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CLASS : M.SC COMPUTER SCIENCE

SEMESTER : III

SUBJECT : OPERTATION TECHNIQUES

SUBJECT CODE : MCS24035

UNIT 1 : Operation Research

Introduction :

The term operation research was first Coined by mcclosky and Trepthen in 1940. This new science Come into existence as a result of research on military operations during world war-II.

OR :

New approach to systematic and scientific study of the operations of the system was called the operations research (or) operational research.

O.R has been variously described as the "science of use", "quantitative Common sense", "scientific approach to decision making problems", etc.,

Nature and features of O.R:

i) O.R is the application of scientific methods, techniques and tools to problems involving the operation of a system. so as the to provide those in control of the system with optimum solution to the problem.

— chruchman, Ackoff and Arnoff.

ii) O.R is a scientific knowledge through interdisciplinary team effort for the purpose of determining the best utilization of limited resources.

— H.A Taha.

Advantages and limitations of models:

i) Through a model, the problem under consider become Controller.

ii) It provides some logical and systematic approach to the problem.

iii) It indicates the limitations and scope of an activity.

iv) It helps in incorporating useful tools that eliminate duplications of method appiled to solve any specific problems.

V) It helps in finding avenues for new research and improvements in a system.

Vi) It provides economic description and explanations of the operations of the system they represent.

Limitations :

i) Models are only an attempt in understanding operations and should never be Considered as absolute in any sense.

ii) Validity of any model with regard to corresponding operation can only be

Verified by carring out the experiment and observing relevant data characteristics.

iii) Construction of models require experts from various disciplines.

objectives of O.R:

i) It aims to decision making and improve the quality of each and every operations of the business.

ii) It aims to maximize the profit and reduce the cost of each and every operation, by optimization of total output.

iii) It aims to increase the productivity in the business by optimization of full output in the business.

iv) TO develop more effective approach to Complete the particular tasks.

v) To learn all about administration and management in social culture for the purpose of effective implementation at every stage.

vi) It also aims to introduced many new digital concepts in operational management.

Scope:

1. In agriculture:

* Increase population result in many Product.

* optimum allocation of land to a Variety of crops as per the climatic Conditions.

* optimum distribution of water from numerous resources like canal for irrigation purpose.

Hence there is a requirement of determining best policies under the given restrictions. Therefore a good quantity of work can be done in this direction.

2. In industry:

* Mostly industry make decisions on past basis and hence chances of serious loss happens. This loss can be compensated through OR techniques.
* Thus O.R is helpful for the industry director in deciding optimum distribution of several limited resources like men, machines, material, etc., to reach at the optimum decision.

3. In production management:

* A production manager can utilize OR techniques to calculate this number and size of the items to be

Produced.

* In scheduling and sequencing the production machines.

* In Computing the optimum product mix.

* To choose, locate and design the sites for the production plans.

4. Finance, Budgeting and Investment:

* cash flow analysis, long range capital requirement, dividend policies, investment portifolious.

* credit policies, credit risks and delinquent account procedures.

* claim and complaint procedure.

5. Marketing:

* product selection, timing, Competitive actions.

* Advertising mean with respect to

cost and time.

* Number of salesman, frequency, of calling, of accounts etc.,

* Effectiveness of market research.

6. personal:

* Forecasting the manpower requirement, recruitment policies and job assignments.

* Selection of suitable personal with the consideration for age and skills etc.,

* Determination of optimum number of persons for each service centre.

Phases:

1. Pre-modeling phase:

i) Identification of problem.

ii) Quantify the problem.

2. Modeling phase:

i) Data Collection.

ii) Formulation a mathematical model of problem.

iii) Identification of possible alternative solutions.

3. Implementation phase:

i) Interpretation of solution.

ii) Model Validation.

iii) Monitor and control.

Models:

A model is an ideal representation of a real system. System can be a problem, process, operation, object or event.

Types of Models:

1. classification based on functions:

i) Normative models.

ii) Predictive models.

iii) Descriptive models.

Normative models:

These models provide the best solution to problems subject to certain limitations. These models are also called optimization models or prescriptive models because they prescribe what have to be done.

Example:

Linear programming,

X-Ray of healthy man,

CPM and PERT planning model.

ii) Predictive model:

These models predict the outcomes regarding certain event due to a given set of alternatives for the problem. They can answer, 'what is type of questions.

Example:

Television network predict the election outcome before counting the votes based on the survey results.

iii) Descriptive models:

These models describe the system under study based on observation, survey, questionnaire results.

Example:

organization chart, plant layout diagram, scale diagram models etc.,

2. classification based on structure:

i) Iconic models:

Iconic models is a physical or pictorial or visual representation of the real system. They are scaled up or scaled down versions of the particular system they represent.

Ex:

Model or blue prints of proposed building, models of sun and planets are scaled down & model of atom, models of cells in human body are scaled up.

ii) Analogue Models:

These models representations a system a set of properties which is different from the original system and the does not resemble it physically.

Example:

A barometer that indicates change in atomospheric pressure through movement of a needle, graphs, flow diagrams, charts etc.,

3. classification based on nature of an environment :

i) Deterministic Models :

In these models all parameters and functional relationship are assumed to be known with certainly when decision is to be made.

Example :

* Linear programming
* Transportation
* Assignment models.

ii) probabilistic Models or stochastic models :

These type of models usually such situation in which outcome of managering action can not be predicted with certaintly.

Linear Programming problem.

Profit $\quad$ loss

Max (or) Min $\quad z = c_1x_1 + c_2x_2 + \dots + c_nx_n$ — ①

objective equ

where c_i's are real constants.

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \le \text{ or } \ge \text{ or } = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \le \text{ or } \ge \text{ or } = b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \le \text{ or } \ge \text{ or } = b_m$$

— ② subject constraint

where a_{ij}'s b_j's are real constants

& $x_j \ge 0 \quad j = 1, 2, \dots n$ → ③

Linear programming problem deals with the optimization (Max (or) Min) of a function of decision variables known as objective function, subject to a set of simultenous linear equations known as Constraints and non-negative constrains is called LPP.

Here ① is called as the objective functions.

② is called the subject Constrains

③ are called the non-negative restrictions.

Procedure for mathematical formulation of LPP.

1. Identify the unknown decision Variable to be determined and assign symbols to them.

2. Identify all the restrictions or Constraints in the problem and express them as linear equations or inequalities of decision Variables.

3. Identify the objects / objective or aim and represent it also as a linear function of decision Variables.

4. Express the Complete formulation of Lpp as a general mathematical model.

Problems:

3. Identify the objects / objective or aim and represent it also as a linear function of decision Variables.

4. Express the Complete formulation of lpp as a general mathematical model.

Problems:

1. A firm manufactures two types of products A and B and sells them at a profit of Rs. 2 on type A and Rs. 3 on type B. Each product is processing time on M_1 and 2 minutes on M2. Type B requires 1 minute on M1 and 1 Minute on M2. Machine M1 is available for not more than 6 hours 40 minutes while machine M2 is available for 10 hours during any working day. Formulate the problem as a LPP so as the maximize the profit.

Soln:

Profit :	Rs : 2	Rs : 3
(Max Z)	A	B
M_1	x_1	x_2
M_2	$2x_1$	x_2

Let the firm decide to produce x_1 units of product A

x_2 units of products B to maximize its profit.

To produce these units of type A and type B products, its require.

$x_1 + x_2$ processing minutes on M_1

$2x_1 + x_2$ processing minutes On M_2

Since machine M_1 is available for not more than 6 hours and 40 minutes and Machine M_2 is available for 10 hours doing any working day, the Constraints are

$x_1 + x_2 \leq 400$

$2x_1 + x_2 \leq 600$

1 hr = 60 min

$60 \times 6 = 360$ min.

$+40$

400 min

$10 \times 60 = 600$ min

Since Maximize the profit from type A is Rs. 2 and profit from type B is Rs. 3, the total profit is $2x_1 + 3x_2$. As the objective is to maximize the profit, the objective function is maximize $z = 2x_1 + 3x_2$.

$\therefore$ The complete formulation of the LPP is

Maximize $z = 2x_1 + 3x_2$

subject to the constraints

$$x_1 + x_2 \leq 400$$

$$2x_1 + x_2 \leq 600$$

$$x_1, x_2 \geq 0$$

2. A firm produces three products. These products are processed on three different machines. The time required to manufacture as unit of each of three products and the daily capacity of the three machines are given in the table below.

Machine	Time per unit (min)			Machine
	Pro 1	Pro 2	Pro 3	Capacity (Min / day)
M1	2	3	2	440
M2	4	-	3	470
M3	2	5	-	430

It is required to determine the number of units to be manufactured for each product daily. The profit per unit for product 1, 2 and 3 is RS. 4, RS. 3 and RS. 6 respectively. It is assumed that all the amounts produced are consumed in the market. Formulate the mathematical model for the problem.

Soln:

Let x_1, x_2 and x_3 be the number units of product 1, 2 and 3 produced respectively.

To produce these amount of products 1, 2 and 3 it requires.

$2x_1 + 3x_2 + 2x_3$ min on M_1

$4x_1 + 3x_3$ min on M_2

$2x_1 + 5x_2$ min on M_3

But the capacity of the machines M_1, M_2 and M_3 are 440, 470 and 430 (Min/day).

∴ The Constraints are

$$2x_1 + 3x_2 + 2x_3 \leq 440$$

$$4x_1 + 3x_3 \leq 470$$

$$2x_1 + 5x_2 \leq 430$$

and $x_1, x_2, x_3 \geq 0$

Since the profit per unit for product 1, 2 and 3 is Rs. 4, Rs. 3 and Rs. 6 respectively, the total profit is $4x_1 + 3x_2 + 6x_3$. As the objective is to maximize the profit, the objective function is maximize $Z = 4x_1 + 3x_2 + 6x_3$

subject to the constraints

$$2x_1 + 3x_2 + 2x_3 \leq 440$$

$$4x_1 + 3x_3 \leq 470$$

$$2x_1 + 5x_2 \leq 430$$

and

$$x_1, x_2, x_3 \geq 0.$$

3. A person wants to decide the constituents of a diet which will fulfill his daily requirements of proteins, fats and carbohydrates at the minimum cost. The choice is to be made from four different types of foods. The yields per unit of these foods are given in the following table.

Food type	Yield / unit			Cost (unit) RS
	Proteins	Fats	Carbohydrates	
1	3	2	6	45
2	4	2	4	40
3	8	7	7	85
4	6	5	4	65
Minimum	800	200	700	

Soln:

Let x_1, x_2, x_3 & x_4 be the unit of food of type 1, 2, 3 & 4 used respectively.

From these units of food of type 1, 2, 3 & 4

$3x_1 + 4x_2 + 8x_3 + 6x_4$ proteins / day.

$2x_1 + 2x_2 + 7x_3 + 5x_4$ Fats / day.

$6x_1 + 4x_2 + 7x_3 + 4x_4$ carbohydrates / day.

Since the minimum requirement of these proteins, fats and carbohydrates are 800, 200 and 700 respectively, the Constraints are

$3x_1 + 4x_2 + 8x_3 + 6x_4 \geq 800$

$2x_1 + 2x_2 + 7x_3 + 5x_4 \geq 200$

$6x_1 + 4x_2 + 7x_3 + 4x_4 \geq 700$

& $x_1, x_2, x_3, x_4 \geq 0$.

$\because$ The costs of these foods of type 1, 2, 3 & 4 are RS. 45, RS. 40, RS. 85

and RS. 65 per unit. The total cost is RS. $45x_1 + 40x_2 + 85x_3 + 65x_4$. As the objective is to minimize the total cost, the objective function is

$$\text{Minimize } z = 45x_1 + 40x_2 + 85x_3 + 65x_4$$

Subject Equation

$$3x_1 + 4x_2 + 8x_3 + 6x_4 \geq 800$$

$$2x_1 + 2x_2 + 7x_3 + 5x_4 \geq 200$$

$$6x_1 + 4x_2 + 7x_3 + 4x_4 \geq 700$$

&

$$x_1, x_2, x_3, x_4 \geq 0.$$

Formulation of LPP (Graphical solution of LPP).

The major steps in the solution of a LPP by graphical method

1) Identify the problem - the decision Variable, the objective and the restrictions.

2. set up the mathematical formulation of the problem.

3. plot a graph representing all the Constraints of the problem and identify the feasible region (solution space). The feasible region is the intersection of all the regions represented by the constraints of the problem and is restricted to the first quadrant only.

4. The feasible region obtained in step 3 may be bounded (or) unbounded. Compute the coordinates of all the corner points of the feasible region.

5. Find out the value of the objective function at each corner (solution) point determined in step 4.

6. select the Corner point the optimizes (maxi (or) mini) the values of the objective function. It gives the optimum feasible solution.

Problem:

1. solve the following LPP method using graphical method.

$$\text{Max } z = 3x_1 + 2x_2$$

subject to $-2x_1 + x_2 \leq 1$

$x_1 \leq 2$

$x_1 + x_2 \leq 3$ and

$x_1, x_2 \geq 0$.

soln:

First Consider the inequality Constraints as equalities.

$$-2x_1 + x_2 = 1 \longrightarrow (1)$$

$$x_1 = 2 \longrightarrow (2)$$

$$x_1 + x_2 = 3 \longrightarrow (3)$$

and $x_1 = 0 \longrightarrow (4)$

$x_2 = 0 \longrightarrow (5)$

equ (1) $\Rightarrow$ put $x_1 = 0$

$-2(0) + x_2 = 1$

$\boxed{x_2 = 1}$ $(0,1)$

Put $x_2 = 0$

$-2x_1 + 0 = 1$

$-2x_1 = 1$

$\boxed{x_1 = -\frac{1}{2}}$ $(-\frac{1}{2}, 0)$

equ (3) $\Rightarrow$ put $x_1 = 0$

$0 + x_2 = 3$

$\boxed{x_2 = 3}$ $(0,3)$

put $x_2 = 0$

$x_1 + (0) = 3$

$\boxed{x_1 = 3}$ $(3,0)$

B point $\cancel{x_1} + x_2 = 3$

$-\cancel{x_1} = -2$

$x_2 = 1$

$x_1 + x_2 = 3$

$x_1 + 1 = 3$

$\boxed{x_1 = 2}$

C point $-2x + \cancel{x_2} = 1$

$-x_1 \mp \cancel{x_2} = \mp 3$

$-3x_1 = \mp 2$

$$x_1 = \frac{2}{3}$$

$$x_1 + x_2 = 3$$

$$\frac{2}{3} + x_2 = 3$$

$$x_2 = 3 - \frac{2}{3}$$

$$= \frac{9-2}{3}$$

$$= \frac{7}{3}$$

$$\boxed{x_2 = \frac{7}{3}}$$

$\therefore$ The vertices of the solution space are $O(0,0)$ $A(2,0)$ $B(2,1)$ $C\left(\frac{2}{3}, \frac{7}{3}\right)$ and $D(0,1)$.

The values of z at these vertices are given by

Vertex	Value of z
	$(z = 3x_1 + 2x_2)$
$O(0,0)$	$3(0)+2(0)$ $z = 0+0 = 0$
$A(2,0)$	$3(2)+2(0)$ $6+0$ $z=6$
$B(2,1)$	$z = 3(2)+2(1)$ $= 6+2$ $\boxed{z = 8}$
$C\left(\frac{2}{3}, \frac{7}{3}\right)$	$z = \cancel{3} \times \frac{2}{\cancel{3}} + 2 \times \frac{7}{3}$ $= 2 + \frac{14}{3}$ $= \frac{6+14}{3} = \frac{\overset{6.}{\cancel{20}}}{\cancel{3}} =$
$D(0,1)$	$z = 3(0)+2(1)$ $= 0+2$ $z = 2$

∴ Since the problem is of Maximization type the optimum solution to the LPP is

Maximum $z = 8$,

$x_1 = 2$

$x_2 = 1$.

∴ The problem have feasible solution.

The problem have feasible solution. The area bounded by all Constraints, called feasible solution.

3. use graphical method for LPP.

Maximize $z = 6x_1 + x_2$

Subject Constraints $= 2x_1 + x_2 \geq 3$

$x_2 - x_1 \geq 0$

$x_1, x_2 \geq 0$

Soln:

First Consider the inequality constraints as equations

$2x_1 + x_2 = 3 \longrightarrow (1)$

$x_2 - x_1 = 0 \longrightarrow (2)$

And $x_1 = 0 \longrightarrow (3)$

$x_2 = 0 \longrightarrow (4)$

equ (1) => put $x_1 = 0$

$2x_1 + x_2 = 3$

$2(0) + x_2 = 3$

$x_2 = 3$

$(0, 3)$

$x_2 = 0 \Rightarrow 2x_1 + 0 = 3$

$x_1 = 3/2$

$(3/2, 0)$

equ ② $\Rightarrow x_2 - x_1 = 0$

Put $x_1 = 0 \Rightarrow x_2 - 0 = 0$

$x_2 = 0$

$(0,0)$

$x_2 = 0 \Rightarrow 0 - x_1 = 0$

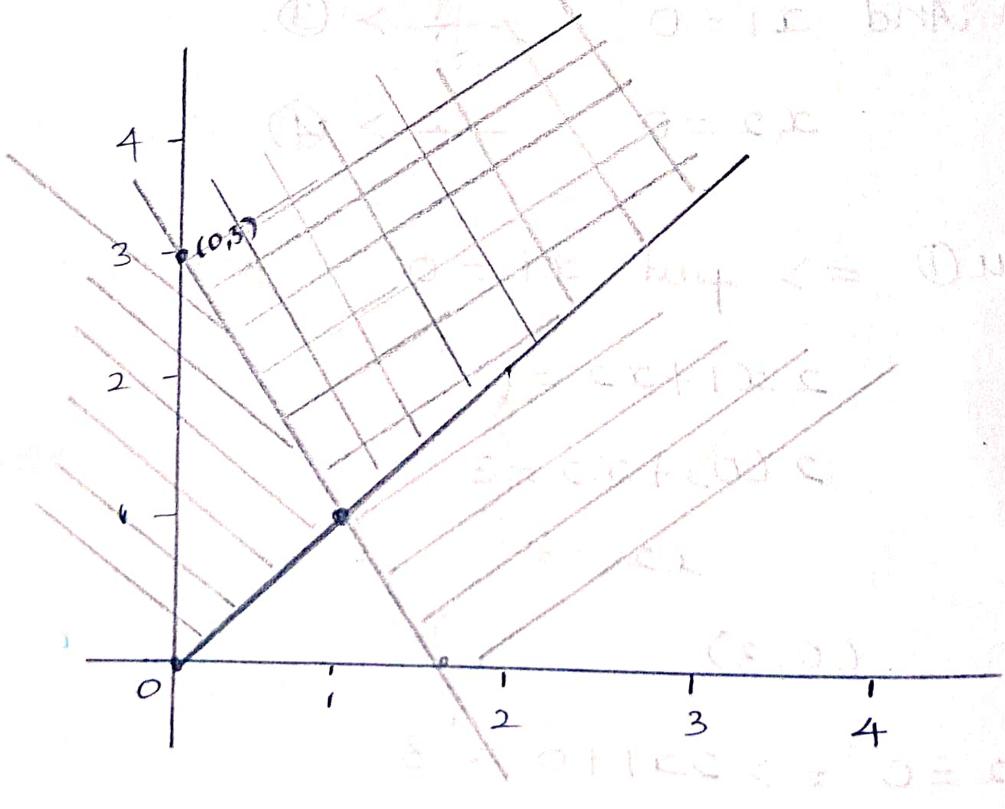
$x_1 = 0$

one value pair
(any pair)

$(0,0)$

$x_1 = x_2$

We take any value $(2,2)$



Two extreme point of the feasible region are A & B.

The feasible region is inbounded.

A point

$$2x_1 + x_2 = 3$$
$$x_1 \mp x_2 = 0$$
$$\overline{3x_1 \quad = 3}$$
$$x_1 = \frac{3}{3}$$
$$x_1 = 1$$

Put x_1 value in any equation

$$2(1) + x_2 = 3$$
$$x_2 = 3 - 2$$
$$\boxed{x_2 = 1}$$

A (1,1)

B point

(0,3)

$\therefore$ The vertices of solution are A(1,1), B(0,3).

Vertex	Value of $z = 6x_1 + x_2$
A (1,1)	$6(1) + 1 = 7$
B (0,3)	$6(0) + 3 = 3$

$Z(A) = 7$

$Z(B) = 3$

But there are points in this convex region for which Z will have much higher values. In fact, the maximum value of Z occurs at infinity. Hence the problem has unbounded solution.

4. Solve graphically the following LPP.

Maximize $\quad Z = x_1 + x_2$

subject constraint $\quad x_1 + x_2 \le 1$

$$-3x_1 + x_2 \ge 3$$

$$x_1, x_2 \ge 0$$

Soln:

$x_1 + x_2 = 1 \quad \longrightarrow (1)$

$-3x_1 + x_2 = 3 \quad \longrightarrow (2)$

Put $\underline{0}$

Equ ① $\Rightarrow x_1 + x_2 = 1$

Put $x_1 = 0$ $\quad 0 + x_2 = 1$

$$\boxed{x_2 = 1}$$

$(0, 1)$

Put $x_2 = 0 \Rightarrow x_1 + 0 = 1$

$$\boxed{x_1 = 1}$$

Equ ② $\quad -3x_1 + x_2 = 3$

Put $x_1 = 0$ $\quad -3(0) + x_2 = 3$

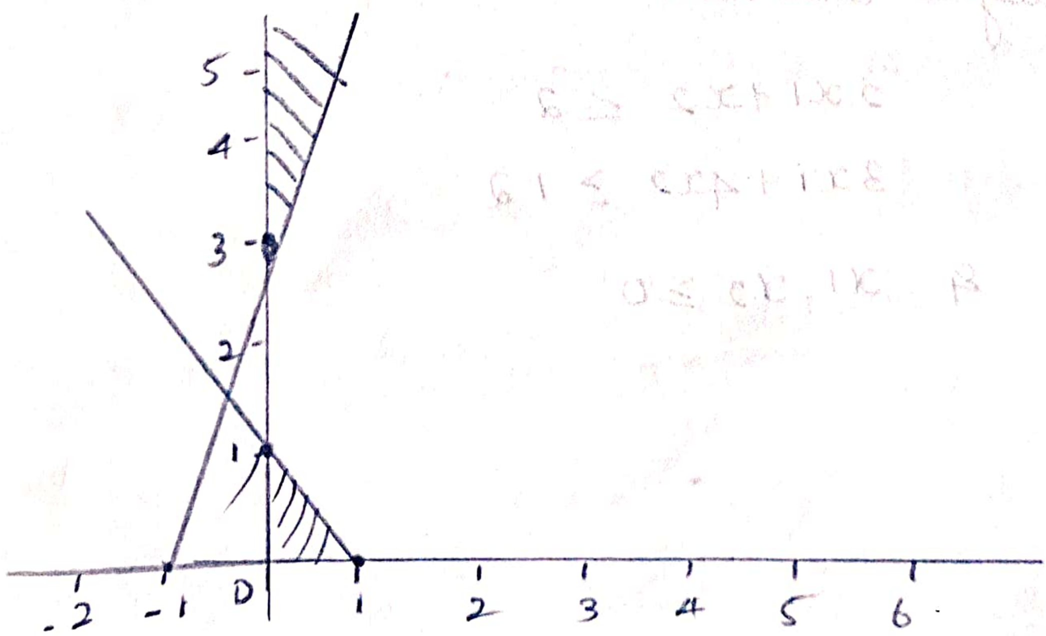
$$\boxed{x_2 = 3}$$

Put $x_2 = 0$ $\quad -3x_1 + 0 = 3$

$$-x_1 = \frac{3}{3}$$

$$\boxed{x_1 = -1}$$

$(-1, 0)$



Hence the Constraints are not, satisfied simultenously.

The given the LPP has no feasible region and hence we get an infeasible region.

5. Find graphical method maximum

$z = 10x_1 + 6x_2$

subject Constraint $5x_1 + 3x_2 \leq 30$

$3x_1 + 2x_2 \geq 18$

and x_1 & $x_2 \geq 0$

6. Find graphical Maximum $z = 3x_1 + 2x_1$

subject Constraint :

$2x_1 + x_2 \leq 2$

$3x_1 + 4x_2 \geq 12$

& $x_1, x_2 \geq 0$

Solution :

An n-tuple $(x_1, x_2, \dots, x_n)$ of real numbers which satisfies the constraints of a general L.P.P is called a solution to the general LPP.

Feasible solution :

Any solution to a general LPP which also satisfies the non-negative restrictions of the problem is called a feasible solution to the general LPP.

optimum solution :

Any feasible solution which optimizes (min (or) max) the objective function of a general LPP is called a optimum solution.

Slack and surplus Variables : $\leq + S1$

Slack :

Let the Constraints of a general

LPP be $\sum_{j=1}^{n} a_{ij}\ x_j \leq b_i \quad i = 1, 2$

Then the non-negative variables x_{n+i} which satisfy $\sum_{j=1}^{n} a_{ij} x_j + \underline{x_{n+i}} = b_i$ are called slack (method) variable.

Surplus: $\geq -S_1$

Let the constraints of a general LPP be $\sum_{j=1}^{a} a_{ij}\ x_j \geq b_i \quad i = K+1, K+2, \ldots$

Then the non-negative variable x_{n+i} which satisfy $\sum_{j=1}^{n} a_{ij}\ x_j - x_{n+i} = b_i$ are called surplus.

Simplex method:

1. Use simplex method to solve the LPP.

Maximum $z = 4x_1 + 10x_2$

subject to
$$2x_1 + x_2 \leq 50$$
$$2x_1 + 5x_2 \leq 100$$
$$2x_1 + 3x_2 \leq 90$$

and $x_1, x_2 \geq 0$

Soln :

(

Introducing the slack Variables S_1, S_2, S_3 the problem in standard form become

$$\text{Max } z = 4x_1 + 10x_2$$

sub to $2x_1 + x_2 + S_1 + 0S_2 + 0S_3 = 50$

$2x_1 + 5x_2 + 0S_1 + S_2 + 0S_3 = 100$

$2x_1 + 3x_2 + 0S_1 + 0S_2 + S_3 = 90$

and $x_1, x_2, S_1, S_2, S_3 \geq 0$

$\because$ There are 3 equations with 5 Variables, the initial basic feasible solution is obtained by equation $(5-3)=2$ Variable to zero.

$\therefore$ The initial basic feasible solution is $S_1 = 50$, $S_2 = 100$, $S_3 = 90$.

The initial simplex table:

CB	YB	XB	x_1 (4)	x_2 (10)	S_1 (0)	S_2 (0)	S_3 (0)	$\theta = \min \frac{x_B}{a_{ri}}$
0	S_1	50	2	1	1	0	0	$\frac{50}{1} = 50$
0	S_2	100	2	(5) pivot point	0	1	0	$\frac{100}{5} = 20^*$
0	S_3	90	2	3	0	0	1	$\frac{90}{3} = 30$
$z_j - c_j$		0	-4	-10	0	0	0	

Namma introduce panna variable

from z coefficient

Choose most negative value.

Here the net evaluation are calculated as $z_j - c_j = c_B a_j - c_j$ → formula

$z_1 - c_1 = c_B a_1 - c_1$ $(0\ 0\ 0)[2\ 2\ 2]^T - 4 = -4$

$z_2 - c_2 = c_B a_2 - c_2 = (0\ 0\ 0)[1\ 5\ 3]^T - 10 = -10$

$z_3 - c_3 = c_B a_3 - c_3 = (0\ 0\ 0)[1\ 0\ 0]^T - 0 = 0$

$z_4 - c_4 = c_B a_4 - c_4 = [0\ 0\ 0][0\ 1\ 0]^T - 0 = 0$

$z_5 - c_5 = c_B a_5 - c_5 = [0\ 0\ 0][0\ 0\ 1]^T - 0 = 0$

since there are some $(z_j - c_j) < 0$, the current basic feasible solution is not optimal.

To find the entering Variable:

Since $(z_2 - c_2) = -10$ is the most negative, the corresponding non-basic Variable x_2 enters the basis. The Column corresponding to this x_2 is called the key column or pivot column.

Find the leaving Variable:

Find the ratio $\theta = \min \left\{ \frac{x_{Bj}}{a_{ir}} \; a_{ir} > 0 \right\}$

$= \min \left\{ \frac{50}{1}, \frac{100}{5}, \frac{90}{3} \right\}$

$= \min \{ 50, 20, 30 \}$

$= 20$

Formula New pivot equ = old pivot equ ÷ pivot element

$= (100 \quad 2 \quad 5 \quad 0 \quad 1 \quad 0) \div 5$

$= (20 \quad \frac{2}{5} \quad 1 \quad 0 \quad \frac{1}{5} \quad 0)$

New s_1 equ = old s_1 equ − (Corresponding column co-efficient) ×

(New pivot equates)

$$= \quad 50 \quad 2 \quad 1 \quad 1 \quad 0 \quad 0$$

$$(-)\left(20 \quad \tfrac{2}{5} \quad 1 \quad 0 \quad \tfrac{1}{5} \quad 0\right) \times 1$$

$$30 \quad 8/5 \quad 0 \quad 1 \quad -\tfrac{1}{5} \quad 0$$

New S_3 equ $= 90 \quad 2 \quad 3 \quad 0 \quad 0 \quad 1$

$$(-)\left(20 \quad \tfrac{2}{5} \quad 1 \quad 0 \quad \tfrac{1}{5} \quad 0\right) \times 3$$

$$= \quad 90 \quad 2 \quad 3 \quad 0 \quad 0 \quad 1$$

$$(-) \quad 60 \quad 6/5 \quad 3 \quad 0 \quad 3/5 \quad 0$$

$$30 \quad 4/5 \quad 0 \quad 0 \quad -3/5 \quad 1$$

$(z_j - c_j) = 0 \quad -4 \quad -10 \quad 0 \quad 0 \quad 0 \quad -$

$$-\left(20 \quad \tfrac{2}{5} \quad 1 \quad 0 \quad \tfrac{1}{5} \quad 0\right) \times -10$$

$$= \quad 0 \quad -4 \quad -10 \quad 0 \quad 0 \quad 0$$

$$(-) \; -200 \quad -\tfrac{20}{5} \quad -10 \quad 0 \quad -\tfrac{10}{5} \quad 0$$

$$200 \quad 0 \quad 0 \quad 0 \quad \tfrac{10}{5} \; (2) \quad 0$$

First iteration :

	C_j		4	10	0	0	0
C_B	y_B	x_B	x_1	x_2	S_1	S_2	S_3
0	S_1	30	$8/5$	0	1	$-\frac{1}{5}$	0
10	x_2	20	$\frac{2}{5}$	1	0	$\frac{1}{5}$	0
0	S_3	30	$\frac{4}{5}$	0	0	$-\frac{3}{5}$	1
$z_j - C_j$		200	0	0	0	2	0

Since all $z_j - C_j \geq 0$ the current basic feasible solution is optimal.

$\therefore$ The optimal solution is

Max $z = 200$

$x_1 = 0$

$x_2 = 20$. (from x_B)

2) Find the non-negative values of x_1, x_2 & x_3 which maximize $z = 3x_1 + 2x_2 + 5x_3$

sub to $x_1 + 4x_2 \leq 420$

$3x_1 + 2x_3 \leq 460$

$x_1 + 2x_2 + x_3 \leq 430$

$x_1, x_2, x_3 \geq 0$

Soln:

Given the LPP, by introducing slack variable,

Max $z = 3x_1 + 2x_2 + 5x_3$

sub to

$x_1 + 4x_2 + 0x_3 + S_1 + 0S_2 + 0S_3 = 420$

$3x_1 + 0x_2 + 2x_3 + 0S_1 + S_2 + 0S_3 = 460$

$x_1 + 2x_2 + x_3 + 0S_1 + 0S_2 + S_3 = 430$

and $x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$

Since there are 3 equation with 6 variable, the initial basic feasible solution is obtained by equating $(6-3) = 3$ variable to zero.

$\therefore$ The initial basic feasible solution is $S1 = 420$, $S2 = 460$, $S3 = 430$

($x1 = x2 = x3 = 0$ non-basic)

The initial simple table.

C_j (3 2 5 0 0 0)

CB	YB	XB	$x1$	$x2$	$x3$	S1	S2	S3	θ
0	S1	420	1	4	0	1	0	0	$\frac{460}{2} = 230$
0	S2	460	3	0	(2)	0	1	0	
0	S3	430	1	2	1	0	0	1	$\frac{430}{1} = 430$
$z_j - c_j$		0	−3	−2	−5	0	0	0	

$z_1 - c_1 = C_B a_1 - c_1 = (0\ 0\ 0)[1\ 3\ 1]^T - 3 = 3$

$z_2 - c_2 = C_B a_2 - c_2 = (0\ 0\ 0)[4\ 0\ 2]^T - 2 = -2$

$z_3 - c_3 = C_B a_3 - c_3 = (0\ 0\ 0)[0\ 2\ 1]^T - 5 = -5$

$z_4 - c_4 = C_B a_4 - c_4 = [0\ 0\ 0)[1\ 0\ 0]^T - 0 = 0$

$z_5 - c_5 = C_B a_5 - c_5 = [0\ 0\ 0)[0\ 1\ 0]^T - 0 = 0$

$z_6 - c_6 = C_B a_6 - c_6 = [0\ 0\ 0)[0\ 0\ 1]^T - 0 = 0$

$(z_j - c_j) < 0$, the common current

basic feasible solution is not optimal.

$\because (Z_3 - C_3) = 5$ is the most negative, the Corresponding non-basic Variable x_3 enters into the basics.

The column corresponding to this x_3 is called the key column or pivot column. To find the leaving Variable.

$$\theta = \min \left\{ \frac{XB_i}{a_{ir}}, a_{ir} > 0 \right\}$$

$$= \min \left\{ \frac{420}{0}, \frac{460}{2}, \frac{430}{1} \right\}$$

$$= \min \{0, 230, 430\}$$

$$= 230.$$

New pivot equation = old pivot element ÷ pivot element.

$$= (460 \quad 3 \quad 0 \quad 2 \quad 0 \quad 1 \quad 0) \div 2$$

$$= (230 \quad \tfrac{3}{2} \quad 0 \quad 1 \quad 0 \quad \tfrac{1}{2} \quad 0)$$

New S_1 equ = old S_1 equ − (Corresponding Column Coefficient) × (new Pivot eqn)

$= (420 \quad 1 \quad 4 \quad 0 \quad 1 \quad 0 \quad 0) -$

$(230 \quad 3/2 \quad 0 \quad 1 \quad 0 \quad \frac{1}{2} \quad 0) \times 0$

$$\begin{array}{ccccccc} = 420 & 1 & 4 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 420 & 1 & 4 & 0 & 1 & 0 & 0 \end{array}$$

New S_3 equ = $430 \quad 1 \quad 2 \quad 1 \quad 0 \quad 0 \quad 1$

$$\begin{array}{ccccccc} & -(230 & 3/2 & 0 & 1 & 0 & \frac{1}{2} & 0) \times 1 \\ \hline & 200 & -\frac{1}{2} & 2 & 0 & 0 & -\frac{1}{2} & 1 \end{array}$$

New $(z_j - c_j)$ equ = $0 \quad -3 \quad -2 \quad -5 \quad 0 \quad 0 \quad 0$

$- (230 \quad \frac{3}{2} \quad 0 \quad 1 \quad 0 \quad -\frac{1}{2} \quad 0) \times -5$

$$\begin{array}{ccccccc} = 0 & -3 & -2 & -5 & 0 & 0 & 0 \\ +1150 & -\frac{15}{2} & 0 & -5 & 0 & -\frac{5}{2} & 0 \\ \hline 1150 & \frac{9}{2} & -2 & 0 & 0 & 5/2 & 0 \end{array}$$

First iteration :

CB	YB	XB	x_1	x_2	x_3	S1	S2	S3	θ
0	S1	420	1	4	0	1	0	0	$\frac{420}{4}=105$
5	x_3	230	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0	$\frac{230}{0}=0$
0	S3	200	$-\frac{1}{2}$	2	0	0	$-\frac{1}{2}$	1	$\frac{200}{2}=100$
z_j-c_j		1150	$\frac{9}{2}$	-2	0	0	$\frac{5}{2}$	0	

$(z_2-c_2)=-2$, the basic feasible solution is not optimal.

∴ Here the non-basic variable x_2 enters into the basic and the basic variable S_3 new pivot element.

New ~~point~~ Pivot eqn = old pivot eqn ÷ pivot element

$= (200 \quad -\frac{1}{2} \quad 2 \quad 0 \quad 0 \quad -\frac{1}{2} \quad 1) \div 2$

$= (100 \quad -\frac{1}{4} \quad 1 \quad 0 \quad 0 \quad -\frac{1}{4} \quad \frac{1}{2})$

New S_1 eqn $= (420 \quad 1 \quad 4 \quad 0 \quad 1 \quad 0 \quad 0) -$

$(100 \quad -1/4 \quad 1 \quad 0 \quad 0 \quad -\frac{1}{4} \quad \frac{1}{2}) \times 4$

$= \quad 420 \quad 1 \quad 4 \quad 0 \quad 1 \quad 0 \quad 0$

$- (400 \quad -1 \quad 4 \quad 0 \quad 0 \quad -1 \quad 2)$

$20 \quad 0 \quad 0 \quad 0 \quad 1 \quad +1 \quad 2$

Second Iteration :

CB	y_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3
0	S_1	20	0	0	0	1	1	-2
5	x_3	230	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0
2	x_2	100	$-\frac{1}{4}$	1	0	0	$-\frac{1}{4}$	$\frac{1}{2}$
$z_j - c_j$		1350	4	0	0	0	$-\frac{1}{4}$	$\frac{1}{2}$

$(z_j - c_j)$ eqn $= 1150 \quad \frac{9}{2} \quad -2 \quad 0 \quad 0 \quad \frac{5}{2} \quad 0$

$= - (100 \quad -\frac{1}{4} \quad 1 \quad 0 \quad 0 \quad -\frac{1}{4} \quad \frac{1}{2}) \times 2$

$$
\begin{array}{rcccccccc}
= & 1150 & 9/2 & -2 & 0 & 0 & \frac{5}{2} & 0 \\
-(& -200 & \frac{2}{4} & -2 & 0 & 0 & \frac{2}{4} & -\frac{2}{2} &) \\
\hline
 & 1350 & 8/2 & 0 & 0 & 0 & 2 & 1
\end{array}
$$

$(z_j - c_j) \geq 0$, the current basic feasible solution is optimal.

$\therefore$ Thus the optimal solution is

Max $z = 1350$, $x_1 = 0$, $x_2 = 100$, $x_3 = 230$.

1. using simplex method, Max $z = x_1 + 4x_2 + 5x_3$ subject to $3x_1 + 6x_2 + 3x_3 \leq 22$, $x_1 + 2x_2 + 3x_3 \leq 14$ and $3x_1 + 2x_2 \leq 14$.

Ans : Max $z = \frac{74}{3}$, $x_1 = 0$, $x_2 = 2$, $x_3 = \frac{10}{3}$

2. using simplex method Min $z = 8x_1 - 2x_2$ subject to $-4x_1 + 2x_2 \leq 1$, $5x_1 - 4x_2 \leq 3$ and $x_1, x_2 \geq 0$ [Hint : Min z type, we shall convert it in to a maximization type. So Max $(-z)$ = Max $z^* = -8x_1 + 2x_2$]

supply & demand equal
ah iruntha adhu TP.

unit - 2

Transportation Model

Mathematical formulation of TP:

$$\text{Min } z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\text{S.C} \quad \sum_{j=1}^{n} x_{ij} = a_i \qquad i = 1, 2, \dots m$$

$$\sum_{i=1}^{m} x_{ij} = b_j \qquad j = 1, 2, \dots n$$

and $x_{ij} \geq 0$ for all i and j

Note 1:

The two sets of Constraints will be Consistent of $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

where

$\sum_{i=1}^{m} a_i$ is total supply

$\sum_{j=1}^{n} b_j$ is total demand.

which is the necessary and sufficient Condition for a transportation problem to have a feasible solution.

Problem satisfying this Condition are called balanced transportation problem.

Note 2:

i) If $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$ then the transportation problem is called to unbalanced

ii) The unbalanced problem can be balanced by adding dummy supply (row) or an dummy demand (column) as the need arises.

Note 3:

If the number of positive allocation at any stage of feasible solution is less than the required number $(m+n-1)$, then the solution is said to be degenerate or non-degenerate

Note 4 :

The transportation table having positive allocation in a cell is called occupied cell otherwise called empty or unoccupied or non-occupied cells.

supply & demand la minimum Paakanum

NWCM:

↳ (North - West Cost Method)

9	8	5	25
6	8	4	35
7	6	9	40
30	25	45	

→ Supply

→ Demand

Soln:

$$\sum_{i=1}^{m} a_i = 30+25+45 = 100$$

$$\sum_{j=1}^{n} b_j = 25+35+40 = 100$$

$$\therefore \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} = 100$$

The solution is a balanced solution.

25 9	8	5	~~25~~ 0
6	8	4	35
7	6	9	40
30 5	25	45	

6 (5)	8	4	~~35~~ 30
7	6	9	40
~~5~~ 0	25	45	

→

8 (25)	4	~~30~~ 5
6	9	40
~~25~~ 0	45	

↓

4 (5)	~~5~~ 0
9 (40)	~~40~~
~~45~~ ~~40~~	

The no. of independent allocation $= m+n-1$

$= 3+3-1$

$= 6-1$

$= 5$

$\therefore$ The solution is non-degenerated basic feasible.

The initial transportation cost =

$25\times 9 + 5\times 6 + 25\times 8 + 5\times 4 + 40\times 9$

$= 225 + 30 + 200 + 20 + 360$

$= 225 + 250 + 360$

$= 835$

$\therefore$ T.P $= 835$

LCM: (Least-Cost method)

2	11	10	3	7	4
1	4	7	2	1	8
3	9	4	8	12	9
3	3	4	5	6	

$$\sum_{i=1}^{m} a_i = 3+3+4+5+6 = 21$$

$$\sum_{j=1}^{n} b_j = 4+8+9 = 21$$

$$\therefore \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = 21$$

$\therefore$ It is a balanced solution

It exists a feasible solution.

2	11	10	3	7	4
$\boxed{3}$ 1	4	7	2	1	~~8~~ 5
3	9	4	8	12	9
~~3~~ 0	3	4	5	6	

$\longrightarrow$

11	10	3	7	4
4	7	2	$\boxed{5}$ 1	~~5~~ 0
9	4	8	12	9
3	4	5	~~6~~ 1	

$\downarrow$

11	10	$\boxed{4}$ 3	7	~~4~~ 0
9	4	8	12	9
3	4	~~5~~ 1	1	

$\longleftarrow$

9	$\boxed{4}$ 4	8	12	~~9~~ 5
3	~~4~~ 0	1	1	

$\boxed{3}$ 9	$\boxed{1}$ 8	$\boxed{1}$ 12

5 4 ~~1~~ 0

~~3~~ 0 ~~1~~ 0 ~~1~~ 0

$\therefore$ Independent allocation $= m+n-1 = 3+5-1 = 7$

$\therefore$ It is an non-degenerated basic feasible solution.

Least Cost $= 3\times1 + 5\times1 + 4\times3 + 4\times4 + 3\times9 + 1\times8 + 1\times12$

$= 3+5+12+16+27+8+12$

L.C = 83

VAM:

1	2	6	7
0	4	2	12
3	1	5	11
10	10	10	

$$\sum_{i=1}^{m} a_i = 10+10+10 = 30$$

$$\sum_{j=1}^{n} b_j = 7+12+11 = 30$$

$$\therefore \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = 30$$

$\therefore$ It is a balanced solution.

1	2	6	7 (1)
0	4	$\boxed{10}$ 2	~~12~~ 2 (2)
3	1	5	11 (2)
10 (1)	10 (1)	~~10~~ 0 (3)	

1	2	7 (1)
~~$\boxed{2}$ 0~~	~~4~~	~~2~~ 0 (4)
3	1	11 (2)
~~10~~ 8 (1)	10 (1)	

1	2	7 (1)
3	$\boxed{10}$ 1	~~11~~ 1 (2)
8 (2)	~~10~~ 0 (1)	

$\boxed{7}$ 1	~~7~~
$\boxed{1}$ 3	~~1~~
~~8~~ ~~1~~	

∴ The no. of independent allocation $= m+n-1$
$= 3+3-1$
$= 5$

∴ It is a non-degenerated basic feasible solution

Transportation Cost $= 10\times2 + 2\times0 + 1\times10 + 7\times1 + 1\times3$
$= 20 + 10 + 7 + 3$

$T.C = 40$

Assignment problem:

i) Hungarian method (Balanced).

Consider the problem of assigning 5 job to 5 person. the assignment Cost are given as

$$\begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ F \end{array} & \begin{pmatrix} 8 & 4 & 2 & 6 & 1 \\ 0 & 9 & 5 & 5 & 4 \\ 3 & 8 & 9 & 2 & 6 \\ 4 & 3 & 1 & 0 & 3 \\ 9 & 5 & 8 & 9 & 5 \end{pmatrix} \end{array}$$

Soln:

The cost Matrix of the given assignment Problem is : step 1 → row reduce

$$\begin{pmatrix} 7 & 3 & 1 & 5 & 0 \\ 0 & 9 & 5 & 5 & 4 \\ 1 & 6 & 7 & 0 & 4 \\ 4 & 3 & 1 & 0 & 3 \\ 4 & 0 & 3 & 4 & 0 \end{pmatrix} \begin{array}{l} R_1 - 1 \\ \\ R_3 - 2 \\ \\ R_5 - 5 \end{array}$$

step 2 -> Column reducing:

$$
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
A & 7 & 3 & \cancel{0} & 5 & \boxed{0} \\
B & \boxed{0} & 9 & 4 & 5 & 4 \\
C & 1 & 6 & 6 & \boxed{0} & 4 \\
D & 4 & 3 & \boxed{0} & \cancel{0} & 3 \\
E & 4 & \boxed{0} & 2 & 4 & \cancel{0}
\end{array}
$$

(3-1.

The optimum assignment schedule:

A -> 5, B -> 1, C -> 4, D -> 3, E -> 2

$= 1+0+2+1+5 = 9$ units.

Finding optimum solution using LCM:

4	1	2	6	9	100
6	4	3	5	7	120
5	2	6	4	8	120
40	50	70	90	90	

$$\sum_{i=1}^{m} a_i = 340 \ (40+50+70+90+90)$$

$$\sum_{j=1}^{n} b_j = 100+120+120 = 340$$

∴ It is a balanced solution.

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = 340$$

4	$\boxed{50}$ 1	2	6	9	~~100~~ 50
6	4	3	5	7	120
5	2	6	4	8	120
40	~~50~~ 0	70	90	90	

→

~~4~~	~~2~~	~~6~~	~~9~~	~~500~~
6	3	5	7	120
5	6	4	8	120
40	~~70~~ 20	90	90	

↓

6	$\boxed{20}$ 3	5	7	~~120~~ 100
5	6	4	8	120
40	~~20~~ 0	90	90	

←

6	5	7	100
5	$\boxed{90}$ 4	8	~~120~~ 30
40	~~90~~ 0	90	

↓

6	7	100
~~$\boxed{30}$ 5~~	~~8~~	~~300~~
~~40~~ 10	90	

$\boxed{10}$ ~~6~~	$\boxed{90}$ 7	~~100~~ 90
~~10~~ 0	90.	

∴ It is a non-degenerated basic feasible soln.

The no. of independent allocation = $m+n-1 = 3+5-1 = 7$

The transportation Cost $= 50\times1 + 50\times2 + 20\times3 + 90\times4 +$
$30\times5 + 10\times6 + 90\times7$
$= 1410$

optimum solution:

	1	2	3	4	5
$U_1 = -1$	4 (-1)	$\boxed{50}$ 1	$\boxed{50}$ 2	6 (2)	9 (3)
$U_2 = 0$	$\boxed{10}$ 6	4 (2)	$\boxed{20}$ 3	5 (0)	$\boxed{90}$ 7
$U_3 = -1$	$\boxed{30}$ 5	2 (1)	6 (4)	$\boxed{90}$ 4	8 (2)
	$V_1 = 6$	$V_2 = 2$	$V_3 = 3$	$V_4 = 5$	$V_5 = 7$

$C_{12} = U_1 + V_2 \Rightarrow 1 = U_1 + V_2 \Rightarrow 1 = -1 + V_2 \Rightarrow V_2 = 1+1 = 2$ $\boxed{V_2 = 2}$

$C_{13} = U_1 + V_3 \Rightarrow 2 = U_1 + 3 \Rightarrow U_1 = 2-3 = -1$ $\therefore \boxed{U_1 = -1}$

$C_{21} = U_2 + V_1 \Rightarrow 6 = 0 + V_1$ $\therefore \boxed{V_1 = 6}$

$C_{23} = U_2 + V_3 \Rightarrow 3 = 0 + V_3$ $\boxed{V_3 = 3}$

$C_{25} = U_2 + V_5 \Rightarrow 7 = 0 + V_5$ $\boxed{V_5 = 7}$

$C_{31} = U_3 + V_1 \Rightarrow 5 = U_3 + 6 \Rightarrow U_3 = 5-6 = -1$ $\boxed{U_3 = -1}$

$C_{34} = U_3 + V_4 \Rightarrow 4 = -1 + V_4$ $\boxed{V_4 = 5}$

$d_{11} = C_{11} - (U_1 + V_1) = 4 - (-1+6) \Rightarrow 4 - (5) \Rightarrow -1$ $\therefore d_{11} = -1$

$d_{14} = 6 - (-1+5) = 6 - 4 \Rightarrow 2$

$d_{15} = 9 - (-1+7) = 9 - 6 = 3$

$d_{22} = 4 - (0+2) = 4 - 2 = 2$

$d_{24} = 5 - (0+5) = 0$

$d_{32} = 2 - (-1+2) = 2 - (1) = 1$

$d_{33} = 6 - (-1+3) = 6 - 2 = 4$

$d_{35} = 8 - (-1+7) = 8 - 6 = 2$

Since $d_{11} = -1 > 0$ ∴ The current solution is not optimal.

	$v_1 = 6$	$v_2 = 2$	$v_3 = 3$	$v_4 = 5$	$v_5 = 7$
$u_1 = -1$	4 (-1) θ	[50] 1	[50] 2 $-\theta$	6 (2)	9 (3)
$u_2 = 0$	[10] 6 $-\theta$	4 (2)	[20] 3 θ	5 (0)	[90] 7
$u_3 = -1$	[30] 5	2 (1)	6 (4)	[90] 4	8 (2)

$\theta + 10$

$\theta - 10$

(Travelling way must be an occupied cell)

	$v_1 = 4$	$v_2 = 1$	$v_3 = 2$	$v_4 = 3$	$v_5 = 5$
$u_1 = 0$	[10] 4	[50] 1	[40] 2	6 (2)	9 (3)
$u_2 = 2$	[0] 6	4 (2)	[30] 3	5 (0)	[90] 7
$u_3 = -1$	[30] 5	2 (1)	6 (4)	[90] 4	8 (2)

$C_{11} = U_1 + V_1 \Rightarrow 4 = 0 + V_1 \quad \boxed{V_1 = 4}$

$C_{12} = U_1 + V_2 \Rightarrow 1 = 0 + V_2 \quad \boxed{V_2 = 1}$

$C_{13} = U_1 + V_3 \Rightarrow 2 = 0 + V_3 \quad \boxed{V_3 = 2}$

$C_{21} = U_2 + V_1 \Rightarrow 6 = U_2 + 4 \Rightarrow U_2 = 6 - 4 \quad \boxed{U_2 = 2}$

$C_{23} = U_2 + V_3 \Rightarrow$

$C_{25} = U_2 + V_5 \Rightarrow 7 = 2 + V_5 \Rightarrow V_5 = 7 - 2 = 5 \quad \boxed{V_5 = 5}$

$C_{31} = U_3 + V_1 \Rightarrow 5 = U_3 + 4 \Rightarrow U_3 = 5 - 4 \Rightarrow \boxed{U_3 = 1}$

$C_{34} = U_3 + V_4 \Rightarrow 4 = 1 + V_4 \Rightarrow V_4 = 4 - 1 \Rightarrow \boxed{V_4 = 3}$

$C_{ij} - (U_i + V_j)$

$d_{14} = \cancel{V_3 + V_4} \Rightarrow 6 - (0+3) \Rightarrow 6 - 3 = 3$

$d_{15} = \cancel{V_4 + V_5} \Rightarrow 9 - (0+5) \Rightarrow 9 - 5 = 4$

$d_{22} = 4 - (2+1) \Rightarrow 4 - 3 = 1$

$d_{24} = 5 - (2+3) \Rightarrow 5 - 5 = 0$

$d_{32} = 2 - (1+1) \Rightarrow 2 - 2 = 0$

$d_{33} = 6 - (1+2) \Rightarrow 6 - 3 = 3$

$d_{35} = 8 - (1+5) \Rightarrow 8-6 = 2$

[10] 4	[50] 1	[40] 2	6 (3)	9 (4)
[0] 6	4 (1)	[30] 3	5 (0)	[90] 7
[30] 5	2 (0)	6 (3)	[90] 4	8 (2)

The transportation Cost $= 10\times4 + 50\times1 + 40\times2 + 30\times3 + 90\times7 + 5\times30 + 90\times4$

$= 40 + 50 + 80 + 90 + 630 + 150 + 360$

$= 260 + 630 + 150 + 360$

T.C $= 1400$

Finding optimal solution using YAM.

11	20	7	8	0	50
21	16	20	12	0	40
8	12	18	9	0	70
30	25	35	40	30	

11	20	7	8	0	50	(7)
21	16	20	12	[30] 0	~~40~~ 10	(12)
8	12	18	9	0	70	(8)
30	25	35	40	~~30~~ 0		
(3)	(4)	(11)	(1)	(0)		

11	20	[35] 7	8	15 ~~50~~ (1)
21	16	20	12	10 (4)
8	12	18	9	70 (1)
30 (3)	25 (4)	~~35~~ (11)	40 (1)	

$\longrightarrow$

11	20	8	15 (3)
21	16	12	~~100~~ (4)
8	12	9	70 (1)
30 (3)	25 (4)	~~40~~ 30 (1)	

$\downarrow$

11	20	8	15 (3)
8	[25] 12	9	~~70~~ 45 (1)
30 (3)	~~25~~ (8)	30 (1)	

11	[15] 8	~~15~~ (3)
8	9	45 (1)
30 (3)	~~30~~ 15 (1)	

[30] 8	[15] 9	~~45~~
30	~~15~~	

Transportation Cost $= 30\times0 + 35\times7 + 10\times12 + 25\times12 +$
$15\times8 + 30\times8 + 15\times9$
$= 0 + 245 + 120 + 300 + 120 + 240 + 135$
$= 1,160$

11	20	[35] 7	[15] 8	0	$U1 = -1$
21	16	20	[10] 12	[30] 0	$U2 = 3$
[30] 8	[25] 12	18	[15] 9	0	$U3 = 0$
$V1 = 8$	$V2 = 12$	$V3 = 8$	$V4 = 9$	$V5 = -3$	

$C_{13} = U1 + V3 \Rightarrow 7 = -1 + V3 \quad \therefore \boxed{V3 = 8}$

$C_{14} = U_1 + V4 \Rightarrow 8 = U1 + 9 \quad \therefore U_1 = 8 - 9 \quad \boxed{U_1 = -1}$

$C_{24} = U_2 + V_4 \Rightarrow 12 = U2 + 9 \Rightarrow U2 = 12 - 9 = 3$

$C_{25} = U_2 + V_5 \Rightarrow 0 = 3 + V5 \quad \boxed{V5 = -3}$

$C_{31} = U_3 + V_1 \Rightarrow 8 = 0 + V_1 \quad \boxed{\therefore V_1 = 8}$

$C_{32} = u_3 + v_2 \Rightarrow 12 = 0 + v_2$ $\boxed{\therefore v_2 = 12}$

$C_{34} = u_3 + v_4 \Rightarrow 9 = 0 + v_4$ $\boxed{\therefore v_4 = 9}$

UNIT-3 PERT Critical Path method

Project scheduling by pert / CPM :

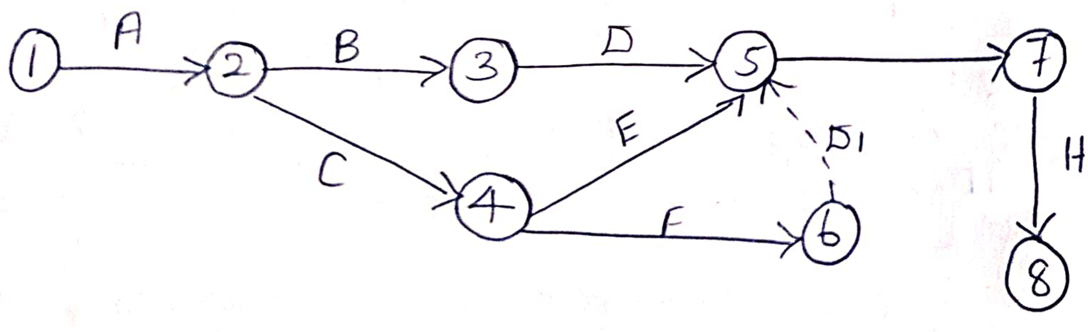
Program evaluation

i) Planning
ii) scheduling
iii) Control
iv) Activity
V) Predecessor
Vi) successor
Vii) dummy activity

1.

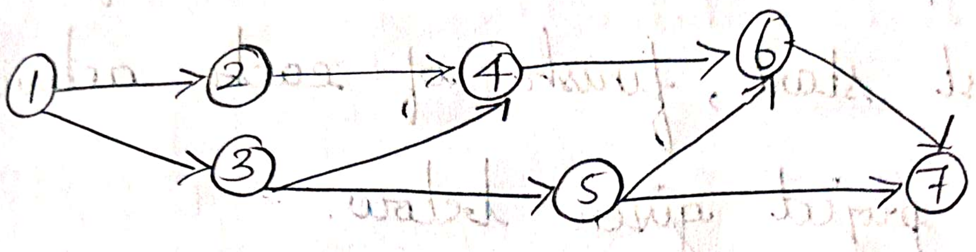
Activity	A	B	C	D	E	F	G	H
Predecessor	-	A	A	B	C	C	D,E,F	G

Soln:



2. Draw the event - oriented network for the following data.

Event no	1	2	3	4	5	6	7
Predecessor	-	1	1	2, 3	3	4, 5	5, 6

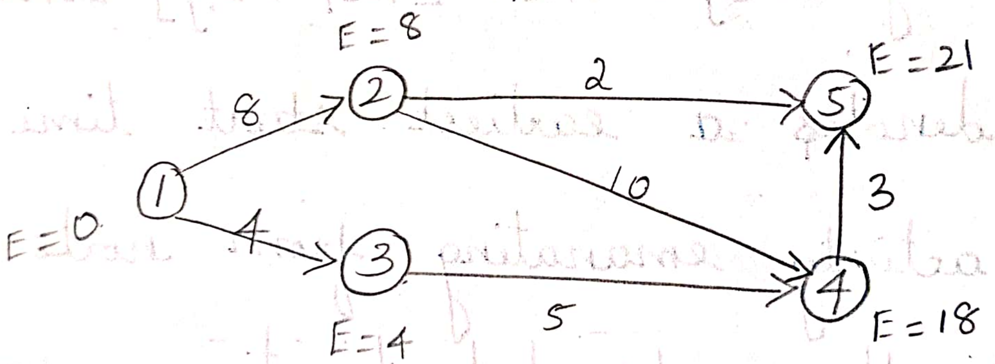


Formula for earliest - start time of an activity i-j in a project network is given by $ES_j = Max [ES_i + t_{ij}]$ where ES_j denotes a earliest start time of all the activity emanating from node i and t_{ij} is the estimated duration of the activity i-j.

Latest - start time of all the activities emanating from the event i of the activity i-j $LS_i = Min [LS_j - t_{ij}]$ for all define i-j activities where t_{ij} is the estimated duration of the activity i-j.

1. Compute the earliest start, finish and latest start, finish of each activity of the project given below.

Activity	1-2	1-3	2-4	2-5	3-4	4-5
Duration	8	4	10	2	5	3



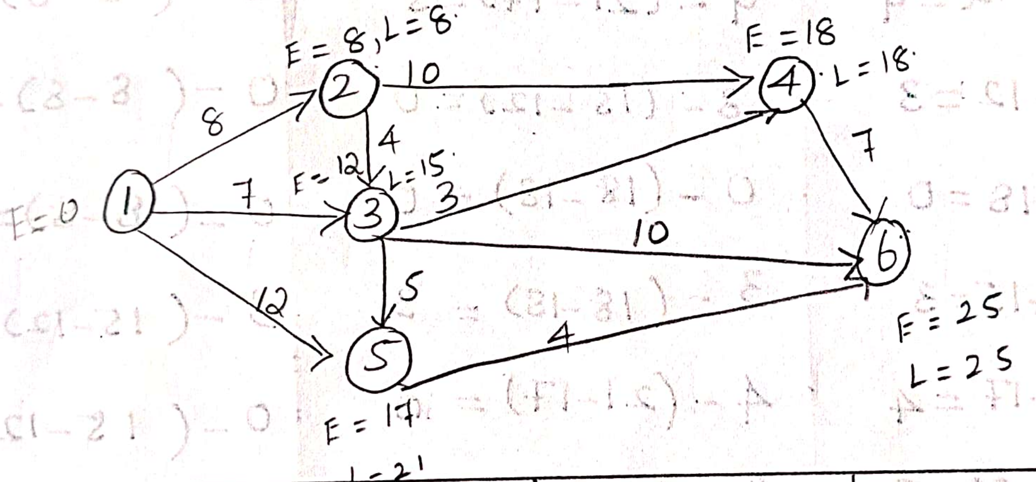
(Take Max value for c.p)

ACT	DURATION	EARLIEST SHORT (ES)	EARLIEST FINISH (EF) $(ES+t_{ij})$	LATEST START $(Lf-t_{ij})$	LATEST FINISH (LF)
1-2	8	0	8+0=8	8-8=0	8
1-3	4	0	4+0=4	13-4=9	13
2-4	10	8	10+8=18	18-10=18	18
2-5	2	8	2+8=10	21-2=19	21
3-4	5	4	5+4=9	18-5=13	18

4-5	3	18	3+18=21	21-3=18	21

2. Calculate the total float, free float, independent float for the project whose activities are given below.

ACTIVITY	1-2	1-3	1-5	2-3	2-4	3-4	3-5	3-6	4-6	5-6
DURATION	8	7	12	4	10	3	5	10	7	4



ACT	DURATION	EARLIEST START (ES)	EARLIEST FINISH (ES+tij)	LATEST START(Lf-tij)	LATEST FINISH
1-2	8	0	8+0=8	8-8=0	8
1-3	7	0	7+0=7	15-7=8	15
1-5	12	0	12+0=12	21-12=9	21
2-3	4	8	4+8=12	15-4=11	15
2-4	10	8	10+8=18	18-10=8	18
3-4	3	12	12+3=15	18-3=15	18

3-5	5	12	12+5 = 17	21 - 5 = 16	21
3-6	10	12	10+12 = 22	25-10 = 15	25
4-6	7	18	18+7 = 25	25-7 = 18	25
5-6	4	17	17+4 = 21	25-4 = 21	25

T.F = (LF-EF)	F.F = TF-(L-E) event j	I.F = F.F - (L-E) event i
8-8 = 0	0-(8-8) = 0	0-(0-0) = 0
15-7 = 8	8-(15-12) = 5	5-(0-0) = 5
21-12 = 9	9-(21-17) = 5	5-(0-0) = 5
15-12 = 3	3-(15-12) = 0	0-(8-8) = 0
18-18 = 0	0-(18-18) = 0	0-(8-8) = 0
18-15 = 3	3-(18-18) = 3	3-(15-12) = 0
21-17 = 4	4-(21-17) = 0	0-(15-12) = -
25-22 = 3	3-(25-25) = 3	3-(15-12) = 0
25-25 = 0	0-(25-25) = 0	0-(18-18) = 0
25-21 = 4	4-(25-25) = 4	4-(21-17) = 0

Critical point (path) = 1 → 2 → 4 → 6

= 8 + 10 + 7

= 25

∴ Critical path = 25

Nodes may be numbered using the rule given below :

(Ford and Fulkerson's Rule)

1. Number the start node which has no predecessor activity, as 1.

2. Delete all the activities emanating from this node 1.

3. Number all the resulting start nodes without any predecessor as 2, 3, ...

4. Delete all the activities originating from the start nodes 2, 3, ... in step 3.

5. Number all the resulting new start nodes without any predecessor next to the last number used in step (3).

6. Repeat the process until the terminal node without any successor activity is reached and number this terminal node suitably.

10.7 Basic differences between PERT and CPM

PERT

1. PERT was developed in a brand new R and D Project it had to consider and deal with the uncertainties associated with such projects. Thus the project duration is regarded as a random variable and therefore probabilities are calculated so as to characterise it.

2. Emphasis is given to important stages of completion of task rather than the activities required to be performed to reach a particular event or task in the analysis of network. i.e., PERT network is essentially an event – oriented network.

3. PERT is usually used for projects in which time estimates are uncertain. Example : R & D activities which are usually non-repetitive.

4. PERT helps in identifying critical areas in a project so that suitable necessary adjustments may be made to meet the scheduled completion date of the project.

CPM

1. CPM was developed for conventional projects like construction project which consists of well known routine tasks whose resource requirement and duration were known with certainty.

2. CPM is suited to establish a trade off for optimum balancing between schedule time and cost of the project.

3. CPM is used for projects involving well known activities of repetitive in nature.

However the distinction between PERT and CPM is mostly historical.

$\boxed{\text{Example 1}}$ **: Construct the network for the project whose activities and the three time estimates of these activities (in weeks) are given below. Compute**

(a) Expected duration of each activity

(b) Expected variance of each activity

(c) Expected variance of the project length

Most likely time estimate : (t_m or m) is the duration of any activity when sometimes things go on very well, sometimes things go on very bad while doing the project.

Two main assumptions made in PERT calculations are

(i) The activity durations are independent. i.e., the time required to complete an activity will have no bearing on the completion times of any other activity of the project.

(ii) The activity durations follow β – distribution.

β distribution is a probability distribution with density function $k(t-a)^{\alpha}\ (b-t)^{\beta}$ with mean $t_e = \frac{1}{3}\left[2t_m + \frac{1}{2}(t_o - t_p)\right]$ and the standard deviation $\sigma_t = \frac{t_p - t_o}{6}$

PERT Procedure

(1) Draw the project net work

(2) Compute the expected duration of each activity $t_e = \frac{t_o + 4t_m + t_p}{6}$ Mean

(3) Compute the expected variance $\sigma^2 = \left(\frac{t_p - t_0}{6}\right)^2$ of each activity.

(4) Compute the earliest start, earliest finish, latest start, latest finish and total float of each activity.

(5) Determine the critical path and identify critical activities.

(6) Compute the expected variance of the Project length (also called the variance of the critical path) σ_c^2 which is the sum of the variances of all the critical activities.

(7) Compute the expected standard deviation of the project length σ_c and calculate the standard normal deviate $\frac{T_S - T_E}{\sigma_c}$ where

T_S = Specified or Scheduled time to complete the project

T_E = Normal expected project duration

σ_c = Expected standard deviation of the project length.

(8) Using (7) one can estimate the probability of completing the project within a specified time, using the normal curve (Area) tables.

Note : (2), (3) are valid because of assumption (ii). (6) is valid because of assumption (i).

15.6 Programme Evaluation Review Technique : (PERT)

This technique, unlike CPM, takes into account the uncertainty of project durations into account.

PERT calculations depend upon the following three time estimates.

Optimistic (least) time estimate : **(t_0 or a)** is the duration of any activity when everything goes on very well during the project. i.e., labourers are available and come in time, machines are working properly, money is available whenever needed, there is no scarcity of raw material needed etc.

Pessimistic (greatest) time estimate : **(t_p or b)** is the duration of any activity when almost every thing goes against our will and a lot of difficulties is faced while doing a project.

There are three other types of floats for an activity, namely, Free float, Independent float and interference (interfering) float.

Free Float of an activity (F.F.) is that ***portion of the total float*** which can be used for rescheduling that activity without affecting the succeeding activity. It can be calculated as follows :

Free float of an activity $i-j$ = Total float of $i-j-$ (L – E) of the event j

= Total float of $i-j$ – Slack of the head event j

= Total float of I - J – Slack of the head event j

where L = Latest occurrence

E = Earliest occurrence

Obviously Free Float ≤ Total float for any activity.

Independent float (I.F) of an activity is the amount of time by which the activity can be rescheduled without affecting the preceding or succeeding activities of that activity.

Independent float of an activity $i-j$ = Free float of $i-j-$ (L –E) of event i.

= Free float of $i-j-$ Slack of the tail event i.

Clearly,

Independent float ≤ Free float for any activity

Thus I.F ≤ F.F ≤ T.F.

Interfering Float or Interference Float of an activity $i-j$ is nothing but the slack of the head event j.

Obviously,

Interfering Float of $i-j$ =Total Float of $i-j-$ Free Float of $i-j$

10.5 Floats

Total float of an activity (T.F) is defined as the ***difference*** between the ***latest finish*** and the ***earliest finish of the activity*** or the difference between the ***latest start*** and the ***earliest start*** of the activity.

$$\text{Total float of an activity } i-j = (LF)_{ij} - (EF)_{ij}$$

$$\text{or} = (LS)_{ij} - (ES)_{ij}.$$

Total float of an activity is the amount of time by which that particular activity may be delayed without affecting the duration of the project. If the total float is positive then it may indicate that the resources for the activity are more than adequate. If the total float of an activity is zero it may indicate that the resources are just adequate for that activity. If the total float is negative, it may indicate that the resources for that activity are inadequate.

Note : (L – E) of an event of $i-j$ is called the ***slack*** of the event j.

Find the minimum value of the function $f(x,y) = x^2+y^2$

$\alpha = 0.1$:

steps:

1. Initialize:

starting point (x_0, y_0)

Let's say $\boxed{(x_0, y_0) = (1,1)}$

2. Calculate the gradient:

The gradient of $f(x,y)$ is given by:

$$\nabla f(x,y) = (2x, 2y)$$

starting point $(1,1)$, the gradient is $\boxed{\nabla f(1,1) = (2,2)}$

3. Determine the $\underbrace{\text{search direction}}_{(-)\text{ of the gradient}}$:

(–) of the gradient.

$$-\nabla f(1,1) = -(2,2) = (-2,-2)$$

4. Update the point

step size α

$\alpha = 0.1$

Updated point

$$(x_1, y_1) = (x_0, y_0) - \alpha \nabla f(x_0, y_0)$$

$$(x_1, y_1) = (1,1) - 0.1\ (2,2)$$

$$(x_1, y_1) = (1,1) - (0.1(2), 0.1(2))$$

$$(1,1) - (0.2, 0.2)$$

$$= (1-0.2)\ (1-0.2)$$

$$= (1.0-0.2) - (1.0-0.2)$$

$$\boxed{(x_1, y_1) = (0.8, 0.8)}$$

5. Repeat: step 2-4 until a desired level of accuracy is achieved.

Iteration 2:

1. Calculate the gradient:

$f(x, y) = x^2 + y^2$ at the point $(0.8, 0.8)$

$\nabla f(0.8, 0.8) = (2(0.8), 2(0.8)) = (1.6, 1.6)$

2. Determine the $\underbrace{\text{search direction}}_{(-)\text{ gradient}}$:

$-(1.6, 1.6) = (-1.6, -1.6)$

3. Update the point.

$(x_2, y_2) = \underbrace{(x_1, y_1)}_{\alpha\text{ - Step Size}} - \alpha \nabla f(x_1, y_1)$ → previous point

put value are:

$(x_2, y_2) = (0.8, 0.8) - 0.1(-1.6, -1.6)$

$= (0.8, 0.8) - (0.1(-1.6) + (0.1)(-1.6))$

$= (0.8, 0.8) + (0.16, 0.16)$

$$\begin{array}{r} 0.8 \\ 0.16 \\ \hline 0.96 \end{array}$$

$\boxed{\text{Updated point 2 is } (0.96, 0.96)}$.

Iteration 3:

1. Calculate the gradient:

$f(x, y) = x^2 + y^2$ at the point $(0.96, 0.96)$

$\nabla f(0.96, 0.96) = (2(0.96), 2(0.96)) = (1.92, 1.92)$

2. Determine $\underbrace{\text{search direction}}_{(-)\text{ gradient}}$:

$-(1.92, 1.92) = (-1.92, -1.92)$.

3- Update the point:

$$(x_3, y_3) = \underbrace{(x_2, y_2)}_{\text{Previous point}} - \alpha \nabla f(x_2, y_2)$$

α - step size

$$\begin{array}{r} 0.96 \\ 82 \\ \hline 14 \end{array} \qquad \begin{array}{r} -1.92 \\ 0.1 \\ \hline -0.22 \end{array}$$

put values are:

$$(x_3, y_3) = (0.96, 0.96) - 0.1(-1.92, -1.92)$$
$$= (0.96, 0.96) - (0.1(-1.92), 0.1(-1.92))$$
$$= (0.96, 0.96) - (0.192, -0.192)$$

$$\boxed{(x_3, y_3) = (1.152, 1.152)}$$

Keep repeating until the gradient becomes sufficiently small or the function value converges to a minimum.

$f(x, y) = x^2 + y^2$ at minimum at $(0, 0)$

gradually converge.

Gradient Method

→ It is also called as steepest Ascent method. The idea is to generate successive points in the direction of the gradient of the function

→ Termination of gradient method occurs at the point where the gradient vector becomes null

① The maximizing function is

$f(x)$ $f(x_1, x_2) = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$ →

absolute optimum occurs at $(x_1^* , x_2^*) = (1/3, 4/3)$

$= 0.3333, 1.3$

Sol:-

initial point $x^0 = (1,1)$

$$\nabla f(x) = (4 - 4x_1 - 2x_2, 6 - 2x_1 - 4x_2) \rightarrow ②$$

First Iteration (Substitute $x^0 = (1,1)$ in equation 2)

$$\Delta f(x^0) = (-2, 0)$$

$$\boxed{x^1 = x^0 + r \nabla f(x^0)}$$

$$= (1,1) + r(-2,0)$$

$$= (1,1) + (-2r, 0)$$

$$x^1 = (\overset{x_1}{1-2r}, \overset{x_2}{1}) \longrightarrow ③$$

$$\boxed{h(r) = f(x^1)}$$

$h(r) = 4(1-2r) + 6(1) - 2(1-2r)^2 + 2(1-2r)(1) - 2($

$h(r) = 4 - 8r^2 + 4r$

$$\frac{-r}{16} + \frac{1}{64} = 0$$

$$\frac{-r}{16} = \frac{-1}{64}$$

$$\boxed{r = 1/4}$$

Substitute $1/4$ in eqn x^v

$$x^v = \left(\frac{3-r}{8}, \frac{21}{16} \right)$$

$$= \left(\frac{3-(1/4)}{8}, \frac{21}{16} \right)$$

$$\boxed{x^v = \left(11/32, 21/16 \right)}$$

Sixth iteration

Substitute $x^v = (11/32, 21/16)$ in eqn ②

$\nabla f(x^v) = (0, 1/16)$

Because $\nabla f(x^v) \approx 0$, the process can be terminated at this Point. The approximate maximum Point is given by

$$x^v = (0.34375, 1.3125)$$

The exact optimum is $x^* = (0.3333, 1.3333)$

$f)^2$

$$= \frac{3}{2} + \frac{30}{4} + \frac{6\gamma}{4} - \frac{18}{64} - \frac{60}{32} + \frac{12\gamma}{32} - \frac{50}{16} - \frac{2\gamma^2}{16} - \frac{[illegible]}{4}$$

$$= -\frac{1}{8}\gamma^2 + \frac{3\gamma}{2} + \frac{3}{8}\gamma - \frac{5\gamma}{4} - \frac{60}{32} + \frac{30}{4} - \frac{50}{16} - \frac{18}{64}$$

$$= -\frac{1}{8}\gamma^2 + \frac{24\gamma + 6\gamma - 20\gamma}{16} - \frac{149}{32}$$

$$\boxed{h(\gamma) = -\frac{1}{8}\gamma^2 - \frac{10}{16}\gamma + \frac{149}{32}}$$

$h'(\gamma) = 0$

$$-\frac{1}{4}\gamma + \frac{1}{16} = 0$$

$$\boxed{\gamma = 1/4}$$

Sub $\gamma = 1/4$ in eqn x^{iv}.

$$x^{iv} = (3/8,\ 5/4 + \gamma/4)$$

$$= \left(3/8,\ 5/4 + \frac{(1/4)}{4}\right)$$

$$\boxed{x^{iv} = (3/8,\ 21/16)}$$

Fifth iteration.

Sub $x^{iv} = (3/8, 21/16)$ in eqn ②

$$\nabla f(x^{iv}) = (-1/8, 0)$$

$$\boxed{x^{v} = x^{iv} + \gamma \nabla f(x^{iv})}$$

$$= (3/8, 21/16) + \gamma(-1/8, 0)$$

$$= (3/8, 21/16) + (-\gamma/8, 0)$$

$$\boxed{x^{v} = (3/8 - \gamma/8,\ 21/16)}$$

Let $h(\gamma) = f(x^{v})$

$$= 4\left(\frac{3-\gamma}{8}\right) + 6\left(\frac{21}{16}\right) - 2\left(\frac{3-\gamma}{8}\right)^2 - 2\left(\frac{3-\gamma}{8}\right)\left(\frac{21}{16}\right) - 2\left(\frac{21}{16}\right)^2$$

$$\boxed{h(\gamma) = \frac{-2\gamma^2}{64} + \frac{1}{64}\gamma + \frac{597}{38}}$$

Let $h'(\gamma) = 0$

$$-\frac{4\gamma}{64} + \frac{1}{64} = 0$$

$$= 4\left(\frac{1-\gamma}{2}\right) + \frac{15}{2} - 2\left(\frac{1-\gamma}{2}\right)^2 - \frac{2}{2}(1-\gamma)\left(\frac{5}{4}\right) - \left(\frac{25}{8}\right)$$

$$= 2(1-\gamma) + \frac{15}{2} - 2\left(\frac{1}{4} + \frac{\gamma^2}{4} - \frac{\gamma}{2}\right) - \left(\frac{5}{4} - \frac{5\gamma}{4}\right) - \frac{25}{8}$$

$$= 2 - 2\gamma + \frac{15}{2} - \frac{1}{2} - \frac{\gamma^2}{2} + \gamma - \frac{5}{4} + \frac{5\gamma}{4} - \frac{25}{8}$$

$$= \frac{-\gamma^2}{2} - \gamma + \frac{5\gamma}{4} + \frac{37}{8}$$

$$\boxed{h(\gamma) = -\frac{\gamma^2}{2} + \frac{1}{4}\gamma + \frac{37}{8}}$$

Let $h'(\gamma) = 0$

$$\frac{-2\gamma}{2} + \frac{1}{4} = 0$$

$$-\gamma + \frac{1}{4} = 0$$

$$\boxed{\gamma = \frac{1}{4}}$$

Substitute $\gamma = 1/4$ in eqn x'''

$$x''' = \left(\frac{1}{2} - \frac{1/4}{2}, \frac{5}{4}\right)$$

$$x''' = \left(\frac{1}{2} - \frac{1}{8}, \frac{5}{4}\right)$$

$$\boxed{x''' = \left(\frac{3}{8}, \frac{5}{4}\right)}$$

Fourth Iteration

Sub $x''' = (3/8, 5/4)$ in eqn ②

$$\nabla f(x''') = f(0, 1/4)$$

$$x^{iv} = x''' + \gamma \nabla f(x''')$$

$$= (3/8, 5/4) + \gamma(0, 1/4)$$

$$= (3/8, 5/4) + (0, \gamma/4)$$

$$\boxed{x^{iv} = \left(3/8, 5/4 + \gamma/4\right)}$$

$$h(\gamma) = f(x^{iv})$$

$$= 4(3/8) + 6(5/4 + \gamma/4) - 2(3/8)^2 - 2(3/8)(5/4 + \gamma/4) - 2(5/4 + \gamma/4)^2$$

$$= \frac{3}{2} + \frac{30}{4} + \frac{6\gamma}{4} - \frac{18}{64} - \left(\frac{6}{8}\right)\left(\frac{10}{4} + \frac{2\gamma}{4}\right) - 2\left(\frac{25}{16} + \frac{\gamma^2}{16} + \frac{10\gamma}{4}\right)$$

$$\boxed{h(\gamma) = \gamma - 2\gamma^2 + 9/2}$$

Let

$$h'(\gamma) = 0$$

$$1 - 4\gamma = 0$$

$$\boxed{\gamma = 1/4}$$

Substitute $\gamma = 1/4$ in x''

$$x'' = (1/2, 1 + 1/4)$$

$$\boxed{x'' = (1/2, 5/4)}$$

Third iteration

Sub $x'' = (1/2, 5/4)$ in eqn ②

$$\nabla f(x'') = (-1/2, 0)$$

$$\boxed{x''' = x'' + \gamma \nabla f(x'')}$$

$$= (1/2, 5/4) + \gamma(-1/2, 0)$$

$$= (1/2, 5/4) + (-\gamma/2, 0)$$

$$\boxed{x''' = (1/2 - \gamma/2, 5/4)}$$

$$h(\gamma) = f(x''')$$

$$= 4(1/2 - \gamma/2) + 6(5/4) - 2(1/2 - \gamma/2)^2 - 2(1/2 - \gamma/2)(5/4) - 2(5/4)^2$$

$$= 4\left(\frac{1-\gamma}{2}\right) + 15/2 - 2\left(\frac{1-\gamma}{2}\right)^2 - 2\left(\frac{1-\gamma}{2}\right)\left(\frac{5}{4}\right) - 2\left(25/16\right)$$

Let $h'(\gamma) = 0$

$$-16\gamma + 4 = 0$$

$$-16\gamma = -4$$

$$\boxed{\gamma = 1/4}$$

Substitute $\gamma = 1/4$ in eqn (3)

$$x' = (1 - 2\gamma, 1)$$

$$= (1 - 2(1/4), 1)$$

$$x' = (1/2, 1)$$

Second iteration

Sub $x' = (1/2, 1)$ in eqn (1)

$$\nabla f(x') = (0, 1)$$

$$\boxed{x'' = x' + \gamma \nabla f(x')}$$

$$= (1/2, 1) + \gamma (0, 1)$$

$$= (1/2, 1) + (0, \gamma)$$

$$x'' = (\overset{x_1}{1/2}, \overset{x_2}{1 + \gamma})$$

$$\boxed{h(\gamma) = f(x'')}$$

$$= 4(1/2) + 6(1+\gamma) - 2(1/2)^2 - 2(1/2)(1+\gamma) - 2(1+\gamma)^2$$

$$= 2 + 6 + 6\gamma - 1/2 - 1 - \gamma - 2 - 2\gamma^2 - 4\gamma$$

$$= 6 + \gamma - 2\gamma^2 - 1/2 - 1$$

$$= 5 + \gamma - 2\gamma^2 - 1/2$$

$$= \gamma - 2\gamma^2 + 9/2$$

Find the maximum of the func $f(x) = -x^2 + 2x + 11$ in the range $-2 \le x \le 2$ using PSO method. Use 4 particles $(N=4)$ with the initial positions $x_1 = -1.5$, $x_2 = 0.0$, $x_3 = 0.5$, & $x_4 = 1.25$. Show the detailed computations for iteration 1. Assume $w = 0.8$ & $C_1 = C_2 = 2.05$.

→ Given:

Step 1: $f(x) = -x^2 + 2x + 11$ range : $-2 \le x \le 2$.

Step 2: 4 Particles $(N=4)$ with initial positions.

$x_1 = -1.5$

$x_2 = 0.0$

$x_3 = 0.5$

$x_4 = 1.25$

1
-1.5

3
0.5

2
0.0

4
1.25

Weight inertia $(W) = 0.8$.

Acceleration Coefficients $C_1 = C_2 = 2.05$.

Step (3):

Evaluate performance using objective func

$$\boxed{f(x) = -x^2 + 2x + 11} \quad -2 \leq x \leq 2.$$

$\Rightarrow f(x_1) = -(1.5)^2 + 2(-1.5) + 11$

$= 5.75$

Fitness calculation $\Rightarrow f(x_2) = -(0.0)^2 + 2(0.0) + 11$

$= 11$

$\Rightarrow f(x_3) = -(0.5)^2 + 2(0.5) + 11$

$= 11.75$

$\Rightarrow f(x_4) = -(1.25)^2 + 2(1.25) + 11$

$= 11.93$

Step (4). Update personal best position.

$p\,best\,(x_1) = -1.5$

$p\,best\,(x_2) = 0.0$

$p\,best\,(x_3) = 0.5$

$p\,best\,(x_4) = 1.25$ maximum

step 5: Update Global Best position

$G(best) = (x_4) = 1.25.$

step 6: Update Velocity & Position for each particle.

iteration: 1

For particle (x_1)

Formula:

New Velocity:

$$V_i^{t+1} = \omega * (V_i^t) + rand * C_1 * (P_{best} - X_i^t) + rand * C_2 * (G_{best} - X_i^t)$$

New Position:

$$X_i^{t+1} = X_i^t + V_i^{t+1}$$

∴ Consider initial Velocity $\boxed{V_i = 0}$ & assume random value from 0 to 1

$$V_1 = 0.8 * (0) + (0.3)(2.05)(-1.5 - (-1.5)) + (0.6)(2.05)(1.25 - (-1.5))$$

random value within range 0 to 1

$$= 0 + (0.3)(2.05)(0) + (0.6)(2.05)(2.75)$$

$$= 0 + 0 + 3.3825$$

$$\boxed{V_1 = 3.3825}$$

$$X(1) = -1.5 + 3.3825$$

$$\boxed{X(1) = 1.88}$$

$\Rightarrow V_2 = 0.8(0) + 0.2(2.05)(0.0-0.0) + 0.6(2.05)(1.25-0.0)$

$= 1.537$

$x(2) = 0.0 + 1.537$

$= \underline{1.537}$

$\Rightarrow V_3 = 0.8(0) + 0.4(2.05)(0.5-0.5) + 0.1(2.05)(1.25-0.5)$

$= 0.1537$

$x(3) = 0.5 + 0.1537$

$= \underline{0.6537}$

$\Rightarrow V_4 = 0.8(0) + 0.9(2.05)(1.25-1.25) + 0.2(2.05)(1.25-1.25)$

$= 0$

$x(4) = 1.25 + 0$

$= \underline{1.25}$

1
1.88

2
1.53

3
0.65

4
1.25

step 7 : Evaluate new solution performance using objective function.

$$f(x) = -x^2 + 2x + 11 \qquad -2 \le x \le 2$$

$$f(x_1) = -(1.88)^2 + 2(1.88) + 11$$

$$= 11.23$$

$$f(x_2) = -(1.53)^2 + 2(1.53) + 11$$

$$= 11.72$$

$$f(x_3) = -(0.65)^2 + 2(0.65) + 11$$

$$= 11.87$$

$$f(x_4) = -(1.25)^2 + 2\,1(25) + 11$$

$$= 11.93$$

step 8 : Update pBest positions.

pbest (x_1) = 1.88

pbest (x_2) = 1.53

pbest (x_3) = 0.65

pbest (x_4) = 1.25

step 9 : Update Gbest positions

Gbest $= (x_4) = 1.25$ 　 ① 1.88 　 ② 1.53 　 ③ 0.65 　 ④ 1.25

Check fitness (old – Gbest) > fitness (New – Gbest)

(11.93) > (11.23)
(11.93) > (11.72)
(11.93) > (11.87)
(11.93) > (11.93)

old	New
$f(x_1) = 5.78$	$f(x_1) = 11.23$
$f(x_2) = 11$	$f(x_2) = 11.72$
$f(x_3) = 11.75$	$f(x_3) = 11.87$
$f(x_4) = 11.93$	$f(x_4) = 11.93$

step 10 :

Check stopping Condition :

Check (current iteration <= Max iteration)

iteration 2

$$\Rightarrow V_1 = 0.8(3.3825) + 0.3(2.05)(1.88 - 1.88) + 0.6(2.05)(1.88 - 1.25)$$

$$= 3.489.$$

$$X_1 = 1.88 + 3.489$$

$$= 5.36$$

$$\Rightarrow V_2 = 0.8(1.537) + 0.2(2.05)(1.53 - 1.53) + 0.6(2.05)(1.25 - 1.53)$$

$$= 0.8852$$

$$x_2 = 1.53 + 0.8852$$

$$= 2.41$$

$$\Rightarrow V_3 = 0.8(1.1537) + 0.4(2.05)(0.65 - 0.65) + 0.1(2.05)(1.25 - 0.65)$$

$$= 1.045$$

$$= \cancel{0.2066}$$

$$x_4 = 1.25 + 1.045$$

$$= 1.695$$

$$\Rightarrow V_4 = 0.8(0.2583) + 0.9(2.05)(1.25 - 1.25) + 0.2(2.05)(1.25 - 1.25)$$

$$= 0.2066$$

$$x_4 = 1.25 - 0.2066$$

PSO Initialization:-

- PSO is initialized by group of random Particles (Each particle is solution).

- Each particle searches for the optimum value by updating generation [iteration].

- In each iteration every particle is updated [by following 2 BEST Values]

1. First Best one is the best Solution [FITNESS].

2. Second best is tracked by Particle Swarm optimizer.

- After finding 2 Best values.

- Particle updates its velocity and position.

- Particle can update their Position by:

$$x_i^{t+1} = x_i^t + v_i^t * t$$

- Velocity of particle is given by:

$$v_{k+1}^i = wv_k^i + c_1 r_1 (xBest_i^t - x_i^t) + c_2 r_2 (gBest_i^t - x_i^t)$$

- xBest = best particle position

- g Best = best group position.

- parameters w [inertia weight];
- c_1, c_2 = two positive constants
- r_1 and r_2 = two random parameters within [0,1]

- For Position Update:

Present = old position + velocity (V)

V = Particle velocity

Present = Current Position.

Step 1: Initialization

Initialize parameters

Initialize population

- Initialize position (x_i) Random for each particle.
- Initialize velocity (v_i) Random for each particle.

Step 2: Evaluate Fitness $f(x_i^t)$

Calculate fitness value for each particle.

If Fitness value is better than Best Fitness value (gBest).

Than

Set New value as new (gBest)

Choose particle with Best Fitness value as gBest.

Step 3:

For each particle calculate velocity and position.

- Calculate particle position by:

$$x_i^{t+1} = x_i^t + v_i^t * t$$

- Calculate Velocity by:

$$v_{k+1}^i = wv_k^i + c_1 r_1 (xBest_i^t - x_i^t) + c_2 r_2 (gBest_i^t - x_i^t)$$

Step 4:

Evaluate Fitness $f(x_i^t)$

Find Current Best [gBest]

Step 5:

Update $t = t+1$

Step 6:

Output gBest & x_i^t