Welcome

MATHEMATICAL ECONOMICS

Topic : Uses of derivatives

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Contents :

1. Derivatives :

- The rate of change of quantity y with respect to quantity x
- Varying rate of change of a function with respect to an independent variable

2. Uses :

- Derivatives are widely applied in economic modeling and theory, because they are an invaluable tool used to measure the effects and rates of change in economic variables and find the maximum and minimum values of functions.
- Uses of derivatives were explained in upcoming slides

I. <u>Increasing and decreasing functions :</u>

The first derivative measures the rate of change and slope of a function, a positive first derivative indicates the function is increasing, a negative first derivative indicates it is decreasing

f' (a) > 0 => increasing function

f' (a) < 0 => decreasing function

II. Concavity and convexity :

A positive second derivative denotes the function is convex,

A negative second derivative denotes the function is concave

$f''(a) > 0$	convex
$f''(a) < 0$	concave

III. Relative extrema :

maximum or minimum

the first derivative of the function must '=' to '0'

to distinguish mathematically between a relative maximum and minimum, the second derivative test is used. If,

$f'(a) = 0 \quad f''(a) > 0 \quad \text{minimum}$

$f'(a) = 0 \quad f''(a) < 0 \quad \text{maximum}$

IV. Marginal concepts :

marginal cost and marginal revenue can each be expressed mathematically as derivatives,

$$MC = \frac{dTQ}{dQ}$$

if $TC = Q^2 + 7Q + 23$, then $MC = dTC/dQ$

$$MC = 2Q + 7$$

V. Optimizing economic functions :

helps in the form of techniques to maximize profits and levels of physical output and productivity as well as to minimize cost.

Eg : Maximize profits π for a firm, given total revenue $R = 400Q - 33Q^2$ and total cost $C = 2Q^2 - 3Q^2 + 400Q + 5000$, assuming $Q > 0$

Sol:

$$\pi = 400Q - 33Q^2 - (2Q^2 - 3Q^2 + 400Q + 5000)$$

$$= -2Q^2 - 30Q^2 + 3600Q - 5000$$

$$\pi' = -6Q^2 - 60Q + 3600$$

$$= -6(Q^2 + 10Q - 600)$$

$$= -6(Q+30)(Q-20)$$

$Q = -30$ $\quad\quad\quad\quad$ $Q = 20$

$$\pi'' = -12Q - 60$$

$$\pi''(20) = -12(20) - 60 = -300 < 0$$

Profit is maximised at Q=20

$$\pi(20) = -2(20)^3 - 30(20)^2 + 3600(20) - 5000$$

$$= 39000$$

VI. Relationship among total, marginal and average concepts :

the relationship between the total, average, and marginal products of an input can easily sketched by using derivatives,

Eg:

$$TP = 90K^2 - k^3$$

Sol:

$$TP' = 180K - 3k^2$$

$$= 3k(60-K)$$

K=0   K=60

$$TP'' = 180-6K$$

$$= -6<0$$

$$Ap_k = TP/K$$

$$= 90K-K^2$$

$$MP = 180-6K$$

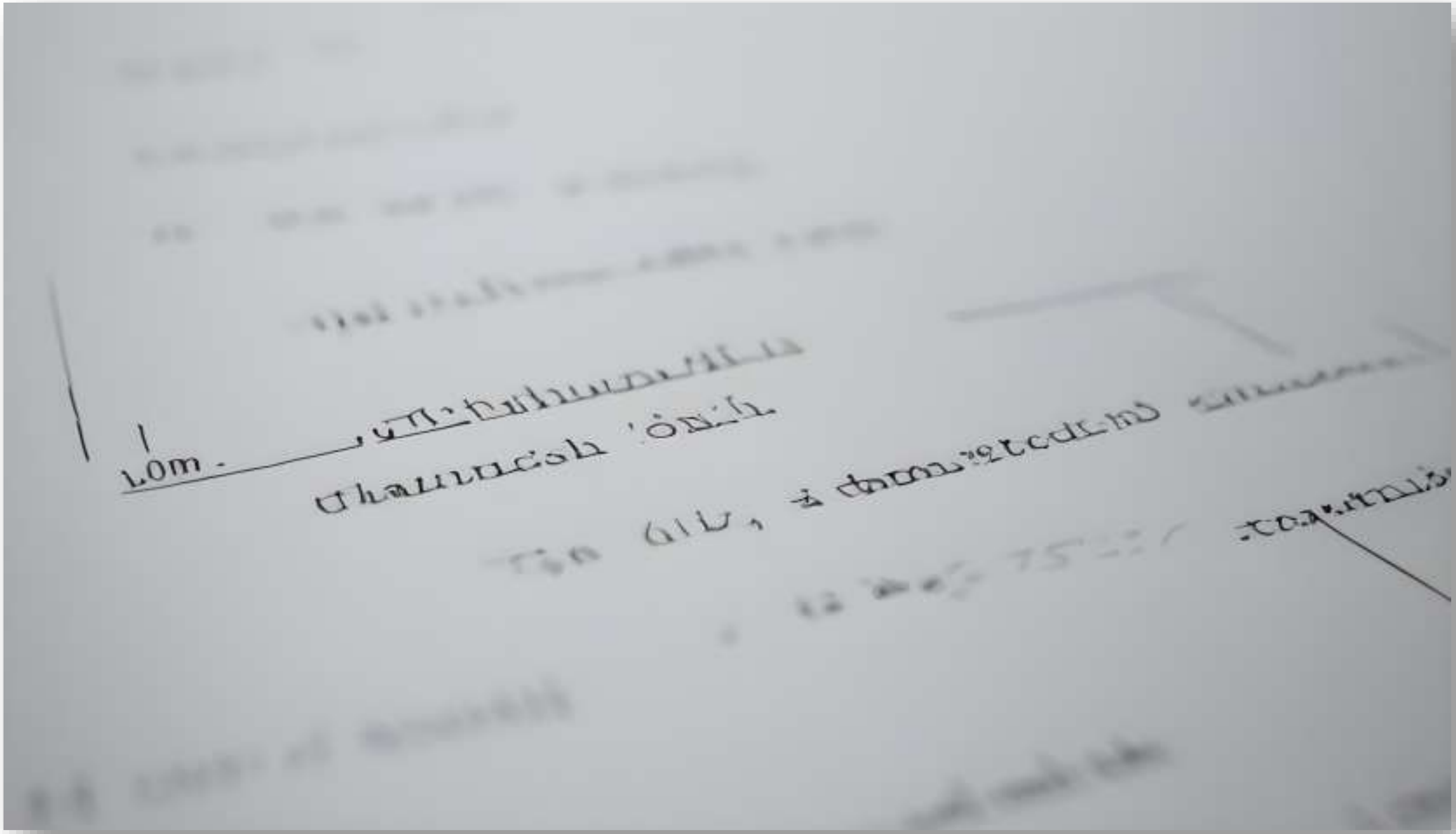
3. Conclusion:

With the help of the derivatives, we can find the optimum points of economic functions, if any. For example, the use of derivatives is helpful to compute the level of output at which the total revenue is the highest, the profit is the highest and (or) the lowest, marginal costs and average costs are the smallest, etc.

References:

- Introduction to Mathematical Economics, Edward T. Dowling, 2011
- https://economics.uwo.ca/math/resources/derivatives/7-derivative-uses-in-economics/content/#:~:text=With%20the%20help%20of%20the,costs%20are%20the%20smallest%2C%20etc.

Thank you



Any doubt

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Mathematical Economics

Topic : Properties of Determinants

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Content :

→ Properties of Determinants :

- Property of interchange
- Sign property
- Zero property
- Multiplication property
- Sum property
- Property of Invariance
- Triangular property

What Are the Properties of Determinants?

The features of determinants aid in quickly calculating the value of a determinant with the fewest steps and calculations possible. The following are the 7 most important properties of determinants.

1. Property of Interchange:

When the rows or columns of a determinant are swapped, the determinant’s value remains unchanged.
Det(A) = Det(A’)

As a result of this property, if the rows and columns of the matrix are swapped, the matrix is transposed, and the determinant value and the determinant of the transposition are both equal.

$$A = \begin{vmatrix} 4 & 5 \\ 3 & 2 \end{vmatrix}$$

$$A' = \begin{vmatrix} 4 & 3 \\ 5 & 2 \end{vmatrix}$$

2. Sign Property:

If any two rows or columns are swapped, the sign of the determinant's value

$$A = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$

$R1 \rightleftarrows R3$

$$B = \begin{bmatrix} 1 & 5 & -7 \\ 6 & 0 & 4 \\ 2 & -3 & 5 \end{bmatrix}$$

If the row or column is swapped once, the determinant's value changes the sign. To obtain matrix B, the first row of matrix A has been swapped with the third row, and we have Det(A) = -Det(A) (B). If the determinant's value is D and the rows or columns are swapped n times, the new determinant's value is (-1)nD.

3. Zero Property:

If the elements in any two rows or columns are the same, the determinant’s value is zero.

$$A = \begin{bmatrix} 1 & 5 & 7 \\ 1 & 5 & 7 \\ 2 & -3 & 5 \end{bmatrix}$$

The items in the first and second rows are identical in this case. As a result, the determinant’s value is zero.

Det(A) = 0

4. Multiplication Property:

If each of the elements of a given row or column is k times the earlier value of the determinant, the deciding value becomes k times the earlier value of the determinant. If each element of a given row or column is multiplied by a constant k, the determining va[...]comes [...]s the earlier value of the determinant.

$$A = \begin{bmatrix} 1 & 5 & 7 \\ 6 & 0 & 4 \\ 2 & -3 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 5 & 7 \\ 24 & 0 & 16 \\ 2 & -3 & 5 \end{bmatrix}$$

Det(B) = k× Det(A)

The second row of A is multiplied with a constant k. Here, k = 4.

The second row’s elements are multiplied by a constant k, and the determinant value is multiplied by the same constant. This characteristic aids in the extraction of a common factor from each determinant row or column. In addition, the value of the determinant is 0 if the corresponding elements of any two rows or columns are equal.

5. Sum Property:

The determinant can be expressed as a sum of two or more determinants if a few items of a row or column are expressed as a sum of words.

6. Property of Invariance:

When each element of a determinant’s row and column is multiplied by the equimultiples of the elements of another determinant’s row or column, the determinant’s value remains unchanged. This can be stated as a formula as follows:

$$R_i \to R_i + \alpha R_j + \beta R_k \text{ or } C_i \to C_i + \alpha C_j + \beta C_k$$

7. Triangular Property:

The value of the determinant is equal to the product of the components of the diagonal of the matrix if the elements above and below [illegible]l are both equal to zero.

$$\begin{vmatrix} a1 & a2 & a3 \\ 0 & b1 & b3 \\ 0 & 0 & c3 \end{vmatrix}$$

$$= \begin{vmatrix} a1 & 0 & 0 \\ a2 & b1 & 0 \\ a3 & b2 & c3 \end{vmatrix}$$

= a1.b2.c3


Thank you