rate follows a poisson distribution. Inter arrival time follows an exponential distrubtion

Notation:

λ = arrival rate, poisson distribution; Random in nature, average no. of customers arriving per unit of t

μ = service rate,

$1/\lambda$ = interarrival time $1/\mu$ = interservice time

Poisson queueing system:

Queues that follow the poisson arrivals and poisson arrivals are called the poisson queues.

Notation:

μ = average no. of customers completing service per unit of time.

$\boxed{\rho = \frac{\lambda}{\mu}}$ => traffic intensity (or) server (or) utilization factor (or) server busy.

P_n = Probability distri/ of queue length

P_0 = pro/ for the server to be idle in P_0

L_s = Averge queue length

L_q = Averge no. of customers in the system (both in waiting & servicing)

W_s = Avergge waiting time of a customer in the queue

W_q = Average waiting time of a customer in the system.

1) The goods trains are coming in a yard at the rate of 30 trains per day & the serive time for each train is assumed to be exponential with an average of 36 min. If the yard can admit 9 trains at a times, then the proba/ that the yard is empty is?

solu:

$\lambda = 30/\text{day} \quad \mu = \frac{1}{36}\text{ min}$

$\mu = \frac{1}{36} \times 60 \times 24 = 40 \text{ day}$

$\rho = \frac{\lambda}{\mu} = \frac{30}{40} = 3/4 \qquad N = 9$

Then the proba/ that the yard is empty (P_0) $= \frac{1-\rho}{1-\rho^{N+1}}$

$$= \frac{1-\frac{3}{4}}{1-\left(\frac{3}{4}\right)^{9+1}} = \frac{1-\frac{3}{4}}{1-\left(\frac{3}{4}\right)^{10}} = \frac{\frac{4-3}{4}}{\frac{4^{10}-3^{10}}{4^{10}}}$$

$$= \frac{\frac{1}{4}}{4^{10}-3^{10}} \times 4^{10} = \frac{4^{-1} \times 4^{10}}{4^{10}-3^{10}} = \frac{4^{9}}{4^{10}-3^{10}}$$

$$= 0.2619$$

2) If for period of 2hr in a day (8-10 AM) trains arrive at the yard every 20 min. but the ~~ser~~ service time continues to remain 36 min, then calculate for this period.

a) the proba./ that the yard is empty.

b) average queue length, assuming that capacity of the yard is 4 trains only

Solu:

Here $\lambda = 20\,min$ $\cancel{\mu = \frac{1}{36}}$ $\mu = 36$

$\cancel{\rho = \frac{\lambda}{\mu} = \frac{20}{36}} = \cancel{36}$

$\rho = \frac{\lambda}{\mu} = \frac{20}{36} = \frac{5}{9}$

$N = 4$

a) $P_0 = \frac{1-\rho}{1-\rho^{N+1}} = \frac{1-\frac{5}{9}}{1-(5/9)^{4+1}} = \frac{\frac{9-5}{9}}{1-(5/9)^5}$

$= \frac{\frac{4}{9}}{\frac{9^5-5^5}{9^5}} = \frac{4}{\cancel{9}} \frac{\times 9^{\cancel{5}4}}{9^5-5^5}$

$= \frac{4 \times 9^4}{9^5 - 5^5} = \frac{4 \times 6561}{59049 - 3125}$

$= \frac{26244}{55924}$

P_0

$\boxed{P_0 = 0.469}$

b) $L_s = P_0 \sum_{n=0}^{N} n\rho^n$

$= 0.4 \sum_{n=0}^{4} = 0.4 [0 \cdot \rho^0 + 1 \cdot \rho^1 + 2 \cdot \rho^2 + 3\rho^3 + 4\rho^4]$

$= 0.4 [\rho + 2\rho^2 + 3\rho^3 + 4\rho^4]$

$= 0.4 \left[\frac{5}{9} + 2[5/9]^2 + 3[5/9]^3 + 4[5/9]^4\right]$

$= 0.4 \times \frac{5}{9}\left[1 + 2 \times \frac{5}{9} + \cancel{3}(5/\cancel{9}_3)^2 + 4(5/9)^3\right]$

$= 0.4 \times \frac{5}{9}\left[1 + \frac{10}{9} + \frac{25}{27} + 4\left(\frac{125}{729}\right)\right)$

$= \frac{2}{9}[1 + 1.11 + 2.74 \; 0.68]$ 0.82

$= \frac{11.12}{9} = 1.23$

3) At a railway station only one train is handled at a time. The railways yard is sufficient only for two trains to wait while other is given signal to leave the station. Trains arrive at the station at an average rate of 6 per hr & the railways station can handle them on an average of 12/hr i) find the proba. that there is no train in the system. ii) find the average no. of customer in the system.

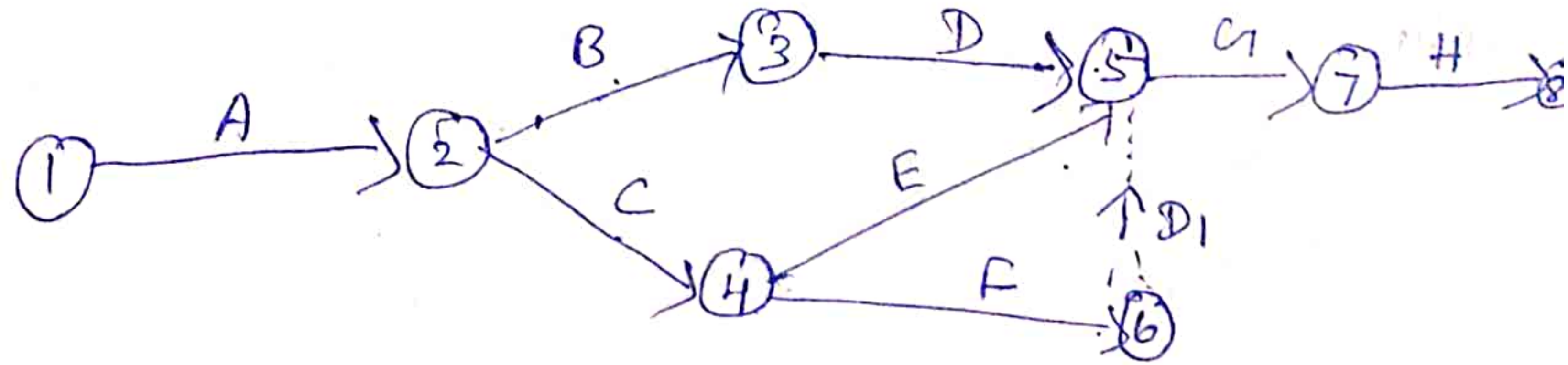
UNIT – IV
PERT AND CPM

1) Activity: A B C D E F G H

predecessor: – A A B C C D,E,F G

Activity	A	B	C	D	E	F	G	H
predecessor	–	A	A	B	C	C	D,E,F	G

Soln:



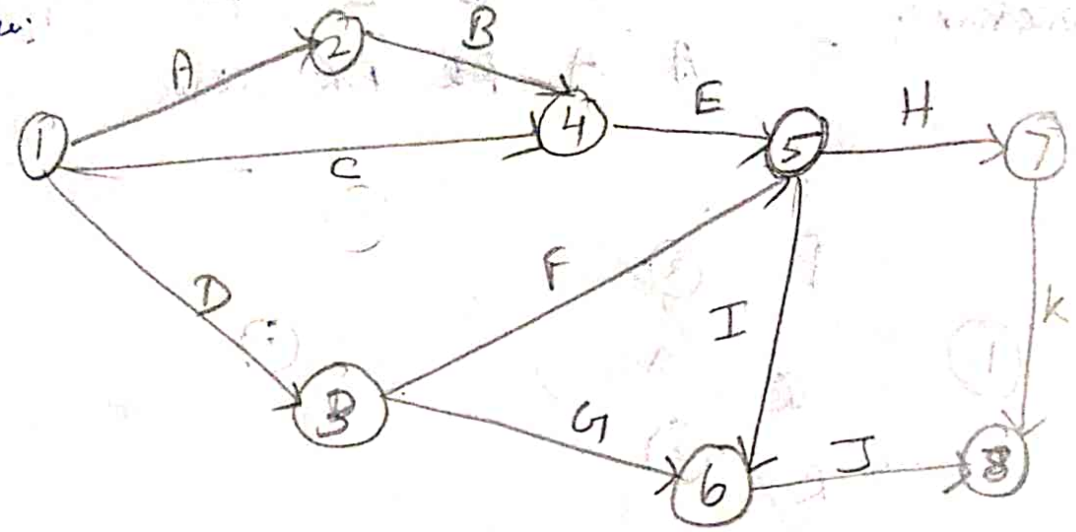
2) Draw the network for the project ... activities with their predecessor relationships are given below.

A, C, D can start simultaneously:

E > B, C; F, G > D; H, I > E, F; J > I, G;

K > H; B > A

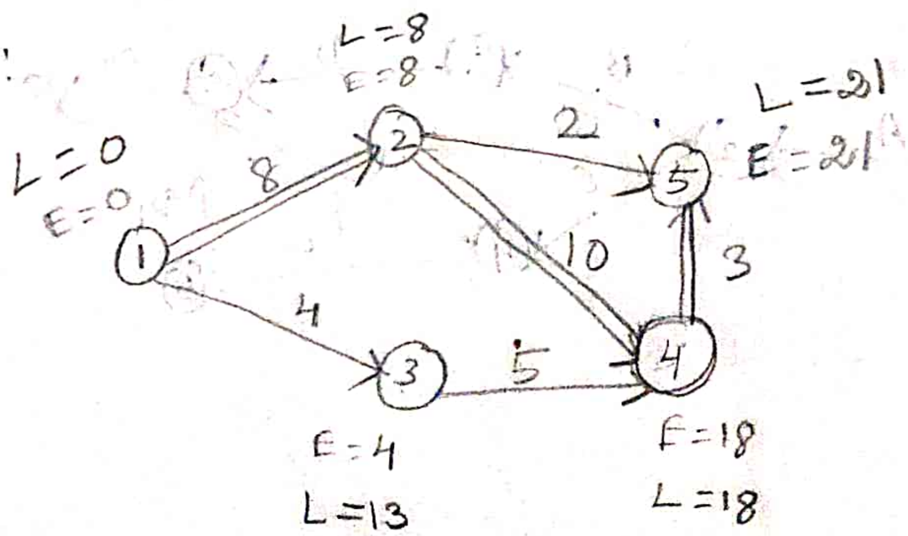
Soln:



3) Compute the earliest start, earliest finish latest start and latest finish of each activity of the project given below.

Activity:	1 - 2	1 - 3	2 - 4	2 - 5	3 - 4	4 - 5
Duration:	8	4	10	2	5	3

Soln:



Formula for Earliest start of an activity i-j in a project network is given by

$$ES_j = \text{Max}\,[ES_i + t_{ij}]$$

where ES_i denotes the earliest start time of all the activities emanating from node i & t_{ij} is the estimated duration of the activity i-j

Formula for the latest start time of all the activities emanating from the event i of the activity i-j, $LS_i = \text{Min}\,[LS_j - t_{ij}]$ for all defined i-j activities where t_{ij} is the estimated duration of the activity i-j.

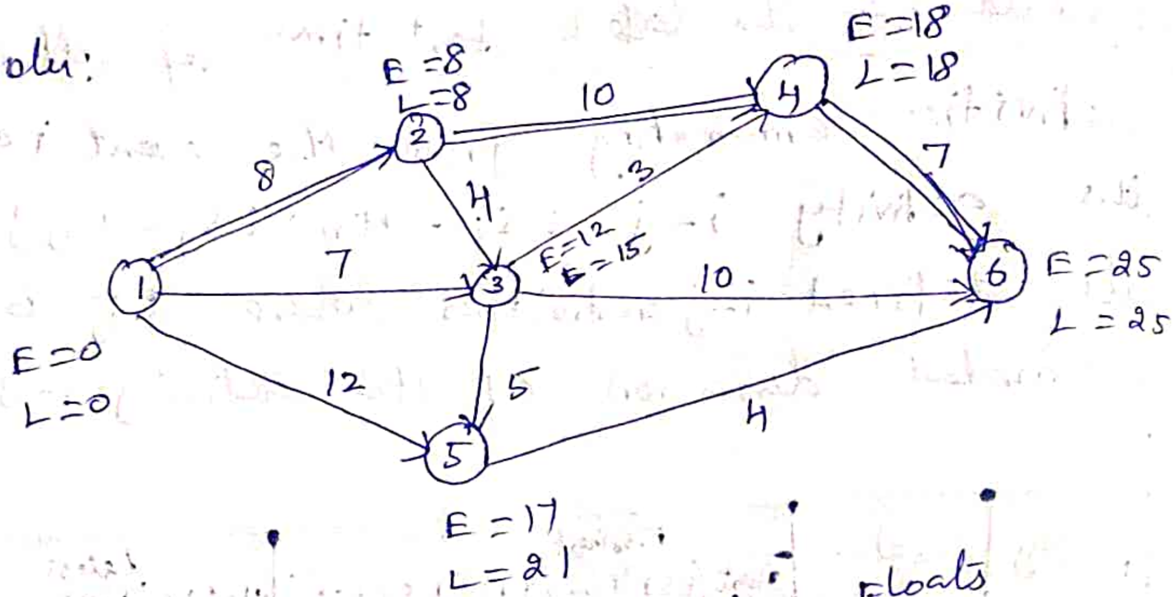
Activity	duration	Earliest		Latest	
		Start (ES)	Finish (EF) = ES + t_{ij}	Start LS	Finish (LS) = LF − t_{ij}
1-2	8	0	8	8	0
1-3	4	0	4	13	9
2-4	10	8	18	18	8
2-5	2	8	10	21	19
3-4	5	4	9	18	13
4-5	3	18	21	21	18

critical path is 1 — 2 — 4 —> 5

4) Calculate the total float, free float and independent float for the project whose activities are given below.

Acti :	1-2	1-3	1-5	2-3	2-4	3-4	3-5	3-6	4-6	5-6
duration:	8	7	12	4	10	3	5	10	7	4

Solu:



Acti	dura	Earliest Start	Earliest Finish	Latest Sta	Latest Fini	Floats TF $LF_{ij} - EF_{ij}$	FF	IF
1-2	8	0	8	0	8	0	0	0
1-3	7	0	7	8	15	8	5	5
1-5	12	0	12	9	21	9	5	5
2-3	4	8	12	11	15	3	0	0
2-4	10	8	18	8	18	0	0	0
3-4	3	12	15	15	18	3	3	0
3-5	5	12	17	16	21	4	0	-3
3-6	10	12	22	15	25	3	3	0
4-6	7	18	25	18	25	6	0	0
5-6	4	17	21	21	25	4	4	0

Total float : $(LF)_{ij} - (EF)_{ij}$ (or) $(LS)_{ij} - (ES)_{ij}$

Free float : Total float(i-j) − (L − E) of the event j

Independent float : Free float i-j − (L − E) of event i

Acti	T·F	F·F	I·F
1-2	8 − 8 = 0	0 − (8-8) = 0	0 − (0-0) = 0
1-3	15 − 7 = 8	8 − (15-12) = 5	5 − (0-0) = 5
1-5	21 − 12 = 9	9 − (21-17) = 5	5 − (0 − 0) = 5
2-3	15 − 12 = 3	3 − (15−12) = 0	0 − (8-8) = 0
2-4	18 - 18 = 0	0 − (18−18) = 0	0 − (8-8) = 0
3-4	18 - 15 = 0	3 − (18 − 18) = 3	3 − (15-12) = 0
3-5	21 − 17 = 4	4 − (21 − 17) = 0	0 − (15-12) = −3
3-6	25 − 22 = 3	3 − (25-25) = 3	3 − (15-12) = 0
4-6	25 − 25 = 0	0 − (25-25) = 0	0 − (18-18) = 0
5-6	25 − 21 = 4	4 − (25-25) = 4	4 − (21-17) = 0

critical path is 1 → 2 → 4 → 6.

5) construct the network for the project whose activities and the three time estimates of these activities (in weeks) are given below. Compute

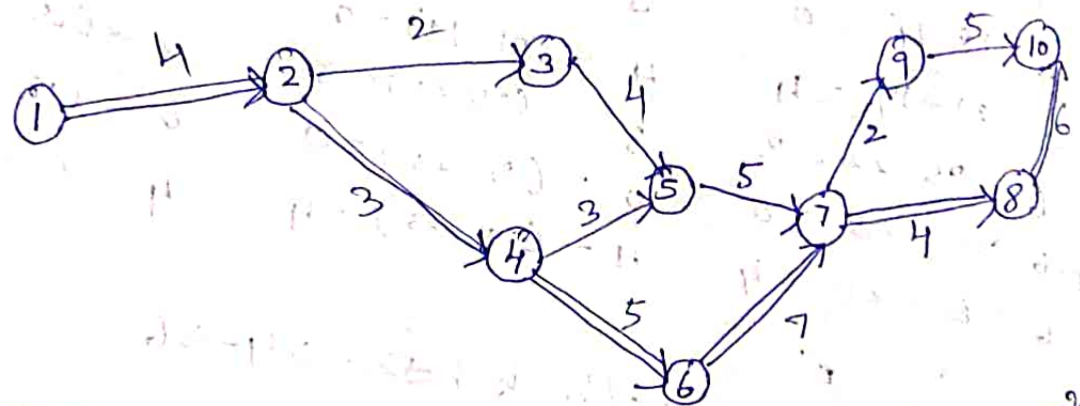
a) Expected duration of each activity

b) Expected variance of each activity

c) Expected variance of the project length.

Activity:	1-2	2-3	2-4	3-5	4-5	4-6	5-7	6-7	7-8	7-9	8-10	9-10
t_o	3	1	2	3	1	3	4	6	2	1	4	3
t_m	4	2	3	4	3	5	5	7	4	2	6	5
t_p	5	3	4	5	5	7	6	8	6	3	8	7

Solu: a) & b)



Activity	t_o	t_m	t_p	$t_e = t_o + 4t_m + t_p / 6$	$\sigma^2 = \left(t_p - t_o / 6\right)^2$
1-2	3	4	5	4	1/9 = 0.11
2-3	1	2	3	2	1/9 = 0.11
2-4	2	3	4	3	1/9 = 0.11
3-5	3	4	5	4	1/9 = 0.11
4-5	1	3	5	3	4/9 = 0.44
4-6	3	5	7	5	4/9 = 0.44
5-7	4	5	6	5	1/9 = 0.11
6-7	6	7	8	7	1/9 = 0.11
7-8	2	4	6	4	4/9 = 0.44
7-9	1	2	3	2	1/9 = 0.11
8-10	4	6	8	6	4/9 = 0.44
9-10	3	5	7	5	4/9 = 0.44

critical path $1-2-4-6-7-8-10$

Expected project duration $\Big\} = 4+3+5+7+4+6 = 29$ weeks

c) Expected variance of the project length = sum of the expected variances of all the critical activities.

$$= \frac{1}{9} + \frac{1}{9} + \frac{4}{9} + \frac{1}{9} + \frac{4}{9} + \frac{4}{9} = \frac{15}{9} = \frac{5}{3} = 1.67$$

UNIT - 3

Inventory Theory

Notation:

Q^o = optimum order quantity (EOQ)

C_{min} = minimum total annual inventory cost

n^o = optimum no. of orders

t^o = optimum length of time b/w order

T = The total time period.

K = No. of items production rate per unit of time

r = demand rate per unit of time

C_2 = shortage cost.

D (or) r = no. of items required per unit time (demand)

	Inventory problem with no shortage			Inventory problem with shortage	
	Fundamental Problem	with several production rate	with finite production	Fundamental problem	with finite production
Q°	$\sqrt{\frac{2DC_s}{C_1}}$	$\sqrt{\frac{2DC_s}{C_1 T}}$	$\sqrt{\frac{2DC_s}{C_1}\left(\frac{k}{k-r}\right)}$	$\sqrt{\frac{2DC_s}{C_1}\left(\frac{C_1+C_2}{C_2}\right)}$	$\sqrt{\frac{2C_s}{C_1}\frac{C_1+C_2}{C_2}\frac{kr}{k-r}}$
C_{min}	$\sqrt{2DC_1C_s}$	$\sqrt{\frac{2DC_1C_s}{T}}$	$\sqrt{2DC_1C_s\left(\frac{k-r}{k}\right)}$	$\sqrt{2DC_1C_s\left(\frac{C_2}{C_1+C_2}\right)}$	$\sqrt{2C_1C_s\left(\frac{C_2}{C_1+C_2}\right)\left(\frac{r(k}{k}\right.}$
n°	$\sqrt{\frac{DC_1}{2C_s}} = \frac{D}{Q^\circ}$	$\sqrt{\frac{DC_1}{2TC_s}}$	$\sqrt{\frac{DC_1}{2C_s}\left(\frac{k-r}{k}\right)}$	$\sqrt{\frac{DC_1}{2C_s}\left(\frac{C_2}{C_1+C_2}\right)}$	$\sqrt{\frac{C_1}{2C_s}\frac{C_2}{C_1+C_2}\frac{r(k-r)}{k}}$
t°	$\sqrt{\frac{2C_s}{DC_1}} = \frac{Q^\circ}{D} = \frac{1}{n^\circ}$		$\sqrt{\frac{2DC_s}{C_1k(k-r)}}$	$\sqrt{\frac{2C_s}{DC_1}\left(\frac{C_1+C_2}{C_2}\right)}$	$\sqrt{\frac{2C_s}{C_1}\frac{C_1+C_2}{C_2}\frac{k}{r(}}$

Cost associated with inventories:

1) setup cost (C_s) 2) ordering cost (C_s)
3) purchase cost (C_s) 4) processing cost (C_s)
5) procurement cost (C_s).

1) Holding cost
2) carrying cost $\Big\} = C_1$
3) storage cost

1) Inventory carrying cost
2) unit cost $\Big\} = C$

1) Shortage cost $= C_2$

1) A manufacturer has to supply his customers 600 units of his product per year. Shortage are not allowed & the storage cost amounts to Rs. 0.60 per unit per year. The set up cost per run is Rs. 80 i) The optimum order quantity. ii) the min. average yearly cost. iii) optimum no. of order per year iv) optimum period of supply per optimum year.

Solu:

$D = 600$ units. Storage cost $C_1 = 0.60$

setup cost $C_s = 80$

i) Optimum order quantity $Q_0 = \sqrt{\frac{2DC_s}{C_1}}$

$$= \sqrt{\frac{\cancel{2} \times \cancel{600}^{1000} \times 80}{\cancel{0.60}\ \cancel{0.10}\ \cancel{0.05}\ 0.01}}$$

$$= \sqrt{\frac{20 \times 80}{0.01}} = \sqrt{\frac{1600}{0.01}} = \sqrt{160000}$$

$\boxed{Q_0 = 400 \text{ unit}}$

ii) A $C_{min} = \sqrt{2DC_SC_1} = \sqrt{2\times600\times0.\ldots}$

$= Rs\ 240$

iii) $n_o = \frac{D}{Q^o} = \frac{600}{400} = 3/2$

iv) $t_o = \frac{1}{n_o} = 2/3$ of a year.

H.w

2) A certain item costs Rs. 235 per ton. The monthy requirement are 5 tons, & each time the stock is replacement. there is a setup cost of RS 1000. The cost of carrying inventory has been estimated at 10% of the average inventory per year. what is the optimum order quantity [Ans: 71.458 tons]

3) An item is produced at 50 units / day & the demand occurs at the rate of 25 units / day. if the setup cost is Rs. 100/ order & the holding cost is Rs 0.01 / unit per day. Assumed that no shortage find i) EOQ

ii) optimum order time iii) minimum annual cost.

ii) A $C_{min} = \sqrt{2DC_SC_1} = \sqrt{2 \times 600 \times 0.60 \times 80}$

$= Rs\ 240$

iii) $n_0 = \frac{D}{Q^0} = \frac{600}{400} = 3/2$

iv) $t_0 = \frac{1}{n_0} = 2/3$ of a year.

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ii) optimum order time iii) minimum annual cost.

Solu:

$k = 50 \quad r = 25 \quad D = 25 \quad C_S = 100$

$C_1 = 0.01$

i) $Q^0 = \sqrt{\frac{2DC_S}{C_1} \frac{k}{k-r}} = \sqrt{\frac{2 \times 25 \times 100}{0.01} \times \frac{50}{50-25}}$

$= \sqrt{\frac{2 \times 2\cancel{5} \times 100}{0.0\cancel{1}} \times \frac{50}{\cancel{25}}}$

$= \sqrt{\frac{10000}{0.01}} = \sqrt{1000000}$

$Q^0 = 1000$ units

ii) $t^0 = \sqrt{\frac{2DC_S}{C_1 k(k-r)}} = \sqrt{\frac{2 \times 2\cancel{5} \times 100}{0.01 \times 50 \times \cancel{25}}} = \sqrt{\frac{2 \times 100 \times 100}{1 \times 50}}$

$= \sqrt{400}$

$t^0 = 20$

iii) $C_{min} = \sqrt{2DC_S C_1 \cdot \left(\frac{k-r}{k}\right)} = \sqrt{2 \times 25 \times 100 \times 0.01 \times \frac{25}{50}}$

$= \sqrt{25} = 5$

4) The demand for an Item is 18,000 units per year. The holding cost per unit time is Rs. 1.20 & the cost of shortage is Rs. 5.00 the production cost is Rs. 400. Assuming the replacement rate is Instantaneous,

i) determine the optimal order quantity.

ii)

ii) No. of order per cost iii) Time b/w orders.

Soln: $D = 181000$ $C_1 = Rs. 1.20$ $C_2 = Rs. 5.00$

$C_S = 400$

i) $Q^o = \sqrt{\frac{2 C_S D}{C_1}} \left(\frac{C_1 + C_2}{C_2}\right) = \sqrt{\frac{2 \times 400 \times 18000}{1.20}} \times \left(\frac{1.}{}\right.$

$= 1.113 \times 3,464.10$

$Q^o = 3856$ unit

ii) $N^o = \frac{D}{Q^o} = \frac{18000}{3856} = 4.668$

iii) $t^o = \frac{Q^o}{D} = \frac{3856}{18000} = 0.214$

5) The demand for an item in a company 18,000 units / year, & the company can produce the items at a rate of 3000/month. The cost of one set up is Rs. 500 & the holding cost of one unit per month is 1.15 paise. The shortage cost of one unit is Rs. 20 per month. Determine the i) Optimum manufacturing quantity & ii) the no. of shortages: Also determine iii) the

manufacturing time and iv) time b/w set-ups.

solu:

$C_1 = 0.15$ / month $\quad C_2 = 20. \quad C_s = 500$

$K = 3000$ units / month $\quad r = 18000$ unit / year

$r = \frac{18000}{12}$ unit / month $= 1500$

i) $Q^0 = \sqrt{\frac{2C_s}{C_1} \frac{C_1+C_2}{C_2} \frac{Kr}{K-r}}$

$= \sqrt{\frac{2 \times 500}{0.15} \times \frac{0.15+20}{20} \times \frac{3000 \times 1500}{3000-1500}}$

$= \sqrt{\frac{1000}{0.15} \times \frac{20.15}{20} \times \frac{4500000}{1500}}$

$= \sqrt{6666.66 \times 1.0075 \times 3000}$

$= \sqrt{20149979.85}$

$= 4488.87$ unit

$\cancel{N^0 = \frac{D}{Q^0} = \frac{r}{Q^0}} = \frac{1500}{4488.87} = 0.334$

ii) Time b/w setup $= \frac{Q^0}{D} = \frac{Q^0}{r} = \frac{4488.87}{18000} = 0.2493$

iii) manufacturing time $= \frac{Q^0}{K} = \frac{4488.87}{3000 \times 12} = \frac{4488.87}{36000}$

$= 0.12446.$

ii) No of Shortages $s = \frac{C_1}{C_1+C_2} Q^0 \left(1 - \frac{r}{k}\right)$

$$= \frac{0.15}{0.15+20} \times 4488.87 \left(1 - \frac{1500}{3000}\right)$$

$$= \frac{0.15}{20.15} \times 4488.87 \left(\frac{3000-1500}{3000}\right)$$

$$= \frac{0.15}{20.15} \times 4488.87 \times \left(\frac{1500}{3000}\right)$$

$$= 0.0074 \times \frac{67333.05}{30}$$

$$= 0.0074 \times 2244.435$$

$$= 16.60 \text{ units}$$

UNIT - 5

Dynamic programming

Dynamic programming approach for priority Management employment smoothening:

procedure Adopted in DPP:

* Define the variables, objective function & constraints
* Divide the problem into no. of sub-problem
* Develop recursive relationship for optimality.
* Decide whether to follow the forward or the backward method to solve the problem.
* Make tabular presentation to shown the required values & calculation for each Stage.
* Find optimal policy at each stage & then the overall optimal policy

1) A firm has divided its marketting area into three zones. The amount of sales depends upon the no. of salesman in each zero. The firm has been collecting the data regarding sales and salesmen in each area over a no. of past year. The information is summarized in table. For the next year firm has only 9 salesmen & the problem is to allocate these salesmen to three different

zone so that the total sales are maximum.

No. of Salesmen	Profit in thousands of rupees		
	Zone 1	Zone 2	Zone 3
0	30	35	42
1	45	45	54
2	60	52	60
3	70	64	70
4	79	72	82
5	90	82	95
6	98	93	102
7	105	98	110
8	100	100	110
9	90	100	110

Soln:

let x_1, x_2 & x_3 be the no. of salesmen allocated to zone 1, zone 2 & zone 3 res.)

$f_1(x_1)$ $f_2(x_2)$ & $f_3(x_3)$ are profit from zone 1, zone 2 & zone 3 respectively.

Stage: 1 zone 1

Stage: 2 zone 1 + zone 2.

Stage: 3 zone 1 + zone 2 + zone 3

stage : 1

No. of salesman x_1	0	1	2	3	4	5	6	7	8	9
profit in thousand of rupees $f_1(x_1)$	30	45	60	70	79	90	98	105	100	90

stage : 2 zone 1 + zone 2

zone 1 →	x_1	0	1	2	3	4	5	6	7	8	9
	$f_1(x_1)$	30	45	60	70	79	90	98	105	100	90
zone 2	↓										
x_2	$f_2(x_2)$			$f_1(x_1)$ +	$f_2(x_2)$						
0	35	65*	80*	95*	105*	114	125*	133	140	135	125
1	45	75	90	105*	115*	124	135*	143*	150	145	—
2	52	82	97	112	122	131	142	150	157		
3	64	94	109	124	134	143*	154*	162			
4	72	102	117	132	142	151	162				
5	82	112	127	142	152	161					
6	93	123	138	153	163*						
7	98	128	143	158							
8	100	130	145								
9	100	130									

stage : 3 (zone 1 + zone 2) + zone 3

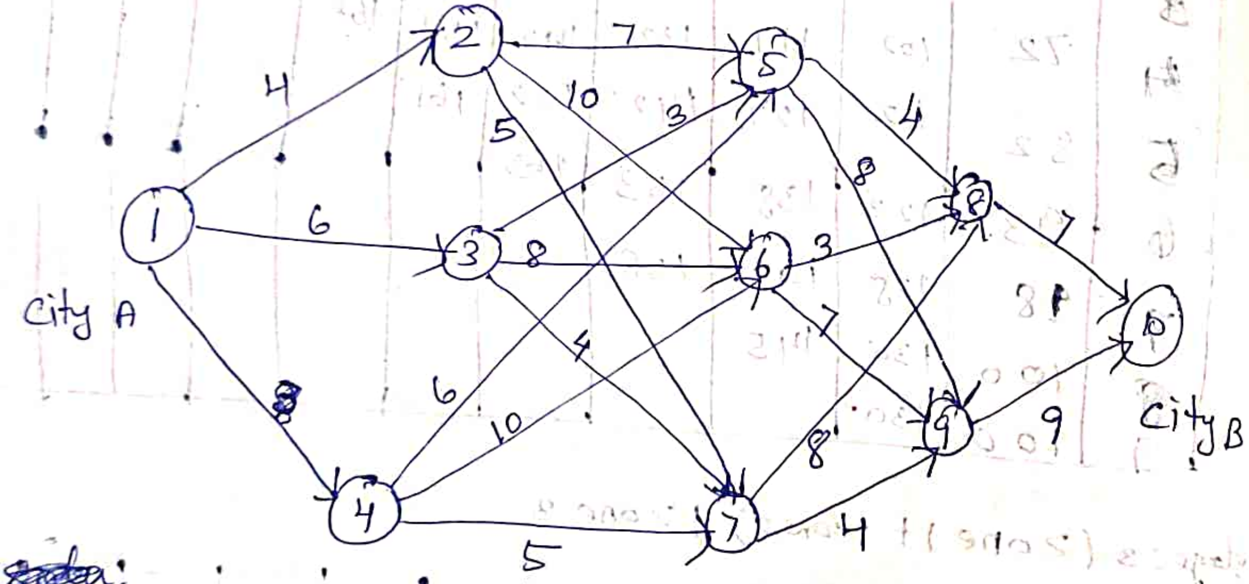
No. of salesmen :	0	1	2	3	4	5	6	7	8	9
Max of $f_1(x_1)+f_2(x_2)$	65	80	95	105	115	125	135	143	154	163
salesmen in (x_1+x_2) zone 1 + zone 2	0+0	0+1	0+2	0+3 1+2	1+3	0+5	1+5	1+6 3+4	3+5	6+3
salesmen in zone 3 x_3	9	8	7	6	5	4	3	2	1	0
profit $f_3(x_3)$	110	110	110	102	95	82	70	60	54	42
zone (1+2+3) $f_1(x_1)+f_2(x_2)+f_3(x_3)$	175	190	205	207	210*	207	205	203	208	205

Maximum profit for 9 Salesmen is $= 210 \times 1000$
$= 2,10,000$

2,10,000 if 5 salesmen are allotted to zone 3 and from the remaining four, 1 is allotted to zone 2 & 3 to zone 1.

Stage coach / Shortest Path:

1) Find the shortest path city 1 to city 10 in the diagram show below using recursive principle of Dynamic programming



~~Soln~~:

Stage: 5 → 1 stage: 4 → 2, 3, 4 stage: 3 → 5, 6, 7
Stage: 2 → 8, 9 stage: 1 → 10.

- According to diagram the distance have been given in the unit of km.

city of origin = city A
city of destination = city B

Method: 2

Current location	Possibility	distance	Total distance
Stage: I			
10	8 - 10	7	
	9 - 10	9	
Stage: II			
8	5 - 8	4	4+7=11
	6 - 8	3	3+7=10
	7 - 8	8	8+7=15
9	5-9	8	8+9=17
	6-9	7	7+9=16
	7-9	4	4+9=13
Stage: III			
5	2-5	7	7+11=18
	3-5	3	3+11=14
	4-5	6	6+11=17
6	2-6	10	10+10=20
	3-6	8	8+10=18
	4-6	10	10+10=20
7	2-7	5	5+13=18
	3-7	4	4+13=17
	4-7	5	5+13=18
Stage: IV			
2	1-2	4	4+18=22
3	1-3	6	6+14=20
4	1-4	3	3+17=20

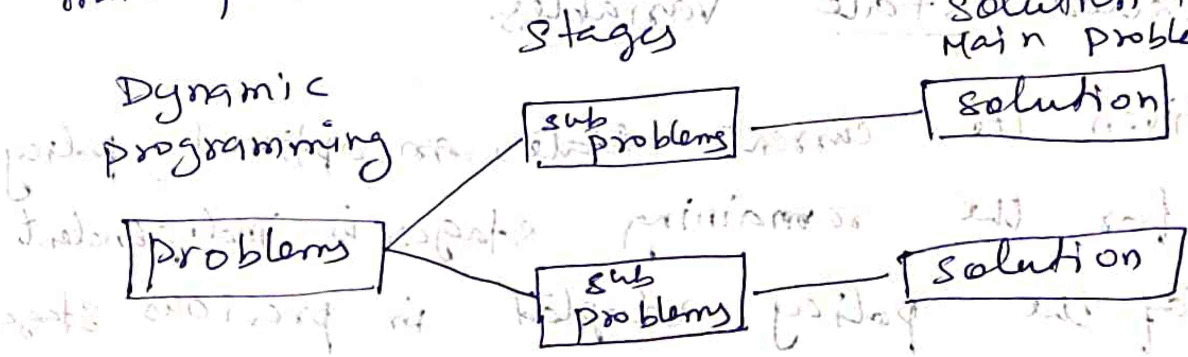
$1 \xrightarrow{6} 3 \xrightarrow{3} 5 \xrightarrow{4} 8 \xrightarrow{7} 10 = 20$

$1 \xrightarrow{3} 4 \xrightarrow{6} 5 \xrightarrow{4} 8 \xrightarrow{7} 10 = 20$

What is Dynamic programming?

* It is the technique which is used in optimization of multi-stage decision problem.

* In dynamic programming, the original problem is subdivided into sub problems & the solution of these sub problem are integrated to attain the solution for the main problem.



Characteristics of Dynamic programming:

1) Stages:

It is device to sequence the decisions. That is, it decomposes a problem into sub-problems such that an optimal solution to the problem can be obtained from the optimal solu. to sub-problem.

2) States:

* Every stage consists of a number of states associated with it.

* The states are the different possible conditions of the problem.

* Decision at each stage converts the current stage into state associated with the next stage.

3) State variables:

* The state of the system at a stage is described by a set of variables, called <u>state variables</u>.

4) * Given the current state, an optimal policy for the remaining stages is independent of the policy adopted in previous stages.

* For dynamic Prog. / Problem, in general knowledge of the current state of the system conveys all of the information about ~~the~~ its previous behaviour necessary for determining the optimal policy hence forth. This is the <u>Markovian property</u>

5) The solu/ procedure begins by finding the optimal policy for each state of the last stage.

6) A recursive relationship which identifies the optimal policy for each state with n stages remaining, given the optimal policy for each state with (n-1) stages left.

7) using this recursive relationship, the solution procedure moves backward stage-by-stage, each time finding the policy when starting at the initial stage.

Application of DPP:

* capital budgeting
* cargo loading problem
* Reliability improvement
* Linear Programming
* shortest / stage coach
* optimal - sub diving problem
* Minimizing total tardiness in single machine scheduling.

capital budgeting:

A capital budgeting problem is a problem in which a given amount of capital is allocated to a set of plants by selecting the most promising alternative for each selected plant such that the total revenue of the organization is maximized.

Cargo loading problem:

1) In a cargo loading problem, there are 4 items of diff. weights / unit & different value / unit as given below

Item(i)	weight / unit (w_i kg / unit)	value / unit (P_i ₹/unit)
1	1	1
2	3	5
3	4	7
4	6	11

The maximum cargo load is restricted to 17. How many units of each item be loaded to maximize the value?

Solu:

It is a four problem, each item represents a stage. The state of the system is represented by the weight capacity available for allocation to stages 1, 2, 3, 4 & is denoted by x_i which varies from 0 to 17. If a_i is the ~~max~~ the number of item i, then the problem is

$$\text{maximize } Z = \sum_{i=1}^{4} a_i p_i$$

$$\text{subject to } \sum_{i=1}^{4} a_i w_i \leq W$$

Stage : 1 Here

$w_1 = 1$ kg/unit $p_1 =$ Re. 1/unit

$$\frac{W}{w_1} = \frac{17}{1} = 17 \quad \therefore a_1 = 0, 1, 2 \ldots 17$$

Stage : 2 Here $w_2 = 3$ kg/unit $P_2 =$ Re. 5/unit

$$\frac{W}{w_2} = \frac{17}{3} = 5.67 = (5 \text{ integral value})$$

$$\therefore a_2 = 0, 1, 2 \ldots 5$$

Stage : 3 Here $w_3 = 4$ kg/unit $P_3 =$ Rs. 7/unit

$$\frac{W}{w_3} = \frac{17}{4} = 4.25 \quad \therefore a_3 = 0, 1, 2, 3, 4$$

Stage : 4 Here $w_4 = 6$ kg/unit, $P_4 =$ Rs. 11/unit

$$\frac{W}{w_4} = \frac{17}{6} = 2.83 \quad \therefore a_4 = 0, 1, 2$$

Let $f_1(x_1)$, $f_2(x_2)$, $f_3(x_3)$ & $f_4(x_4)$ be the value of the loaded items at stage 1, 2, 3 & 4 respectively.

x_i	Stage: 1		Stage: 2		Stage: 3		Stage: 4		$f_i^*(x_i)$
	$w_1=1$, $P_1=1$, $a_1=0,1,\dots 17$		$w_2=3$, $P_2=5$, $a_2=0,1,\dots 5$		$w_3=4$ $P_3=7$ $a_3=0,1,2,3,4$		$w_4=6$, $P_4=11$ $a_4=0,1,2$		
	a_1	$f_1(x_1)$	a_2	$f_2(x_2)$	a_3	$f_3(x_3)$	a_4	$f_4(x_4)$	
0	0	0*	0	—	0	—	0	—	0
1	1	1*	0	—	0	—	0	—	1
2	2	2*	0	—	0	—	0	—	2
3	3	3	1	5+0=5*	0	—	0	—	5
4	4	4	1	5+1=6	1	7+0=7*	0	—	7
5	5	5	1	5+2=7	1	7+1=8*	0	—	8
6	6	6	2	10+0=10	1	7+2=9	1	11+0=11*	11
7	7	7	2	10+1=11	1	7+5=12*	1	11+1=12*	12
8	8	8	2	10+2=12	2	14+0=14*	1	11+2=13	14
9	9	9	3	15+0=15	2	14+1=15	1	11+5=16*	16
10	10	10	3	15+1=16	2	14+2=16	1	11+7=18*	18
11	11	11	3	15+2=17	2	14+5=19*	1	11+8=19*	19
12	12	12	4	20+0=20	3	21+0=21	2	22+0=22*	22
13	13	13	4	20+1=21	3	21+1=22	2	22+1=23*	23
14	14	14	4	20+2=22	3	21+2=23	2	22+2=24*	24
15	15	15	5	25+0=25	3	21+5=26	2	22+5=27*	27
16	16	16	5	25+1=26	4	28+0=28	2	22+7=29*	29
17	17	17	5	25+2=27	4	28+1=29	2	22+8=30*	30

As seen from the table, for total load of 17kg, the maximum value of cargo items is Rs. 30 (22+8 = 22+7+1) which is achieved if we load 1 unit of item 1, 1 unit of item 3 and 2 units of item 4.